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A Framework to Support Teacher Noticing of Students' Mathematical Thinking in Technology-mediated Environments

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A framework to support teacher noticing of students' mathematical thinking in technology-mediated environments

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Abstract
The practice of teacher noticing students' mathematical thinking often includes three interrelated components: attending to students' strategies, interpreting students' understandings, and deciding how to respond on the basis of students' understanding. This practice gains complexity in technology-mediated environments (i.e., using technology-enhanced math tasks) because it requires attending to and interpreting students' engagement with technology. Current frameworks implicitly assume the practice includes noticing the ways students use tools (including technology tools) in their work, but do not explicitly highlight the role of the tool. While research has shown that using these frameworks supports preservice secondary mathematics teachers (PSTs) developing noticing practices, it has also shown that PSTs largely overlook students' technology engagement when they are working on technology-enhanced tasks (*Journal for Research in Mathematics Education*, 2010; 41(2):169–202). In this article, we describe our adaptation of Jacobs et al.'s framework for teacher noticing student mathematical thinking to include a focus on making students' technology-tool engagement explicit when noticing in technology-mediated environments, the Noticing in Technology-Mediated Environments (NITE) framework. We describe the theoretical foundations of the framework, provide a video case example, and then illustrate how the framework can be used by mathematics teacher educators to support PSTs' noticing when students are working in technology-mediated environments.

KEYWORDS
professional development, teacher education, teachers and teaching, technology/calculators

1 | INTRODUCTION

Current frameworks for teacher noticing of students' mathematical thinking (e.g., Jacobs et al., 2010; Leatham et al., 2015) implicitly assume the practice includes noticing the ways students use tools (including technology tools) in their work, but they do not explicitly highlight the importance of doing so. In fact, researchers such as Jacobs et al. (2010), Wilson et al. (2011), Chandler (2017), and Lovett et al. (2019a, 2019b) have posited that noticing student thinking when tools are present is difficult and often results in teachers overlooking ways that the tool(s) informs the students' thinking. This becomes especially important and powerful when students are working with technology tools (i.e., in technology-mediated environments).

Imagine a preservice secondary mathematics teacher (PST) watching a video clip of a student working on a technology-enhanced task in which students are provided a dynamic dotplot and asked to drag the data points so that the mean and the median are the same (Figure 1). The goal of the task is to build an understanding of the mean as a balance point. In the video, the student is seen dragging one point at a time, first selecting the point just to the left of the mean and furthest above the axis and moving it to the right toward the mean and seeing both the mean and median increase and move closer together. Then the student selects the point that is now just to the left of the mean and drags it to the far right. This action resulted in the mean over taking the median. The student says "whoa that's too much." Then the student moves the point they had just dragged to the far right and drags it back toward the mean until they are close together. At this point the student goes back and forth between the two points they had previously dragged and works to adjust them each a little bit at a time, with one on each side of the mean, until the mean and median are both 3.8 (Figure 1).

To an experienced viewer, it is clear that the student is expressing much of their thinking through their interactions with the technology tool. In fact, the student hardly said anything at all. However, the way in which the student initially dragged the points and then subsequently moved back and forth between the two points, one on each side of the mean, to adjust until the mean and median were as close as they could get them to matching, suggests that they have a sense that the two points are balancing each other out. However, when a PST was asked to attend to and interpret the student's thinking present within this video clip the PST responded:

> The student moved the data points and found a way to organize the points so that the mean and median are both 3.8. The student understands that you can make the mean and median the same if you are careful, but there is no evidence that the student understands the mean as a balance point.

The PST did note that the student dragged (i.e., moved) the data points and was successful in finding a solution to the challenge that was posed, but did not consider the intention with which the student dragged the points to be indicative of the student's thinking or their understanding of the mean. This type of response was typical when we first began asking PSTs to notice students' mathematical thinking in technology-mediated environments.

Walkoe et al. (2017) note that technology-mediated environments not only have the potential to support students' mathematical thinking, but also teacher noticing of students' mathematical thinking. In our example above, interacting with the dynamic dotplot supported the students' developing understanding of mean and median, including what each measure of center represents, and how each reacts to both big and small changes in the data set. Furthermore, being able to watch the student engage with the technology provided us (the mathematics teacher educators) insight into the student's thinking about how changes in data affect the mean and median. In fact, Walkoe et al. (2017) wonder "how can teachers learn to look for key student thinking practices, [...], through the lens of technology-mediated student work? (p. 67)." This question is a critical undertaking for mathematics teacher educators in order for PSTs to fully understand student thinking in technology-mediated

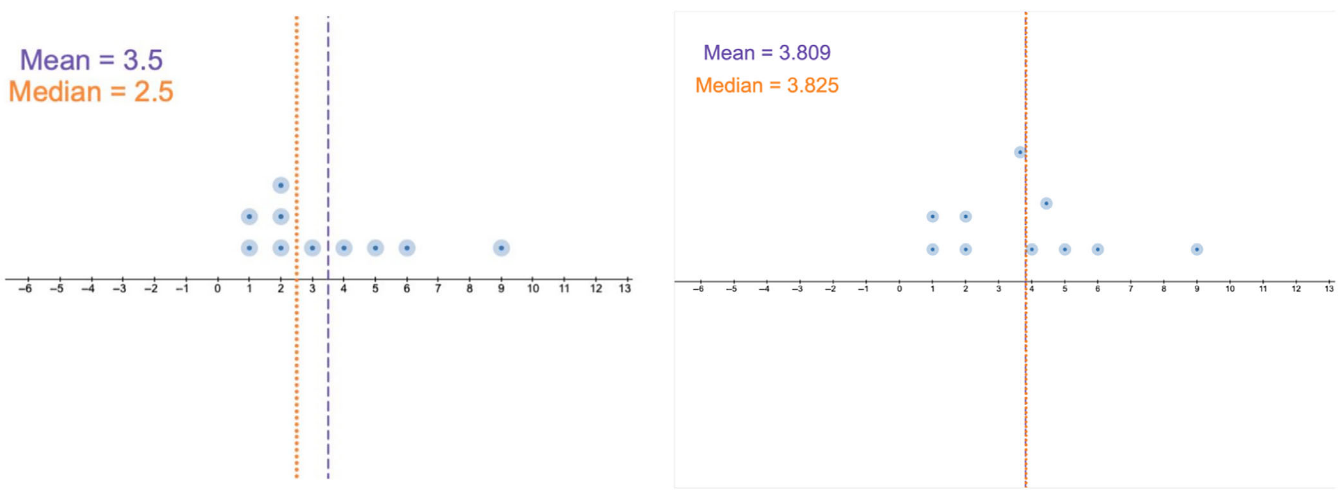


FIGURE 1 The mean balance point task starting position (left) and final student response (right).

environments. It is imperative that we find ways to support PSTs' noticing of all external representations of student thinking—including those expressed through the use of technology tools. The purpose of this article is to address this critical need. In what follows, we describe an adaptation of Jacobs et al.'s (2010) framework for teacher noticing of student mathematical thinking to include a focus on making students' technology engagement explicit when noticing student thinking in technology-mediated environments. We share the framework, describe its theoretical foundations, and then illustrate how the framework can be used by mathematics teacher educators to support PSTs' noticing when students are working in technology-mediated environments.

2 | TECHNOLOGY AND TEACHER NOTICING

Often when we write about teacher noticing in technology-mediated environments our intent is misunderstood—rightfully so—as there are many different ways that technology and noticing have been intertwined. A search for technology and teacher noticing will result in articles that include video recordings of all types, video tagging software, course management systems, animation, and math action technologies. The literature related to each of these intersections among technology and noticing are described below.

2.1 | Video and teacher noticing

First and foremost there is the use of video recording technology, which has allowed teachers and researchers to capture videos of individual students engaged in doing mathematics or whole-class mathematics instruction via purposefully selected video-case artifacts for the purpose of noticing. This includes traditional digital video cases of classroom vignettes (e.g., Fisher et al., 2019; Jacobs et al., 2010; McDuffie et al., 2014; Sherin & van Es, 2005), in-the moment selective video capture of one's own teaching (e.g., Sherin et al., 2011; Stockero, 2013), 360 video (Kosko et al., 2022), and smartphone video (Chao et al., 2016). Santagata et al. (2021) completed a systematic review of teacher noticing studies that utilize video in which they found whole-class video of other teachers' lessons to be the most common, while clips focused on students solving a mathematics problem was the least common. Despite the video type, they found most projects provided teachers with structured guides to support their noticing while viewing videos (Santagata et al., 2021).

2.2 | Video annotation and teacher noticing

Related to the use of video cases is the use of various technologies to create instructional activities in which video is the object being considered through a noticing lens. This includes the use of learning management systems (e.g., Fisher et al., 2018; Johnson et al., 2019) and video annotating technology (Larison et al., 2022; Sherin & van Es, 2005). The former allows for video to be embedded as a learning object that teachers can answer questions about or discuss in online forums (Santagata et al., 2021). The latter includes tools for engaging with the video itself by tagging particular moments after—the fact with labels and/or comments (e.g., Larison et al., 2022; Stockero et al., 2017; Walkoe, 2015; Walkoe et al., 2020). Collectively this work has shown that technology can not only be leveraged in such ways to support teachers in identifying and discussing significant events in video, but also results in increased noticing (Larison et al., 2022). Walkoe et al. (2020) noted, that video tagging technology "affords researchers and teacher educators the ability to examine the relationship between what grabs a teachers' attention and how teachers interpret the thinking displayed in those moments and instructionally actionable feedback about how teachers are progressing in their attending to and interpreting student thinking."

2.3 | Animation and teacher noticing

Another technology that has been used to support teacher noticing is animation. Researchers have used animation to depict hypothetical classroom interactions (or to recreate actual classroom interactions) and then used them in ways similar to videocases (e.g., Chieu et al., 2011; González, 2018; González & Dejarnette, 2018; González & Skultety, 2018). When researchers and teacher educators do not have access to authentic classroom video to be used with teachers, animations are a useful substitute. Furthermore, when researchers or teacher educators want to put forth particular types of classroom interactions for noticing assignments, but have not been able to capture them on video, animation tools make that possible. In this way, teacher educators can create animations that focus teachers' attention on specific elements of the classroom (Herbst & Kosko, 2014; Hollebrands et al., 2018). Animation creation has also been used as a way for PSTs and teachers to depict how they would respond after attending to and interpreting (e.g., Amador et al., 2016; de Araujo et al., 2015; Lee, 2021) an instructional artifact. Research in this area has found that animation technologies help PSTs communicate their instructional follow up more precisely (Amador et al., 2016) and helps them immerse themselves in the moment of teaching (Lee, 2021).

2.4 | Technology-enhanced math tasks and teacher noticing

The lens of teacher noticing has also been applied to teacher noticing of technology-enhanced math tasks (e.g., Smith et al., 2017; Yeo & Webel, 2019). This area of research builds on work related to curricular noticing (Dietiker et al., 2018) and focuses on noticing to evaluate technology tasks for teaching mathematics. Technology-enhanced math tasks are tasks (sometimes embedded in curriculum resources) that include math action technology—technology that responds to user actions in mathematically defined ways (Dick & Hollebrands, 2011).

Smith et al. (2017) developed the Mathematical Cognitive Technology Noticing Framework to examine how teachers notice when evaluating technology-enhanced math tasks. Building on curricular noticing, they used attending, interpreting, and responding to describe teachers' evaluation. Specifically, they noted that attending refers to the features of the technology that teachers identify, interpreting refers to the ways that teachers interpret and anticipate how students might interact with those features, and responding refers to whether teachers adapt, choose, incorporate, or redesign the technology. Yeo and Webel (2019) extended this work to consider how PSTs notice technology resources when they see students engaged with the resources. Though they asked PSTs to watch students engage with the technology, the focus was on the technology itself (i.e., attending, interpreting, and responding as described by Smith et al., 2017). Yeo and Webel found that watching students engage with the technology resource definitely influenced PSTs' interpretation and ultimate response—sometimes positively and other times negatively. This influence was based on how they saw students thinking about the mathematical ideas embedded in the task that used the technology. Together this body of work provides insight into the ways teachers evaluate and ultimately make decisions about the use of math action technologies.

2.5 | Technology mediated environments and teacher noticing

While there exists a large body of work related to noticing with technology, our interests are not focused on using technology to notice, or noticing the technology itself, but rather noticing student thinking in technology mediated environments—that is, when students are engaging with technology-enhanced mathematical tasks involving math action technologies.

Research into how teachers notice students' mathematical thinking when they are working in technology-mediated environments is limited (e.g., Abdu & Slakmon, 2022; Bailey et al., 2022; Chandler, 2017; Dick et al., 2022; Hollebrands et al., 2018; Lovett et al., 2019a; Wilson et al., 2011; Yeo & Webel, 2019). In 2011, simultaneous to the introduction of the noticing construct, Wilson and colleagues engaged PSTs with examining students' work when solving statistical problems using a dynamic statistical tool (i.e., TinkerPlots). Through this work they developed a framework that describes the process teachers engage in when examining student thinking which includes describing (attending), inferring (assumptions about student thinking or motivation), comparing (direct comparison to one's own thinking), and restructuring (interpreting that learning has occurred). They found that PSTs could describe students' actions with technology, but found little evidence of the PSTs connecting the students' actions with the technology to the students' mathematical thinking. Overall the PSTs drew on their own mathematical content knowledge to interpret the students' thinking, which often hindered their ability to unpack the students' understandings. More recently, Hollebrands et al. (2018) studied the ways in which teachers noticed students' mathematical thinking (as expressed in discussion posts) in the context of observing animations of classroom practices and videos of students' work in a Massive Open Online Course (MOOC-Ed). Using the same framework as Wilson et al. (2011), they found that when analyzing the animations and videos, less than 50% of teacher discussion posts were focused on student thinking. Those that were focused on student thinking typically did so by describing and/or inferring. Furthermore, for those posts that were not focused on student thinking, teachers discussed the task, but there was less attention to issues related to teaching and technology.

Using Jacobs et al.'s (2010) framework for noticing student thinking, Chandler (2017) compared PSTs' noticing students' mathematical thinking on geometry tasks presented in two different mediums: written work and technology-mediated work (the technology in use was The Geometer's Sketchpad). She found both groups of PSTs focused their noticing on the students' mathematical thinking, but struggled to provide evidence to back up their interpretations; there was not much variation in how the PSTs noticed the students' thinking between the two mediums.

In our previous work, we have studied PST noticing of students' mathematical thinking in various technology-mediated environments and found it important to explicitly forefront the role of students' engagement with math action technologies (Lovett et al., 2019a). When we first began working with PSTs on noticing student thinking on technology-enhanced tasks, we introduced Jacobs et al.'s (2010) professional noticing construct (i.e., attend, interpret, and decide how to respond) and asked PSTs to notice students' mathematical thinking. Without emphasizing the importance of noticing students' engagement with the technology, PSTs rarely referenced the role technology played in student thinking in their responses and instead focused on what the students said and wrote (Lovett et al., 2019b).

More recently, Abdu and Slakmon (2022) examined an experienced teacher's in-the-moment noticing when engaging students in a technology-enhanced math task (in this case using GeoGebra). They focused on the teacher's noticing of a particular pair of students in the context of the whole class. They noted that in-the-moment noticing when using a technology-enhanced task is extremely complex because as you move around the room you not only miss what the students are saying to each other, but also their actions with the technology and what they see as a result of their actions. They concluded, "The case study demonstrates that noticing students' narratives of exploratory learning with [math action] simulations requires developing listening practices and disciplinary knowledge toward attentive listening suited for the setting." In other words, teachers need to not only listen with their ears, but also with their eyes. They must attend to how students are engaging with the technology, and seeing as a result of their engagement, as that is also an aspect of students' thinking. Overall, what little research there is in the space of noticing student thinking in technology-mediated environments has shown time and again that the work is complex, and that including student thinking directly related to their technology engagement is not something that practicing teachers or PSTs tend to do naturally.

2.6 | Context of this work

To better understand PSTs' noticing practices on technology-enhanced tasks we developed an empirical framework adapting Jacobs et al.'s (2010) framework to highlight the importance of engagement with math action technologies (Dick et al., 2022). While we had an analytical tool to make sense of PSTs' noticing of student thinking with technology-enhanced tasks, the results were the same—PSTs rarely noticed student engagement with the technology rather they were focused on what students said and wrote. We then conjectured that providing the framework as a means of scaffolding PSTs' noticing would assist PSTs in their development of this core teaching practice. Results from Bailey et al. (2022) confirmed our hypothesis and led to further incorporation of the framework into the development of curriculum materials intended to support the transformative development of PSTs' TPACK (Preparing to Teach Math with Technology: Examining Student Practices in Algebra & Function, 2023). While we have evidence that using the framework to scaffold PSTs' developing practice of teacher noticing is effective, in our published work thus far we have not explicitly unpacked the framework and described how it can be used by mathematics teacher educators to develop PSTs' skill of teacher noticing of student thinking on technology-enhanced tasks. In the following sections, we share the theoretical considerations that ground the framework and discuss how mathematics teacher educators can use it with PSTs.

3 | A FRAMEWORK FOR NOTICING STUDENT THINKING IN TECHNOLOGY MEDIATED ENVIRONMENTS

The Noticing in Technology Mediated Environments (NITE; Dick et al., 2021, 2022) framework was developed with the understanding that students' mathematical thinking can be expressed through their engagement with math action technologies. Research on these technologies has shown that they can help mediate students' mathematical thinking (e.g., Arzarello et al., 2002; Baccaglini-Frank & Mariotti, 2010; Trouche & Drijvers, 2010). Thus, in the context of students working with technology-enhanced math tasks, it is imperative to attend to students' mathematical thinking with a focus on students' engagement with the technology since students' strategic technology use provides insight into their thinking (e.g., Arzarello et al., 2002; Schack et al., 2013; Walkoe et al., 2017; Yeo & Webel, 2019). When interpreting students' understandings, it is again necessary to coordinate students' spoken and written responses with how they engage with the math action technologies in a technology-enhanced mathematics task. If one relies on only part of that information, it is possible to miss important aspects of a student's developing understanding. We have found that taking a semiotic meditation perspective (e.g., Bartolini Bussi & Mariotti, 2008; Jones, 2000; Mariotti, 2000, 2013) of technology use provides a way to make sense of teacher noticing of student thinking in technology-mediated environments.

The premise that students' understandings are shaped by the tools (i.e., here tools are math action technologies) that they use and by their internal relationship with those tools is consistent with socio-cultural theories of learning (e.g., Vygotsky, 1978). When discussing tools, Verillon and Rabardel (1995) made a distinction between a tool as an artifact and a tool as an instrument, the former being the man-made material object, and the latter being a psychological construct. Specifically, an instrument includes the tool and all of the ways a person thinks about using it (i.e., their utilization schemes). This process of coming to understand the potentialities and constraints of an artifact while at the same time developing mathematical knowledge is referred to as instrumental genesis (Artigue, 2002). Mariotti (2000) explains when a student "uses the artifact, according to certain utilization schemes, in order to accomplish the goal assigned by the task; in doing so the artifact may function as a semiotic mediator where meaning emerges from the subject's involvement in the activity" (p. 36). In other words, when a tool is used as a psychological tool, it is an instrument of semiotic mediation, a mediating artifact between the learner and the mathematics (Jones, 2000). Furthermore, it is important that when teachers choose to use a tool they consider the semiotic potential of the tool, for example, the "potentiality that the use of a specific artifact has in fostering mathematical learning" (Mariotti, 2013, p. 442).

Since utilization schemes are internal, they are not observable. However, teachers can observe techniques—the observable interactions between the user and the artifact (Drijvers et al., 2010). Bartolini and colleagues (2008) note that "personal meanings are related to the use of the artifact, in particular in relation to the aim of accomplishing the task; on the other hand, mathematical meanings may be related to the artifact and its use" (p. 754). In other words, in the context of teaching PSTs to attend to student thinking, attending to students' techniques provides insight into their thinking. It follows then that when interpreting students' understandings in a technology-mediated environment, it is necessary for PSTs to coordinate students' spoken and written responses with the observable ways in which they engage with the tool (i.e., their techniques) and what students see as a result of their engagement. If a PST only relies on part of that information, it is possible to miss important aspects of a student's growing understanding. From this perspective, student engagement with a math action technology is a cognitive action (i.e., a physical and psychological action that is a mediating artifact between the learner and the mathematics); however, teachers are only privy to the observable actions students' take (i.e., their techniques). In this light, going forward when we refer to engagement with technology, we are referring to those observable actions—the techniques.

Our conceptualization of teacher noticing of students' mathematical thinking in the context of using technology-enhanced tasks, the NITE framework, is shown in Figure 2. While we acknowledge that all components of noticing are by their nature interrelated (Jacobs et al., 2010), we have separated "attention to and interpretation of student's spoken and written responses" from "attention to and interpretation of students' engagement with the technology" to highlight the importance of including the actions students take with the technology (i.e., their techniques) and what students see as a result of these actions when attending to and interpreting students' mathematical thinking. Thus, the arrows in the NITE framework indicate the importance of both the horizontal coordination of attention and interpretation as well as the vertical integration of both attention and interpretation. The "decide how to respond" component is separated from the other components for two reasons: (a) to indicate the importance of balancing insight gained from attending to and interpreting both students' spoken and written responses and their technology-engagement when making instructional decisions; and (b) when deciding how to respond the teacher must consider how to position the technology (or not) in their response to support the student in moving forward. For our conceptualization, like Jacobs et al. (2010), we emphasize "that the ability to effectively integrate these three component skills is a necessary, but not sufficient, condition for responding on the basis of children's understandings" (p. 197). Hence PST integration of the three noticing components while coordinating attend and interpret within a technology-mediated environment is the goal of this complex teaching practice.

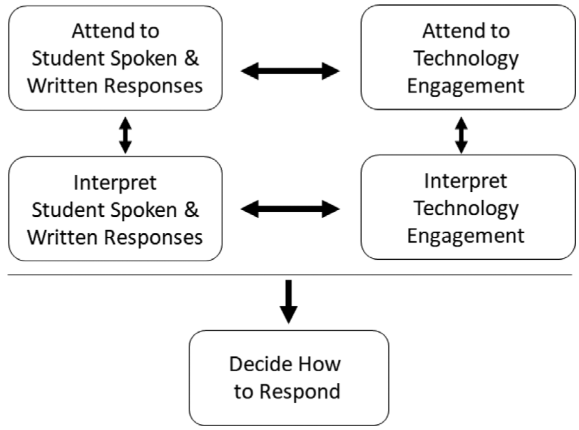
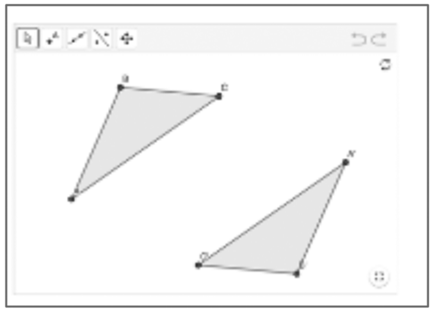


FIGURE 2 Teacher noticing of student's work in a technology tool-mediated environment (the NITE framework).

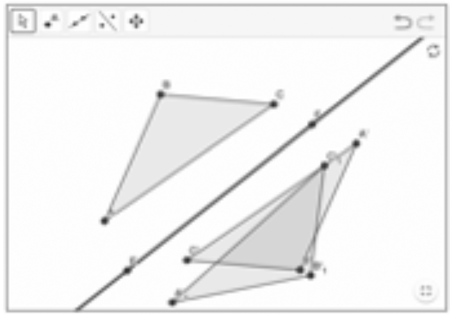
4 | UNPACKING STUDENT THINKING USING THE NITE FRAMEWORK

To illustrate the different components articulated by the NITE framework we share an example of how to use the NITE framework to unpack students' thinking within

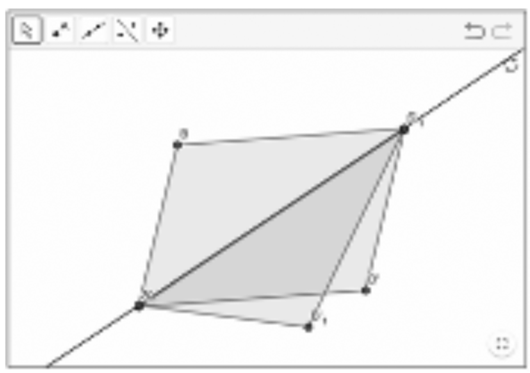
[Abby constructs line EF in between preimage triangle ABC and image triangle A'B'C']
Addison: No, no, no, no, no.
Abby: Stop, stop.
[Addison takes the laptop from Abby. Under the transform menu, she chooses reflection, chooses line EF and then triangle ABC. Image $A'_1B'_1C'_1$ is reflected over the line as shown]



Addison: It's not gonna be exactly the same.
Abby: But it's gonna be close.
Addison: You're not very good at this.
Abby: I know that.
Addison: That's good.
Abby: Look, see.
Addison: But it's not exactly the same.
Abby: Okay. It's not gonna be.
Addison: I can make it exactly the same.
Abby: Oh, wait.
[Abby takes the computer and chooses the select tool.]
Addison: I can make it exactly the same.
[Abby drags triangle ABC little bits trying get the two image triangles to align]



Abby: I don't care… Look at that. It's very close. … It's extremely close.
[Addison takes the laptop back, and clicks undo until the line of reflection, EF, disappears. Addison drags point C onto point A', which causes point C' to lie on point A. She then constructs line AC.]
Addison: Move, please.
Abby: You're so extra.
Addison: You got something to say?
Abby: Yes.
[Addison reflects triangle ABC over line AC and triangle $A'_1B'_1C'_1$ appears as shown below]
Addison: Oh!
Abby: Nice. Real smooth. Look at that.
Addison: No, why isn't it the same?
[Abby takes the laptop back from Addison and undoes the reflection Addison constructed]
Abby: Because you did the line wrong. That's what I was like, you're not that smart.
Addison: I am too!
Abby: The line is not there! That's not where the line is.
Addison: Where is it then?
[Abby drags point A, trying to locate a line of reflection that works.]
Abby: All I know is it's not there…because that did not work.
Addison: Obviously.
Abby: Wait, oh wait, you made, you made a perfect quadrilateral, right?
[After some additional dragging, Abby drags the vertices again so that the two triangles form a quadrilateral.]
Abby: Now that we've done that…Darn it. I get it Addison. Stop giving me grief!
[Abby reflects triangle ABC over the diagonal of the quadrilateral.]
Addison: You still didn't do it. Abby: ***[thumping her hands on the table in frustration]***
Addison: Okay, let's see let's see let's see let's see.
Abby: Okay.



[Addison takes the computer from Abby and drags point B in several directions to see what happens.]
Addison: Okay. Let's see why.
Abby: Wait, wait, wait! Yeah, move it. Yeah. Oh, what are you doing? … What were you saying Addison? … Wait no no no, move it the other way. Other way.
[Addison drags B so that B' and B'_1 almost lie on each other.]
Addison: Hold on I'm trying to make it so it's in screen.
Addison: But it's still not an exact copy.
Abby: No, but it's a rhombus,
Abby: over the line.
Addison: So why is it wrong?

FIGURE 3 Transcript of Addison and Abby working on the mystery transformation GeoGebra activity.

technology-mediated environments. This example is in the context of a technology-enhanced math task focused on building an understanding of geometric transformations using the math action technology, Geogebra.

4.1 | An example: Noticing student thinking on the mystery transformations task

Imagine students have been introduced to reflections, rotations, translations, and dilations. Using a dynamic geometry environment, like GeoGebra, students can drag vertices of the preimage to conjecture and test the transformation that produced the image. By incorporating dynamic transformations in this way, students have the opportunity to explore the properties of rotations, reflections, translations, and dilations not only in terms of congruent and similar figures, but also with respect to symmetry and orientation.

Here we share a pair of Integrated Math 3 students working together on a "mystery transformation" task in which they are given a preimage and image, and are asked to determine the transformation that occurred. The task is created in GeoGebra (https://www.geogebra.org/m/zezbkmbp). The first page of the task shows various pairs of preimage/image points. The students are asked to drag the points and determine what transformation was used to create each image point. On the following page, the students investigate and conjecture which transformation was used to create the image triangle A′B′C′. As the students drag the vertices of the preimage, the corresponding vertices of the image move as well. Students are then asked to draw in the transformation (center, line of reflection, or vector). Scrolling down the page, students test their conjecture by constructing the transformation using GeoGebra's construction tools. The activity reminds students that they know they are correct when "the image you construct is directly on top of triangle A′B′C′." The activity continues with two additional pages similar to page two where students are asked to conjecture and test the mystery transformation.

Below is an excerpt from a pair of students' discussion, Addison and Abby, who are working on page three of the activity (see Figure 3). They have previously conjectured that the transformation is a reflection and they are testing their conjecture by constructing the line of reflection and dragging the points of the preimage, triangle ABC. Addison and Abby are regular partners and friends, they tease each other throughout their work together. So, as you read the transcript keep in mind that the tone of their voices is sarcastic and silly, not mean. (*Note*: The picture-in-picture [showing students and their

Addison and Abby's Words and Actions

Attend to Students' Spoken & Written Responses	Attend to Students' Technology Engagement
• Addison says it is not "exactly the same" but then says "I can make it exactly the same." • Abby says the two images are "pretty close/very close/extremely close." • Abby says that the two images do not match up because "you did the line wrong." • Abby says "all I know is the line is not there because that did not work obviously." • Abby says "you made a perfect quadrilateral." • When the image and preimage still do not line up, Addison says, "let's see why." • Addison says "it's still not an exact copy" and Addison says, "no, but it's a rhombus over the line."	• Abby constructs a line EF between the image and preimage and reflects the preimage over the line EF. • Abby drags the preimage to try to line up the sides of the two images. • Addison deletes the line and drag the points on the preimage to form a quadrilateral in which C and A' and A and C' intersect. • Addison constructs a line that is the diagonal of the quadrilateral and goes through the shared base segment and then reflects the preimage over the line. • Abby continues to drag A bit by bit. • Addison deletes the reflection and drag the points of the preimage to form a quadrilateral. • Abby constructs a line that is the diagonal of the quadrilateral and then reflects the preimage over the line again. • Addison drags point B to determine why the reflection did not work. • They end with B so that they have almost created a rhombus.

FIGURE 4 Addison and Abby's words and actions.

computer screen] video clip can be viewed at: https://youtu.be/-b4pE5tS8FA).

If a teacher only paid attention to what the students said, they would not have a full picture of the students' current understanding. The NITE framework guides the focus on noticing both what we hear the students say or see them write, and to also consider their engagement with the technology and the role it all plays in providing insight into their developing understanding of the concept of geometric transformations. Attending both to the students' spoken and written responses and their engagement with the technology guided by the NITE framework might result in the list provided in Figure 4.

If a teacher only attended to what Addison and Abby said (left column of Figure 4) they could interpret that Addison and Abby understand that a reflection should produce a congruent triangle and that if the line of reflection were the side of the triangle it would be the diagonal of a quadrilateral formed by the preimage and image. However, if one were to coordinate their attention to the students' spoken responses (left column) and their technology engagement (right column) one could construct a more complete description of their understanding. Considering Addison and Abby's technology engagement, there is evidence that they do not appear to understand if the transformation were a reflection the preimage and the image should not only be congruent the triangles, but the image A′B′C′ and their constructed image A″B″C″ need to also have the same orientation. There is no evidence that the students ever considered the orientation of the two images as the vertices they are trying to align are labeled differently (e.g., A′ and C1′) and they do not mention that difference. With respect to reflections, Addison and Abby seem to have an understanding that the preimage and the image should be equidistant from the line of reflection. This was based on their placement of the line of reflection between the two triangle bases. However, since they did not consider point B it does not seem that they understand that all vertices of the triangle and their images must be equidistant from the line of reflection.

After careful coordination of what has been attended to and interpreted about the students' thinking, one would be well informed and could decide how to respond. For example, a non-technology decision might be to ask Addison and Abby to draw a mapping diagram from the vertices of the preimage to the given image and then the preimage to their constructed image, which could help to draw their attention to the orientation of the three figures. A technology dependent strategy might be to probe their understanding related to the preimage and images being equidistant from the line of reflection by asking them to use the Geogebra tools to determine if the vertices of the triangles are in fact equidistant from the line of reflection. How they measure the distance of B, B′, and B1′ from the line of reflection will reveal a lot about their understanding of this property of transformations.

It is important to note that in practice we do not always ask PSTs to separate out attending to students' spoken and written responses from their engagement with the technology. This was done here to emphasize what would be missed if we only attended to what Addison and Abby were saying. However, we have found that ensuring PSTs refer to the NITE framework while taking the time to fully notice all these aspects of the students' thinking (spoken and written words, and technology engagement) is a helpful tool to support PSTs as they work to fully coordinate their integration when attending and interpreting.

5 | USING THE NITE TO SUPPORT PSTS LEARNING TO NOTICE STUDENT THINKING

The Association of Mathematics Teacher Educators (2017) *Standards for the Preparation of Mathematics Teachers* notes the importance of teachers being "proficient with tools and technology designed to support mathematical reasoning and sense making, both in doing mathematics themselves and in supporting student learning of mathematics" (p. 11). An important component of supporting student learning is teacher noticing (Jacobs & Spangler, 2017). When we began our work in this realm, we were initially discouraged by the lack of attention PSTs were giving to students' engagement with technology when they practiced the component skills of teacher noticing. However, the literature base supported this common oversight. It was also through this literature base that we came to realize that a framework that highlighted the importance of attending to and interpreting students' technology-tool use would help us as researchers better understand the obstacles to PST noticing and that such a framework had the potential to support teacher noticing as well. To that end, with the NITE framework we set out to adapt Jacobs et al.'s (2010) conceptualization of teacher noticing of student's mathematical thinking to explicitly highlight the role of student's technology engagement. Addison and Abby's work on the Mystery Transformations task highlights the importance of coordinating the ways in which students engage with technology, and what they see as a result of this engagement, with their other expressed responses when noticing their thinking. However, as we noted earlier, our prior work with PSTs indicates that doing so is not trivial. That led us to considering how we might use the NITE framework to support PSTs development of this important teaching practice.

The literature is rich with examples of the ways in which frameworks have supported PSTs' learning to notice in mathematics classrooms. For example, Mitchell and Marin (2015) used an analysis framework to support PSTs' noticing important aspects of mathematics, Teuscher et al. (2017) used a framework to support PSTs noticing mathematically significant pedagogical opportunities, and Fisher et al. (2019) provided Jacobs et al.'s (2010) framework as part of preservice elementary teachers instruction with the express goal of helping them focus on all three components of noticing. The importance of integrating frameworks into teacher education is emphasized in the Association of Mathematics Teacher Educators' *Standards for Preparing Teachers of Mathematics* (2017): "An effective mathematics teacher preparation program ensures that practice-based experiences, including mathematics methods courses and equivalent learning experiences, provide candidates with experiences using tools and frameworks grounded in research to develop core pedagogical practices and pedagogical content knowledge for teaching mathematics" (p. 35). Noticing frameworks are particularly useful as the practice of noticing is foundational to many other important mathematics teaching practices (e.g., AMTE, 2017; NCTM, 2014; Thomas et al., 2015).

Our research team has created curriculum materials focused on the development of PSTs' noticing when

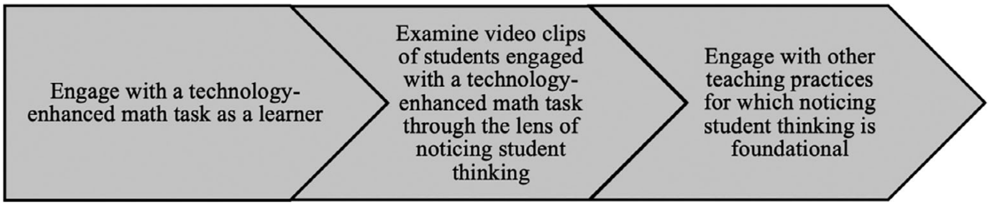


FIGURE 5 PTMT-ESP module design.

students are working on technology-enhanced tasks—and in turn their TPACK (specifically "knowledge of students' understandings, thinking, and learning with technology-enhanced math tasks"; Niess, 2005, p. 511). The NITE framework is foundational to this work. In the curriculum materials, (Preparing to Teach Mathematics with Technology - Examinging Student Practices), in the first module PSTs are explicitly introduced to the NITE Framework and how noticing student thinking is connected to important teaching practices like the NCTM (2014) Mathematics Teaching Practices (e.g., eliciting and using student thinking, posing purposeful questions) and Smith and Stein (2018) Five Practices for Orchestrating Productive Mathematical Discussions. Then in each of seven subsequent modules, PSTs go through a series of tasks as described in Figure 5. Specifically, they begin each module by engaging in a technology-enhanced math task themselves. By doing the task from the perspective of a learner, PSTs engage in anticipating student thinking. Having learned about the NITE framework, PSTs anticipations include the consideration of both how students will respond to prompts in the task as well as how they will engage with the technology. Next, PSTs examine carefully selected video clips (Lovett et al., 2020) of secondary students working on the same technology-enhanced math task through the lens of NITE, writing out and discussing the components with each other. Finally, in the context of the same technology-enhanced task, PSTs have opportunities to decide how to respond in ways that are connected to specific approximated teaching practices (e.g., Grossman et al., 2009; McDonald et al., 2013). Approximations of instructional decisions in our materials include (but are not limited to) posing purposeful questions, monitoring, selecting, and sequencing student responses, scripting class discussions (Campbell & Baldinger, 2022), and selecting (or designing) a next task in a sequence of tasks. As indicated in the NITE framework, when working with technology-enhanced tasks instructional decisions may or may not include the use of the technology. Hence, PSTs are developing practices that are transferable to other contexts. Math teacher educators can find the eight modules at the PTMT website—materials include tasks for PSTs and detailed additional facilitation materials that highlight the role of NITE throughout.

6 | ADDITIONAL CONSIDERATIONS

Thus far, we have discussed how the NITE has been used to scaffold PST noticing when students are working with technology-enhanced math tasks. Empirical results have shown the NITE framework continues to be helpful in focusing PST noticing on students' engagement with technology (Bailey et al., 2022; Dick et al., 2022). Future research considering different ways the NITE informs the work of practicing teachers and how it differs from working with PSTs would benefit MTEs.

While it was not initially intended to do so, it is worth noting that in highlighting the importance of noticing students' engagement with technology and coordinating that engagement with the other ways that students express their thinking, the NITE not only provides insight into student thinking, but there is also a direct connection to design considerations for technology-enhanced tasks. The better we understand the affordances of different technology tools, the more effectively we can design technology-enhanced tasks that promote student learning. Just like PSTs use the NITE as they anticipate student thinking on a technology-enhanced task, we have found that they also refer to it when selecting, adapting, and designing technology-enhanced tasks on their own.

As discussed, the NITE framework has been helpful in our work with both PSTs and practicing teachers to highlight an important aspect of student thinking in technology-mediated environments—technology engagement. We can imagine that a similar strategy—separating out a feature of an instructional situation to push PSTs to consider on the effect of that feature within their noticing—could be helpful for other aspects of an instructional context that might be similarly overlooked. For example, this strategy could be helpful for tools of all types (e.g., manipulatives, measurement tools). These tools are different from math action technologies, as they do not react to students' actions. So rather than considering how students' make sense of how the technology reacts mathematically to actions they take, the focus would be on the ways that students think differently about the mathematical objects due to their own reconfiguration of the objects.

Another important aspect of student thinking in technology-mediated environments worthy of additional lenses is drawing attention to student interactions when noticing pairs of students working together on a technology-enhanced task. The (Preparing to Teach Mathematics with Technology - Examining Student Practices (PTMT-ESP) project) materials intentionally use video clips of pairs of students as students often make more of their thinking visible when working with others (Lovett et al., 2020). While the NITE framework does not explicitly address how to make sense of an individual's mathematical thinking when working in collaboration with others on technology-enhanced tasks. In our work to date, we see PSTs addressing this issue in the ways they "decide how to respond." Specifically, we see PSTs decide how to respond in ways that they envision will help them understand more about an individual student's understanding by suggesting assessing questions they might pose. Drawing on frameworks that consider teacher noticing in the context of students' collaborative mathematical work (e.g., Campbell & Yeo, 2022) could support teachers to understand how groups approach both the mathematical and collaborative nature of a task. In the context of engaging with a technology-enhanced task this is of particular interest as who controls the mouse might be an important feature of the interaction that informs how we interpret developing understandings (Fletcher & Fye, 2022). Suh et al. (2022) note that an important question to consider when selecting a technology task from an equity perspective is in what ways does the technology "allow student ownership and authorship to build positive math identities" (p. 1398). So, separating out ownership and authorship, and considering student dispositions toward each other (see the previous Abby and Addison Mystery Transformation example) when students are working together on a technology-enhanced task could provide important insight into the ways the technology is or is not supporting mathematical identity development.

7 | CONCLUSION

The NITE framework is one answer to Walkoe et al.'s (2017) question, "how can teachers learn to look for key student thinking practices, [...], through the lens of technology-mediated student work? (p. 67)." It provides a way to support teacher noticing as well as other teaching practices (e.g., anticipating student thinking, posing purposeful questions). PSTs that have been introduced to the NITE framework, indicate that they found NITE to be a helpful tool. For example, one PST who was introduced to the NITE framework in his coursework shared that he now keeps the framework up on his monitor when he watches videos of students working technology-enhanced tasks as a reminder to "take in all aspects to truly get a better understanding of what the students know." He shared that when he first started watching such videos he was:

> very focused on what do the students know, what are they telling me and I was so focused in on am I getting what they know, but I kind of needed to like step back and look and it's like here is a completely different area that they are giving you information that you're not even considering just because you are trying to zone in on what they are saying, you're trying to hear what they are saying and what they're writing that you're not looking at how they are actually interacting with it.

For this PST and others with similar reflections, the NITE framework did exactly what we hoped, it drew PSTs' attention to the students' mathematical thinking that was expressed through their engagement with technology-enhanced math tasks and therefore serves as a complement to developing the complex skill of teacher noticing.

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