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Predicting Students' Mathematical Thinking in a Technology-Mediated Environment

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Preservice secondary mathematics teachers (PSMTs) need exposure to noticing students' mathematical thinking when students are engaged with technological tasks. In this paper, we share results from a study in which PSMTs first watched videos of students engaged with a technological mathematical task focused on the concept of function and after watching the videos, were asked to predict the students' technological mathematical thinking as they continued to interact within the technological environment. Results showed that it was difficult for the PSMTs to coordinate the students' mathematical thinking with the students' engagement with the technology. However, for PSMTs who did coordinate both the students' thinking and their engagement with the technology, we identified two components of their predictions that seemed to provide specific insight into the PSMTs' overall noticing of the students' thinking. These were 1) predicting cognitive dissonance or lack thereof, and 2) predicting how the students would interact with each other and with the technology. These results provide mathematics teacher educators with specific additions to potential noticing prompts for PSMTs to express their interpretation of students' technological mathematical thinking when predicting what students may do next when working in a technology-mediated environment.

Teacher noticing of students' mathematical thinking is a skill that involves paying attention to what and how students think and then making instructional decisions based on students' understandings (Jacobs, Lamb, & Philipp, 2010). When working in a technology-mediated environment, teacher noticing also involves noticing how students interact with the technology to develop their mathematical understandings because when using technology, students articulate their thinking through their interactions with the technology itself. Because researchers find that preservice teachers' noticing can be improved with practice (e.g., Schack et al., 2013) and given the important role that technology can play in students' mathematical thinking, there is a need for preservice secondary mathematics teachers (PSMTs) to be provided with opportunities to notice students' thinking in technology-mediated environments.

One way to engage PSMTs with students' mathematical thinking is to provide them with experiences analyzing student work (e.g., Bartell et al., 2012; Casey et al., 2018; Didis et al., 2016; Philipp, 2008). Analyzing students' strategies when examining their work involves interpreting mathematically significant details present in student work. Researchers identified differences between preservice and inservice teachers' interpretations, finding that preservice teachers struggle with identifying key components of students' thinking and generating robust interpretations (e.g., Dick, 2017, Dick, McCulloch, & Lovett, Under Review; Jacobs et al., 2010; Sherin & Star, 2011). To combat this issue, we posit that one way to gain insight into PSMTs' analysis of students' work and provoke more robust interpretations of students' mathematical thinking is asking PSMTs to predict what a student might do next based on the sense they have made of the students' work. The purpose of this paper is to investigate PSMTs' predictions in a technology-mediated environment in which they are asked to predict students' mathematical thinking after making sense of students' work.

BACKGROUND

In their 2017 review of work surrounding the core practices of teaching, Jacobs and Spangler identified teacher noticing as a "hidden core practice of mathematics teaching" (p. 771). With origins in the work of researching one's own teaching practice (Mason, 2002), teacher noticing can be described as the act of paying attention to and making sense of the complexities that occur in the classroom. For PSMTs, teacher noticing is a skill that needs to be purposely developed as "teachers can be responsive only to what has been noticed" (Jacobs and Spangler 2017, p. 772). When studying what teachers' notice, the object of the noticing should be defined and can vary from noticing teacher actions (e.g., Osmanoglu, Isikal, & Koc, 2015) to noticing children's participation (e.g., Wager, 2014). For this paper, we are concerned specifically with a particular component of PSMTs' noticing students' mathematical thinking as conceptualized by Jacobs, Lamb, and Philipp (2010): interpreting their mathematical understandings.

Teacher interpretation of students' mathematical thinking

Within the construct of teacher noticing, the ways that teachers interpret what they notice is as important as what they notice. "Taking an interpretive stance means that teachers focus on understanding why an event occurred or what influence a particular event had on student learning. It also means delving deeply into understanding what students understand about the subject matter and from where that understanding came" (van Es & Sherin, 2002, p. 578). In other words, it is important to determine ways to support the development of the skill of interpretation as this sense making can – and should – inform one's instructional decisions. However, research shows that preservice teachers have difficulty interpreting student thinking (Stahnke, Schueler, & Roesken-Winter, 2016). While we know that teacher noticing is a skill that can be learned (e.g., Schack, et al., 2013) we also know that simply asking PSMTs to repeatedly interpret student thinking does not automatically move them from novice to more expert ways of interpreting (van Es & Sherin, 2002). As a result, it is important to consider ways in which we can elicit PSMTs interpretations so that we can design scaffolds to support their development (Bannister et al., 2018). Since interpreting is an internal process, what PSMTs express when asked to interpret student thinking does not necessarily fully encompass their sense making (Stahnke, Schueler, & Roesken-Winter, 2016). We suggest one way to elicit more detail about PSMTs' interpretations of student thinking is by asking them to make and justify predictions of how they expect students' will continue to work on a similar mathematical task.

Teacher noticing in technology-mediated learning environments

One aspect of noticing student thinking that is understudied is considering how students learn through strategic technology use (e.g., Wilson, Lee, & Hollebrands, 2011). In elementary mathematics classrooms, technology tools may include applets that use digital representations of tens frames or fraction tiles, while in secondary mathematics classrooms technology tools may include, but are not limited to, graphing utilities, dynamic geometry environments, or digital manipulatives. With one-to-one device environments becoming more prevalent in secondary classrooms (Zheng, Warschauer, Lin, & Chang, 2016), technology tools are often employed for developing mathematical understanding. Research findings indicate that the ways in which students engage with such tools mediates their sense-making about the object of their investigation (e.g., Arzarello et al., 2002; Baccaglini-Frank & Mariotti, 2010; Doerr & Zangor 2000; Lee, Angotti, & Tarr, 2010; Lopez-Real & Leung, 2006; Trouche & Drijvers, 2010). Moreover, students' engagement can reveal how they are thinking about the mathematics. For example, Arzarello and colleagues (2002) examined how students engaged with dynamic geometry tools and noted that students used the tools to achieve different goals; the students' engagement provided insight into their cognitive processes. The same has been found across studies of students' use of graphing calculators (e.g., McCulloch, 2011; Doerr & Zangor, 2000) and computer algebra systems (e.g., Artigue, 2002; Kieran & Saldahna, 2005). The research on students' use of technology shows time and again that interpreting students' engagement with these tools provides insight to their mathematical thinking. In fact, Arzarello and colleagues (2002) explicitly call for the importance of teacher's noticing student engagement with tools stating, "Such an analysis is a powerful tool to investigate the cognitive processes of pupils through visible actions" (p. 69).

In 2011, prior to the introduction of the noticing construct, Wilson and colleagues engaged PSMTs with examining students' work solving statistical problems using a dynamic statistical software tool. They found that PSMTs could describe students' actions with the tool but found little evidence of the PSMTs connecting the students' actions with the tool to the students' mathematical thinking. Overall the PSMTs drew on their own mathematical content knowledge to interpret the students' thinking, which often hindered their ability to unpack the students' understanding. More recently, Chandler (2017) compared PSMTs' noticing students' mathematical thinking on geometry tasks presented in two different mediums: written work and tool-mediated work (the tool in use was The Geometer's Sketchpad). She found both groups of PSMTs focused their noticing on the students' mathematical thinking, but struggled to provide evidence to back up their interpretations. This work aligns with that of Jacobs et al. (2010) who found the presence of a tool hindered some prospective teachers' noticing of the students' mathematical thinking. Many prospective teachers focused on pedagogical aspects related to tool-use, but did not connect explicitly to the students' understanding. These results suggest a need for the research community to explicitly ask teachers to consider the role technology tools play in developing mathematical understandings.

CONCEPTUAL FRAMEWORK

Our conceptual framework for teacher noticing of students' work in a technology-mediated environment is shown in Figure 1 (Dick, McCulloch & Lovett, Under Review). While we acknowledge that all components of noticing are by their nature interrelated (Jacobs et al., 2010), we separate both attention to and interpretation of students' expressed (i.e., what they say and/or write) mathematical thinking from attention to and interpretation of the students' engagement with the technology as research indicates that PSMTs struggle to coordinate their understanding of students' work with technology and students' mathematical thinking when in technology-mediated environments (Lovett et al., 2019; Wilson et al., 2011). Going forward we refer to students' mathematical thinking in a technology-mediated environment as technological mathematical thinking.

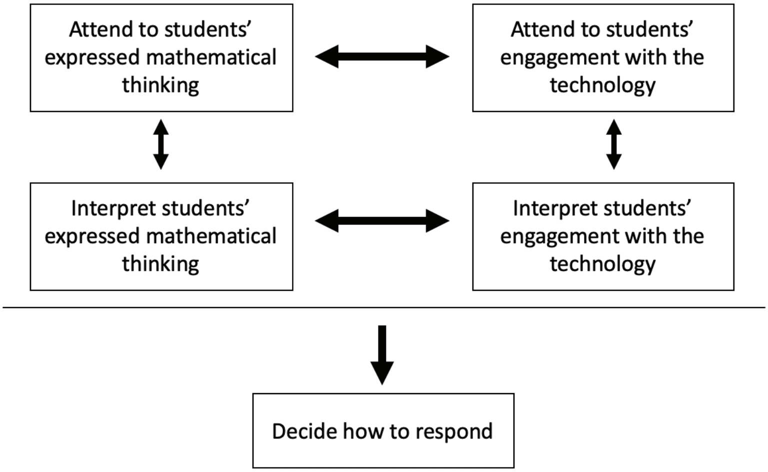


Figure 1. Professional noticing of student's work in a technology-mediated environment.

Our conceptualization involves both the horizontal coordination of attention and interpretation as well as the vertical integration of both attention and interpretation. We separated *decide how to respond* from the other components to balance the importance of focusing on both students' expressed mathematical thinking and technology-engagement prior to making instructional decisions; if one focuses on one more than the other, than the teacher may not be fully informed when making an instructional decision. In addition, when deciding how to respond to a student working in a technology-mediated environment, the teacher must consider how to position the technology (or not) in their response to support the student in moving their mathematical thinking forward. For this reason, deciding how to respond does not necessarily include students' engagement with the technology. Like Jacobs et al. (2010), we emphasize "that the ability to effectively integrate these three component skills is a necessary, but not sufficient, condition for responding on the basis of children's understandings" (p. 197). Hence integration of the three noticing components while coordinating attend and interpret is the goal of this complex teaching practice when in technology-mediated environments.

Specifically, for this paper, we focus on the interpretation components of our framework and consider how predicting student thinking illuminates interpretations. Lee (2013) discusses predictive ability related to professional noticing; she explained that "conjecturing how students might think in a similar context based on existing information about the students, is a key ability for investigating students' mathematical thinking" (p. 1403). Thus, we believe having PSMTs predict students' technological mathematical thinking after viewing the students' work will provide insight into their interpretations. For this study, we consider PSMTs' coordinated predictions (i.e., PSMTs' understanding of both the students' mathematical work and their engagement with the technology) of what students might do next on a similar mathematical task. Thus, the research question we seek to answer is, how do PSMTs' predictions coordinate students' expressed mathematical thinking and engagement with technology?

METHODOLOGY

As part of a larger cross institutional study, PSMTs were asked to complete a professional noticing assignment to attend and interpret students' mathematical technological thinking as expressed in a technology-mediated task (Dick, McCulloch & Lovett, Under Review). Specifically, for this paper, we explore PSMTs' professional noticing through investigating their predictions of students' mathematical thinking in a technology-mediated learning environment to understand more about the PSMTs' coordinated interpretations. The details of this study are described in the following sections.

Context of the study

The focus of this study is using predictions to gain further insight into PSMTs' coordinated interpretations of students' mathematical thinking within a technology-mediated environment. The specific technological context is a vending machine applet that was designed by some of the authors to develop the essential understandings of the concept of function, in particular functions apply to a wide range of situations and their domain and range do not have to be numbers (Cooney, Beckman, & Lloyd, 2010). The vending machine applet, *Middle School Vending Machine,* (https://www.geogebra.org/m/wcuPt43b), utilizes the metaphor of a vending machine and asks the user to classify each machine as a function or non-function (Figure 2). The machines each consist of four buttons – Red Cola, Diet Blue, Silver Mist, and Green Dew (i.e., inputs). When a button is pressed it produces none, one, or more than one of the four different colored cans – red, blue, silver, and green (i.e., outputs). For a full description of the applet see Lovett et al. (2020).

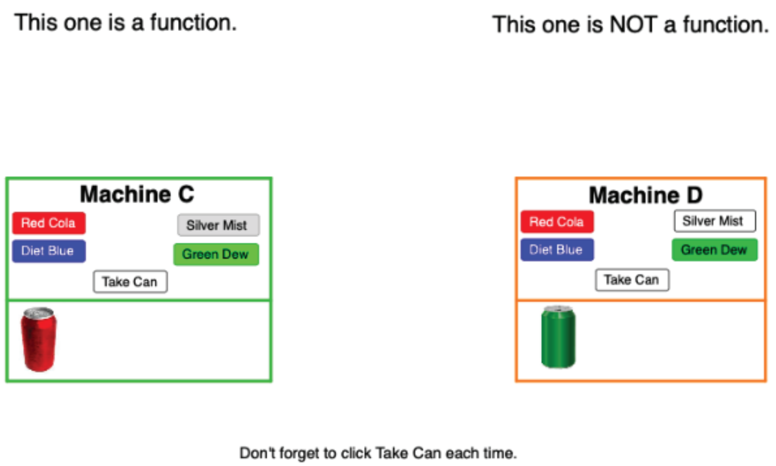


Figure 2. Middle School Vending Machine applet.

The ways in which students engage with the applet provides insight into their expressed understanding of function. For example, to test whether or not a machine could be a function you must click on each button (input) multiple times to see the possible cans (output) that might come from that action. If a student clicks each button exactly once and declares a machine to be a function or not a function, there is evidence, simply from the engagement with the technology, that the student does not understand that a key feature of a function is the consistency of the relationship between an input and its output. In the context of this technology, the ways in which students engage with the tool is an important aspect of their approach to the task.

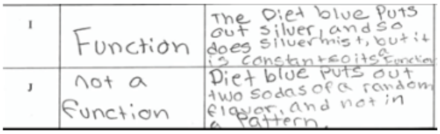
During this task, the PSMTs first engaged with the vending machine applet as a learner and then participated in a class discussion about function as represented in the applet. Next, PSMTs' watched a video of a pair of students, Emma and Calvin (ages 12-14), engaging with two vending machines I and J (Figure 3) in which Machine I is a function since it is consistent even though two buttons output a silver can and in which Machine J is not a function since it is not consistent because it outputs a blue and a random soda can when the Diet Blue button is pressed. This video clip was chosen using the project's design principles (Lovett et al., In Press).

Which One is a Function			
I	Red Cola → red Diet Blue → silver Silver Mist → silver Green Dew → green	J	Red Cola → red Diet Blue → blue & random Silver Mist → silver Green Dew → green
Which One is a Function			
K	Red Cola → random pair Diet Blue → blue Silver Mist → silver Green Dew → green	L	Red Cola → green Diet Blue → green Silver Mist → green Green Dew → green
Are these functions?			
M	Red Cola → red Diet Blue → red Silver Mist → silver Green Dew → silver	N	Red Cola → 2 silver Diet Blue → green Silver Mist → red Green Dew → blue

Figure 3. Configuration of vending machines I through N.

In the video clip (https://youtu.be/AsOItXV6754), the two students justified that Machine J is a non-function because Diet Blue was not consistent and did not discuss that Diet Blue produced two cans. See Figure 4 for the transcript. For the noticing assignment, PSMTs answered three questions with the first two focused on attending to and interpreting the students' engagement with the machines and their thinking about the function concept (See Dick, McCulloch & Lovett, Under Review). For the third question, which is the focus of this study, the PSMTs synthesized their previous analysis to make predictions about how the students would respond to the additional machines (K, L, M, & N). PSMTs were asked to provide a written justification for the remaining four machines that coordinated both how students will engage with the applet and how their understanding of function will lead them to this conclusion (Figure 5).

Emma: Okay. Then do I.
Calvin: I coming right up. Yeah we are good.
Calvin: Well… It is still a function.
Emma: It's still a function.
Calvin: It just doesn't show any blue. That's the only difference.
Calvin: Red.
Calvin: Huh. For Diet Blue…
Emma: No, no, no, no, no!
Calvin: No it pops up two blue. Look! Yeah it pops up two cans.
Emma: Two random cans.
Calvin: Yeah random. Look.
Emma: (Inaudible)
Calvin: I just clicked it once.
Calvin: I think J is not a function.
Emma: Yeah J is not a function.



I	Function	The Diet blue Puts out silver, and so does silvermist, but it is constant so its a function
J	not a Function	Diet blue Puts out two sodas of a random flavor, and not in a pattern.

Figure 4. Transcript of the video clip of Emma and Calvin's work on Machines I and J.

Machine	Prediction (Function/Non-Function)	Justification (Include **BOTH** a discussion of how the students will engage with the applet **AND** how their understanding of function will lead them to this conclusion)
K		
L		
M		
N		

Figure 5. Excerpt from noticing assignment worksheet.

Participants

PSMTs from three public universities in the U.S. participated in this study. All 37 of the PSMTs were enrolled in a secondary mathematics education methods course that focused on developing mathematical knowledge for teaching secondary school mathematics. In each course, PSMTs engaged with written and video artifacts of students' mathematical technological thinking throughout the course.

Data Corpus

The PSMTs' written predictions and accompanying justifications, henceforth referred to as predictions, on the noticing assignment was the sole source of data for this study. Specifically, PSMTs were asked to predict the students' determination of four additional machines as function or non-function and to justify their predictions by discussing both the students' predicted engagement with the machines as well as their expressed understanding of function. Since each PSMT made predictions for four machines, 148 predictions with justification statements were analyzed.

Data Analysis

Data analysis consisted of two components. First, we developed predetermined codes based on our conceptual framework: we coded for PSMT prediction justifications that included only function understanding, included only engagement with the applet, included both but without explicit coordination, and full coordination of the two (See Table A). Using a process as described by DeCuir-Gunby, Marshall, and McCulloch (2011), three researchers worked to reach reliability of code application. After reaching reliability, all data was coded individually by the three researchers with any discrepancies in code application discussed by the team until consensus was met. Next, the three researchers examined the predictions coded as robustly coordinated to search for emergent themes (Creswell, 2013).

Table A

Predetermined Codes and Examples

Predetermined Codes	**Description**	**Example**
Predictions focused only on students' expressed mathematical thinking	Prediction included only a discussion of function understanding with no mention of engagement.	Although each can leads to the same output, the students would recognize that although each color can is not represented in the outputs, the outputs are consistently patterned. So they would deem this a function—as long as they don't get hung up on the fact that each output is the same and leaves other colors of can out. (Machine L, PSMT 5)
Predictions focused only on students' engagement with the technology	Prediction included only a discussion of engagement with no mention of function understanding.	Did not occur.
Minimally coordinated predictions	Prediction included discussion of both function understanding and engagement, but the two were not explicitly coordinated.	The students will interact with it the same way—press each choice a few times and see what each button spits out. I think they will be stumped about the two silver mists given for Red Cola, but eventually talk it out about the fact that the output is still constant. (Machine N, PSMT 37)
Robustly coordinated predictions	Prediction included a coordinated discussion of both function understanding and engagement.	They will conclude that this machine is a function. They will start as usual hitting the red button several times and notice they get a green soda each time. Then they will hit the blue button and notice that they get a green soda each time as well. They will do the same for the silver and green buttons. Although they may be confused that they are getting the same soda I think they understand that a function is "constant" and that fits their definition of a function. (Machine L, PSMT 15)

RESULTS

In what follows we first report the results to the research question. We first share how the PSMTs coordinated their predictions of the students' technological mathematical thinking. As a reminder, full coordination of students' technological mathematical thinking involves consideration of students' expressed mathematical thinking, students' engagement with the technology, and their relationship to each other. Following the results of how the PSMTs' coordinated their predictions, we share the ways the PSMTs articulated their coordinated interpretations of the students' technological mathematical thinking within their predictions.

PSMTs' Coordination of students' technological mathematical thinking

Through our analysis we examined PSMTs' predictions for their coordination of students' technological mathematical thinking. PSMTs predictions could include: predictions focused only on students' expressed mathematical thinking; predictions focused only on technological engagement and not on students' mathematical thinking; and predictions that involved coordination of the two. Our results show instances of PSMTs whose predictions focused only on students' expressed mathematical thinking, however there were no instances of PSMTs' predictions that focused only on technological engagement. In considering PSMTs' coordinated predictions, we identified two levels; minimally coordinated and robustly coordinated predictions. In the following subsections we expand on each of these types of predictions and provide exemplars from the data.

Predictions focused only on students' expressed mathematical thinking. All 37 PSMTs' predictions included a discussion of the students' expressed understanding of function in all four of their predictions. However, 16 PSMTs (43%) never described students' engagement with the applet in any of their predictions. These PSMTs' predictions only included discussions of the students' understanding of function without regard to the ways in which students' interactions with the technology resulted in this understanding. For example, on Machine L, PSMT 17 predicted that the students "will consider this machine as a function." The PSMT went on to explain:

> From the video, they seem to understand that as long as the input gives a consistent output, it is a function. They knew that Machine I was still a function even though no blue cans were included, so I think they will use the same reasoning to conclude this is a function.

This justification for the prediction in no way refers to the students' engagement with the Vending Machine Applet, but did discuss essential understandings of the concept of function.

Minimally Coordinated Predictions. There were only four PSMTs whose predictions always included references to both the students' engagement and the students' understanding of function but did not explicitly discuss the coordination of the two components. For example, for Machine K, PSMT 21 predicted,

> The students will press each button several times and clear the output after each time. The students will say that K is not a function, because the red button gives different answers when pressed several times. They might also think that having an output with multiple cans is indicative of the machine not being a function, but we don't know that for sure.

In this prediction, PSMT 21 discussed how the students would press the buttons and clear the cans, but their prediction did not include a discussion of the reason for the students' engagement, meaning, they did not explicitly discuss what the students were looking for when clicking. Similarly, for Machine L PSMT 11 predicted,

> This machine may trick these students due to the conspicuous nature of all buttons giving one green can. However, Machine I gave silver cans for two buttons and the students agreed that it was still a function. The group's procedure will remain the same, 3-4 clicks per button, and they will likely discuss the unusualness of all the buttons giving a green can. If they identify the consistency of this machine, they should determine that it is a function.

In this prediction, PSMT 11 discussed how they anticipated the students would press the buttons on Machine L and compare the outputs to another machine. However, like PSMT 21, they did not explicitly discuss the way in which the students might make sense of their actions to determine function or non-function. While the PSMT mentioned the idea of consistency of the machine, the PSMT did not explicitly connect the students' use of the technology (clicks) to a search for machine consistency.

Robustly Coordinated Predictions. The remaining 17 PSMTs provided predictions that included a discussion of the students' engagement with the applet that was explicitly connected to their understanding of function for at least some of the machine predictions. Of these, five PSMTs discussed the students' engagement with explicit connections to the student's understanding for all four of their predictions. An example of an explicitly coordinated prediction for Machine M, for PSMT 19 follows:

> The students in group 1 will click on each button several times to see if their results are consistent. They will use the same reasoning as they did for Machine I and Machine L to figure out that this one is a function. They know that as long as the colored can given is the same every time, multiple buttons can give the same colored can. Since 2 buttons gave a red can and the other 2 buttons gave a silver can every time, it is a function.

In this prediction, PSMT 19 clearly connected the students' reasons for clicking each button to their understanding of function as consistent which the students' expressed an understanding of in the video with machines I and J. The PSMTs whose predictions included coordination explicitly connected the engagement of the students with the applet to their essential understandings of function such as consistency or randomness.

Some PSMTs' coordinated predictions were less explicit but their predictions still included both a discussion of the engagement and how the students' engagement related to the understanding of function. For example, for Machine K, PSMT 10

> Students will click on each can multiple times. The first time they click on the red can they will not see anything wrong with it even though there are two cans, but the second time they click on it the two cans will be different than the first time and they will say it is not a function.

In this prediction, PSMT 10 included a play-by-play imagining how the students would think and respond to the Red Cola's different output. While not as explicit as the first example, this PSMTs' prediction connects the students' clicking to their understanding of non-functions as being inconsistent in their outputs.

Components of PSMTs' robustly coordinated predictions

As we worked to better understand characteristics of the predictions coded as robustly coordinated, two themes emerged: 1) predicting cognitive dissonance; and 2) predicting how the students would interact with each other to determine whether or not the machines represented functions. These characteristics provided insight into the PSMTs' interpretations of the students' technological mathematical thinking in ways substantially different than predictions that did not include these components.

Predicting cognitive dissonance. Some of the PSMTs provided predictions that included information about how they anticipated the students would respond to particular machines based on the students' developing understanding of function. In doing so, the PSMTs tended to predict whether or not the students would experience moments of cognitive dissonance. Such predictions were articulated through expressions of anticipated difficulties (or lack thereof). When discussing an anticipated lack of difficulty, some PSMTs predicted anticipated time the students would spend on the machine using the terms like "quickly" in their predictions. Similarly, other PSMTs, such as PSMT 10, who for Machine L, predicted the students would not experience any cognitive dissonance stating,

> I think they will catch this right away. They have enough of an understanding to know that it just has to give them the same soda every time no matter what the color is. They'll know it's not just random because they check over and over.

PSMT 10 predicted the students' understanding of function was robust enough to make sense of Machine L without difficulty.

While there were few such examples, the majority of the PSMTs whose predictions alluded to some type of cognitive dissonance did so in relation to anticipated difficulties for machines. They used terms such as "confused", "struggle", "freak out," "trip up," or "thrown off" to indicate how they predicted the students would develop their understanding of function while engaging with the applet. For example, for Machine K, PSMT 5 predicted,

> Calvin and Emma would recognize, as they did in machine J, that the red can produces two cans of a random color, and this disqualifies it from being a function. They may, however, struggle with their definition of 'consistent'—is it consistent because each output is two cans of the same color, or not?

In this example, PSMT 5 clearly considered how the students' understanding of consistency of function related to the anticipated cognitive dissonance due to the output of Machine K. Similarly, PSMT 30 predicted a "play-by-play" for the students' interactions with Machine K and anticipated where in their engagement would cause the cognitive dissonance,

> Group 1 will conclude that this machine is not a function. They will start by clicking on the red button several times and will see that they get a random pair. Since it is random and not consistent, they will conclude that this machine is not a function. Then they will click the other buttons and notice that they get the same each time for both buttons. But since they get a "random" pair for red, they will conclude the machine is not a function. They may be confused by the 2 cans but since it is random, they will easily decide it is not a function.

In this example, the PSMT first predicted confusion, but also indicated that after questioning the output, the students would easily decide the machine is not a function, thus the PSMT predicted both confusion and simplicity.

Predicting student pair interactions. Another characteristic of the PSMTs' predictions that provided insight into their articulated interpretations of the students' technological mathematical thinking was how PSMTs' predicted discussions or interactions between the student pair. Some PSMTs provided general descriptions of predicted student discussions such as PSMT 29, who for Machine L, predicted, "Students will debate the fact that all outputs are the same, but will settle on it being a function due to the outputs being constant in relation to inputs." For Machine K, PSMT 11's prediction is another example,

> This machine should lead to a discussion of the permissibility of two can outputs, if that conversation hasn't already taken place. This is the first machine that gives a two can output while also being a function. As Calvin was wary of two can outputs in the video, but Emma seemed to disagree, this would be a good time to confirm their definition of a function.

Here, PSMT 11 predicted that the students will have a discussion and also predicted the cognitive dissonance that may arise about the two silver cans.

Other PSMTs were more explicit and included full descriptions of the students' predicted conversations. PSMT 8 provided a hypothetical script for each of their four predictions using the students' names from the video clips. Their prediction for Machine N follows:

> Emma- So each seems consistent.
> Calvin- But what about the two cans?
> Emma- They're not different colors.
> Calvin- So would that mean they are still one?
> Emma- If they were different colors, they would have different options.
> Calvin- So having the same color can but just 2 instead of 1, still applies to consistency?
> Emma- Of course. This is a function.
> Written Conclusion: Each time we press an option, the output is consistent. So, even though there are two cans with the same color, they are **still the same colored cans every time** [emphasis in original].

Within the script, we see the PSMT's thoughts regarding how the students would interact with each other and what the students would write on their worksheet. Other PSMTs' predictions with scripts connected the students' engagement with the machines. For example, PSMT 7 provided predictions such as the one below for Machine L for all four predictions.

> This machine shows each can having a green output. I believe the students will talk through this machine as follows:
>
> Calvin begins clicking the red can, which has a consistent green output. He doesn't say anything, and clicks the blue can, showing a green can as the output. Emma says, "Both green, but not random!" She clicks silver, and green which also produce a green can. She says, "look green every time, hmm, I think it's a function." Calvin, "yes because it is not random, it's always green." Emma says, "yes, it can be green for all of them. This is a pattern!" Calvin adds, "it doesn't show red, blue, or silver, but it is constant, so it's a function." With Machine I, they used this reasoning to explain the reason of it being a function, so they will explain this machine being a function in a similar manner.

In PSMT 7's prediction we see a clear coordination between the students' understanding of function and the students' engagement with the applet. We also gain insight into the PSMTs' prediction of cognitive dissonance which goes beyond "hmm, this might be confusing" to delve into interpreting how Calvin and Emma are thinking and learning both as individuals approaching the task as well as predicting how they will interact with each other as they engage with Machine L.

DISCUSSION

When working in technology-mediated environments, students' thinking is expressed both with written and spoken language as well as within students' engagement with the technology. Thus, teacher noticing of students' mathematical thinking when in technological environments involves more than when students are simply working on a pen and paper task, when in a technological environment, full noticing requires a coordination of students' expressed mathematical thinking and their engagement with the technology. In Dick, McCulloch & Lovett (Under Review), we found that PSMTs struggled with coordinating their attention and interpretation of the students' technological mathematical thinking. However, similar to Bannister et al. (2018), we theorize that asking PSMTs to predict students' thinking after analyzing their work, could act as a scaffold for their teacher noticing. In what follows, we answer the research question regarding PSMTs' coordinated predictions and discuss the implications of these findings for teacher educators.

PSMTs' coordinated predictions

We examined PSMTs' predictions of what students might do next within the technological environment based on the PSMTs' understanding of the students' technological mathematical work. We found that many PSMTs' predictions provided evidence of both the students' function understanding and engagement with the applet, but full coordination of these two components was difficult for the PSMTs and was not as prevalent. These results are similar to our previous study examining the PSMTs' attend and interpretation responses written prior to predicting students' future work, that only a few PSMTs fully coordinated their interpret responses (Dick, McCulloch & Lovett, Under Review). These results suggest that PSMTs need more exposure to noticing students' mathematical technological thinking.

PSMTs' robustly coordinated predictions

In framing this study, we posited that if PSMTs' robustly coordinated their predictions, the predictions would provide insight into the PSMTs' interpretation of the students' technological mathematical thinking differently from just asking PSMTs to answer a traditional interpretation written prompt (e.g. Interpret the students' mathematical thinking). The findings indicate that when asked to predict students' technological mathematical thinking, PSMTs who robustly coordinated often provided predictions that considered what aspects of the technological task were or were not likely to provoke cognitive dissonance, and discussed how students might interact with each other and with the technology.

The finding that the PSMTs' coordinated predictions often included discussions of aspects of the technological task that provoked cognitive dissonance aligns with Krupa et al. (2015) who found that PSMTs often focused on student weaknesses when noticing. However, our results also show the PSMTs focused on student strengths, or what was "smart" in the ways the students' might interact with the technological task which Bannister et al. (2018) noted as an important component of noticing. The presence of both predictions of student misconceptions and student smartness within the PSMTs' coordinated predictions are important additions to the predictions that provide further insight into the PSMTs' overall interpretations of the students' technological mathematical thinking. The fact that some PSMTs focused on student smartness related to the students' use of the technology may be attributed to their own interactions with the technology as a learner, and perhaps their class discussion of the applet, prior to predicting student thinking. The relationship between PSMTs own experiences with technology and what they notice in student work has not been studied; the findings here suggest such work would be beneficial.

In addition, the results also show the prevalence of coordinated predictions that included interactions between the students and the technology. This finding is similar to Amador and colleagues (2016), who found elementary preservice teachers provided more details of their noticing students' mathematical thinking when using an animation platform. While we did not ask the PSMTs to provide scripts within an animation, the PSMTs who thought to provide a script prediction, included greater details and insight into their overall interpretations of the students' technological mathematical thinking. Like Amador et al. (2016), we found that the PSMT predictions "transformed the typical practice of written noticing" and provided specific evidence of how the PSMTs were thinking about the students' understanding of the mathematics, the students' engagement with the technology and the students' dialogue" (p. 146).

LIMITATIONS AND FUTURE WORK

Asking PSMTs to make predictions about students' mathematical technological work did provide insight to their interpretations of students' thinking. However, due to the nature of the noticing prompts, the methods employed in this study did not include a direct comparison of the PSMTs' written predictions to their written interpretations (a prompt simply asking to interpret students' thinking in technology-mediated environments). In the future, we intend to design a noticing task within a different technological environment that would allow us to compare PSMTs' predictions to their interpretations to determine whether or not they elicit similar or different results. In addition, research shows preservice and practicing teachers often struggle with the deciding how to respond component of noticing (Gupta et al., 2018; Jacobs et al., 2010). We wonder if asking PSMTs to first make predictions might support the development of skills related to instructional decisions. A comparison study focused on the deciding how to respond component of noticing with and without predictions would shed light on this open question.

IMPLICATIONS FOR TEACHER EDUCATORS

Noticing students' technological mathematical thinking is a difficult skill for PSMTs due to the complexity of coordinating both mathematical understanding and engagement with technology (Lovett et al., 2019). Despite its difficulty, research shows that PSMTs can improve their noticing skills, including their interpretations of students' mathematical thinking with scaffolded practice and feedback from mathematics teacher educators (Bannister et al., 2017; Jacobs & Spangler, 2017). Our intention in this study was to use predictions to scaffold PSMTs noticing within a technology mediated environment. Results show that asking PSMTs to predict students' technological mathematical thinking elicits aspects of their interpretations, namely predicting cognitive dissonance, and student interactions with the technology and with each other. As such, we suggest mathematics teacher educators add prediction prompts to their repertoire for noticing tasks. Specifically, we suggest asking for PSMTs to express their interpretation of students' technological mathematical thinking, as well as asking them to predict cognitive dissonance and student interactions with the technology and with their peers. For even the most novice of PSMTs, asking them to predict what students will do on a similar task, provides an opportunity for them to both utilize and express their interpretations of students' technological mathematical thinking, and thus provides a scaffold for learning to articulate their noticing.

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