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Analysis of Statically Indeterminate Trusses for Progressive Collapse Using Graphic Statics and Complexity Metrics

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Analysis of Statically Indeterminate Trusses for Progressive Collapse using Graphic Statics and Complexity Metrics

by

Maximilian E. Ororbia

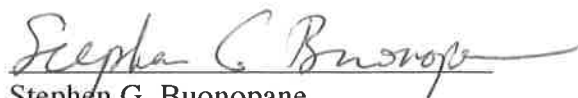
A Thesis

Presented to the Honors Council

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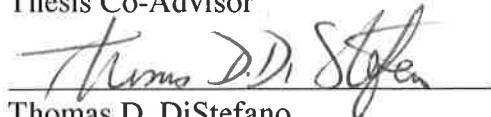
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ABSTRACT

By understanding structural response, structures can be designed to have a desired reliability or robustness to prevent the effects of progressive collapse. With respect to a statically determinate truss structure, if a single primary member or a gusset plate connection of the main truss fails, the truss will immediately collapse over its entire span. To design against progressive collapse, a degree of static indeterminacy is introduced in the structural system. Graphic statics, a geometrical approach used to analyze the form and force interaction of a determinate structure, has not been applied to statically indeterminate structures. By applying graphic statics, a truss can be graphically analyzed through the preparation of accurate and to-scale form and force diagrams. An analysis using graphic statics results in a single diagram, force polygon, that clearly presents the total load paths within a structure. In addition, the force polygon represents all the internal forces acting within the truss. Therefore, this research will use graphic statics as a tool to understand the total load path of indeterminate truss structures and will make the redistribution of forces in a redundant structure subject to damage or member removal more apparent. Furthermore, the research includes a comparative analysis by employing structural complexity. Statically indeterminate truss systems of varying topologies were analyzed by employing both progressive damage and parametric analyses. These analyses account for the local behavior of a truss structure subjected to damage and the overall efficacy of a truss' capability to redistribute load through alternative paths.

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CHAPTER 1: INTRODUCTION

1.1 Thesis Statement

Graphic statics can be applied to the progressive collapse of statically indeterminate trusses to assess member and load path contribution. Overall system performance and the redistribution of forces can be visualized through force polygons, which can be related to deformation work and complexity metrics.

1.2 Research Purpose

Presently, graphic statics, a geometrical approach used to analyze the form and force interaction of a determinate structure, has not been applied to indeterminate structures. Recent use of graphic statics has been used to optimize structural design and as a preliminary form-finding design tool. By understanding structural response, structures can be designed to have a desired reliability or robustness with respect to progressive collapse. Events such as the Minneapolis I-35W Bridge collapse (August 2007) and the Washington I-5 Skagit River Bridge collapse (May 2013) illustrate that progressive collapse is a relevant topic for structural engineering research practice.

Previously published works have analyzed progressive collapse in regards to indeterminate trusses. There also exists research using graphic statics as a means to optimize structures. However, no studies or methods exist that understand behavior or design of a statically indeterminate truss using graphic statics. Therefore, this research will provide tools for a structural engineer to use in understanding the total load path of indeterminate truss structures and will make the redistribution of forces in a redundant structure subject to damage or member removal more apparent. Moreover, the graphical analysis explored through this work will contribute to ongoing research investigating

reliability and complexity design methods used to quantify member contributions of a whole system.

Progressive collapse is the disproportionate failure of a structure initiated by the failure of a single structural component, which then causes the failure to adjoining elements. Structural redundancy is a structure's capability to redistribute loads among its members in order to resist the propagation of failure related to progressive collapse. Redundancy is the existence of alternate load paths. In a truss structure, redundancy can be achieved by designing the truss to be statically indeterminate. A statically determinate truss structure is one in which all the internal member forces can be determined from equilibrium relationships alone. The number of unknown internal forces is equal to the number of equilibrium equations. If a single element is removed the whole truss structure will collapse. However, a statically indeterminate truss structure is one in which the number of unknown forces and reactions exceed the number of equilibrium equations. It necessary to use compatibility equations to solve for the unknowns. Multiple load paths exist within a statically indeterminate truss structure.

CHAPTER 2: BACKGROUND

2.1 “Jenga!”

The intent of the research presented in this thesis is to understand the relationship between form and forces of redundant structural systems as they undergo progressive collapse through member removal or damage. In order to understand the form and force interactions of systems the following methods were investigated: graphic statics and complexity metrics. The concepts of progressive collapse can be understood by considering the well-known Hasbro game of Jenga, created by Leslie Scott in the 1980s. Jenga is played with 54 wooden blocks which are arranged

in groupings of three that are stacked on top of each other to form a tower of 18 levels. Once the tower is constructed, players take turns in removing a single block and then placing the removed block on top of the tower. The player who removes the block that causes the tower to collapse loses. The tower at the beginning of the game is a redundant structure, has multiple load paths in which to transfer its self-weight to the table. If the blocks were not grouped, but instead placed directly on top of each other this would represent no redundancy. These two structural systems related to the Jenga tower are illustrated in Figure 1.

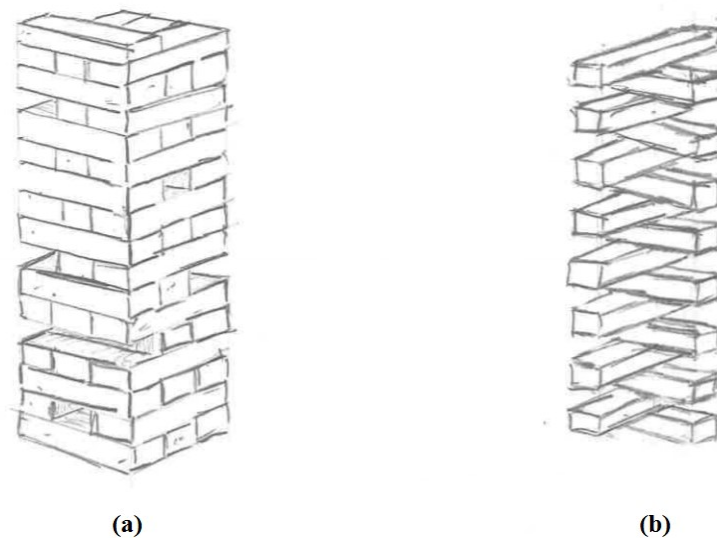


Figure 1. (a) Highly redundant Jenga tower (b) tower with no redundancy

The removal of any one block will not cause the tower to collapse. The block being placed on top of the tower allows for the path of the gravity loads on the tower to remain consistent. Blocks directly surrounding the removed block must carry the unchanged loads that the removed block once supported. The process of block removal is like the progressive collapse of a redundant structural system. As the redundancies are removed the structure must carry the load through alternative paths, through the blocks that were near the removed block. Once enough members, blocks, are removed the structure is unable to support the load and collapses completely. By

understanding the flow of forces through the structure, it would be possible to choose members to remove that will allow the tower to remain standing for as long as possible. There exist several methods that allow the structural engineer (the player) to understand the load paths and keep the structure (the tower) standing even when members are being removed. Two analysis methods are known as graphic statics and complexity metrics.

2.2 Progressive Collapse

The progressive collapse of a structure can be initiated when a single structural element fails and causes the failure of adjoining structural elements. Internal forces stored within a member are redistributed to adjoining members when the initial member fails. Consequently, the other members will fail if the redistributed forces exceed their strength capacity, which will cause a sequence of local failures that will propagate throughout the structural system until its ultimate collapse [8]. Per the American Society of Civil Engineers [4], progressive collapse is defined as the “spread of an initial local failure from element to element, resulting eventually in the collapse of an entire structure or a disproportionately large part of it.”

One of the first notable incidents was the Ronan Point Apartment Tower collapse, London (May 1968), shown in Figure 2(a). A small gas explosion caused by a resident lighting a match in their 18th floor kitchen destroyed a load-bearing precast concrete wall panel, which then caused a chain reaction where 22 corner bay floors collapsed. After careful investigations, the collapse was in part attributed to the lack of alternative load paths [27]. Other instances of progressive collapse that caused much of the structural engineering community and public regulatory agencies to make a concerted effort to prevent this type of collapse are the Alfred P. Murrah Federal Office Building collapse (April 1995) and the World Trade Center twin towers (September 2001). The Murrah Building collapse, in Oklahoma City, was caused by a truck bomb damaging and destroying three

columns at the base of the building. In a sequential manner, several columns were damaged by the explosion, their loads were transferred to the transfer girder spanning over them, the girder became overloaded and failed, and together these failures caused the collapse of the structure above, Figure 2 (b). In the case of the Twin Towers in New York, a Boeing 767 jetliner struck each tower. The impact and intense fire caused the stories where the planes hit to lose their ability to support the loads above, which then caused the upper portion of the building to progressively collapse onto the stories below [26]. These collapses have increased the recognition and the need for verification procedures and design guidelines for progressive collapse resistance.

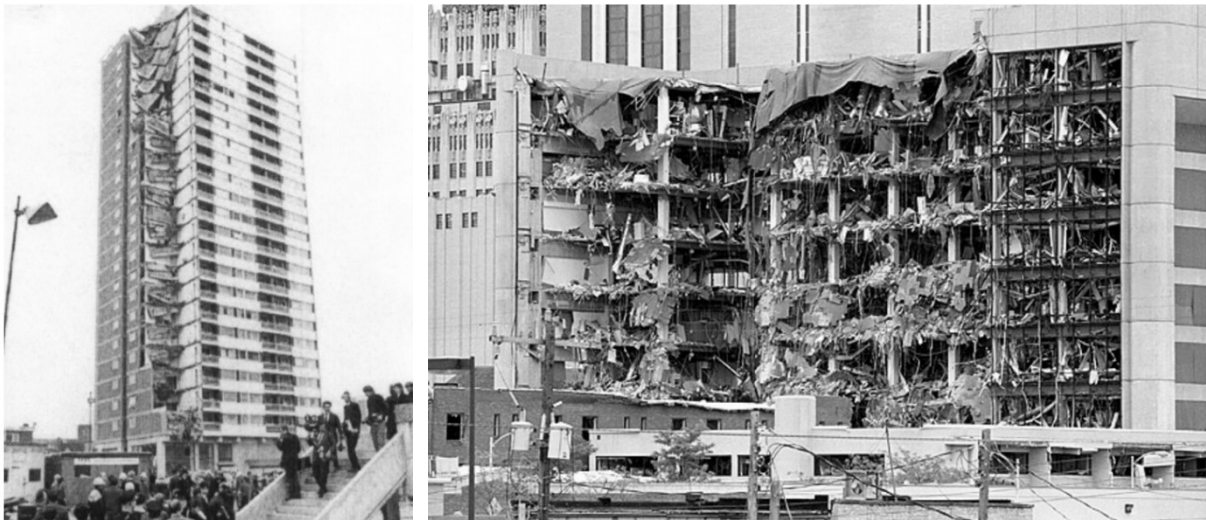


Figure 2. (a) Ronan Point collapse [27] (b) Oklahoma City bombing [10]

Modern design codes and standards used in practice are based either on deterministic or reliability based design procedures and strategies. In reliability based design actions and resistances of a structural system are determined through statistical analysis of empirical data which is dependent on the probabilistic occurrence of an extreme event occurring [29]. One example of a method used for designing against progressive collapse is the alternate load path method. The alternate load path method requires that a structure be designed such that if any single component is damaged or removed, alternate paths exist that will bear the load of the failed

member preventing a domino-effect failure [29]. The implementation of the alternate load path method is simplistic and direct, and is easily understood by practicing engineers [26]. Although, the alternate load path method requires an iterative and systematic analysis of varying damage scenarios. Alternate load paths may be difficult to visualize due to the large or unique topology of the structure and some load paths may be missed when analyzing the structure. Alternatively, by understanding the behavior of a structure as its internal loads are being redistributed, favorable load paths become apparent and the design process could become more efficient. Early in the design of a structural system, an engineer would be able to find the most effective topologies that allow for progressive collapse resistance. The research described herein explores possible tools that could be used in making deterministic design methods more efficient and effective.

2.3 Redundant Truss Structures

Relevant examples concerned directly with truss structures failing instantaneously through the loss of a single structural element can be seen in both the I-35W and I-5 Skagit truss bridge collapses. With regards to the I-35W bridge, the failure of a single gusset plate caused the collapse of the entire bridge, as seen in Figure 3(a). Although other factors played a role in the bridge's collapse, the leading cause was traced to the under-designed thickness of the gusset plate that led to further member failures, and ultimately the bridge's collapse [5]. A more recent occurrence that can be attributed to progressive failure is the I-5 Skagit River Bridge collapse, Figure 3(b). A semi-trailer truck hauling an oversized container struck and damaged a low hanging chord of a through-truss bridge causing further deformation and buckling failures of vertical members. Overloading of the critical upper chord members was initiated by the redistribution of the vertical members' loads which in turn resulted in the collapse of the entire truss span. As explained by Astaneh-Asl

[5], steel deck truss bridges, commonly statically determinate systems, can progressively collapse over the entire span, if a single primary member or a gusset plate connection of the main truss fails.



Figure 3. (a) I-35 Minneapolis bridge collapse [24] (b) I-5 Skagit River Bridge Collapse [16]

Statically determinate truss structures are easily susceptible to progressive collapse because a single load path exists within the structure. Statically determinate structures have been referred to as last-resort or “weakest-link” structures [17]. Following the definition of progressive collapse, if a single member of a statically determinate truss is damaged or removed the truss becomes unstable, a mechanism forms. No alternative load paths exist to carry the load previously supported by the damaged or removed member, which causes the load to be transferred to adjoining members. The surrounding members will be required to carry the additional load and may ultimately reach their capacities thus causing a series of failures to occur until full collapse is achieved. Biondini et al. [8] suggest that the progressive collapse of structural systems can be avoided through the design of robust structures. Robust structures can resist damage that is proportionate to the cause of the initial damage. Key structural elements can be designed to be robust and resist the initial event that would cause collapse progression. Structural robustness is achieved by designing independent structural components to resist potential causes of the initial failure [29].

Another structural design method used to prevent progressive collapse is redundancy. Although the terms robustness and redundancy are commonly used synonymously, their meanings are different. The notable difference between the terms is related to how they define the system to deal with damage. As defined by Biondini et al. [8], “...structural robustness can be viewed as the ability of the system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself. Structural redundancy can instead be defined as the ability of the system to redistribute among its members the load which can no longer be sustained by some other damaged members.” Through redundancy, alternative load paths are created which prevent a large redistribution of forces to transfer to a single member or through a primary load path. Biondini et al. [8] describes redundancy as a system's ability to redistribute loads from other damaged members that can no longer support those forces. An additional method in designing alternate load paths is static indeterminacy. Although related, Frangopol and Curley [17] demonstrated that the degree of static indeterminacy is not always an accurate measure of structural redundancy.

Statically indeterminate structures, “fail-safe” structures [17], may have a level of redundancy that allows them to survive the failure of any of its individual components. The level of indeterminacy can be described through a metric called the degree of redundancy, n . Through the following equation, the degree of redundancy can be solved for

$$n = F - E \quad (\text{Eqn. 1})$$

where F is the number of unknown forces and E is the number of applicable equilibrium equations.

In a truss structure, the degree of static indeterminacy, n , can be calculated as follows

$$M + R - 2J = n \quad (\text{Eqn. 2})$$

where M is the number of axial force members, R is the number of supported degrees of freedom (i.e. reactions), and J is the number of nodal points (i.e. joints). Each nodal point has two degrees

of freedom which is captured by multiplying the variable J by two. It should be noted that the degree of redundancy is independent of the applied loads. When a structure has a degree of redundancy greater than zero it can be considered a indeterminate system. If the degree of indeterminacy is equal to zero the structure is statically determinate, and if less than zero the structure is unstable. A redundant structural system is often safer structure due to its redundancy. Allen and Zalewski [2] remark, "Redundancy is desirable in circumstances where total structural failure is likely and would have dire consequences." The redundancy allows for the structure to form alternate load paths. Although the degree of indeterminacy is beneficial when designing against progressive collapse, the structural system becomes more complex to analyze. More equations are needed to analyze the structural system because there are not enough equations of equilibrium to solve the internal axial forces and reactions. Ultimately, adding structural redundancy can improve a structure's ability to resist collapse from damage or member loss, but increasing the number of members or changing material properties can be costly and difficult to achieve [28]. According to Murtha-Smith [25], even a statically indeterminate truss is susceptible to progressive collapse when a critical member is lost, and the structure is subject to its full-service load.

One way of quantifying a member's contribution to a structure's resistance to progressive collapse is through measuring the structural system's complexity. De Biagi [12, 13, 14, 15] has shown that structural complexity can be used to measure a variety of load paths in structures with high degrees of static indeterminacy. Through his research, De Biagi [15] has proposed a way of analyzing the behavior of statically indeterminate trusses that undergo the effects of progressive damage on random structural elements. By using structural complexity as the leading parameter in analyses, the efficacy of a truss structure's load paths can be quantified and compared the

system's overall response to damage. The analyses presented in De Biagi's works were studied for this research and are further explained in Section 3.3.

De Biagi [13] defines a complex structure to be one made up of many components that interact with each other in a non-simple way under an arbitrary loading scheme. A structure's complexity is related to its topology and how each member transfers loads throughout the overall system. By understanding the structural scheme, a structural engineer can identify load paths and the relative importance an element has in the overall system. For a statically indeterminate structure, if one member is removed the structural system will be unlikely to fail. The loss of a single member will instead be accommodated by the redistribution of the internal forces to other members throughout the structure. The contribution a member has in a structural system can be revealed by quantifying the effect its removal has on the axial forces of adjoining members or the displacement of the structure.

2.4 On the Development of Graphic Statics

Based on the research of Luigi Cremona (a 19th-century Italian mathematician) and James Clerk Maxwell (a 19th-century Scottish physicist), the Cremona-Maxwell method can be used to analyze and design two-dimensional trusses through graphical methods. Acknowledged as the father of graphic statics, the German engineer Karl Culmann published a comprehensive guide to graphical techniques in *Die Graphische Statik* [11]. Further developed by Wilhelm Ritter, Culmann's pupil, their work revealed how graphical methods could be applied to a variety of structural problems [3]. The Cremona-Maxwell method, now called graphic statics, derives a force diagram from a form diagram and externally applied forces [2]. The form and force diagrams are reciprocal figures, which are developed by drawing lines parallel and perpendicular to the lines forming the form or force diagram. Many notable structural engineers were influenced by and

applied graphical analysis methods to what are now considered as great structures of the modern era. Figure 4 presents several examples of famous works that were designed using graphic statics and demonstrates that graphical analysis methods can be applied to a variety of structural systems.

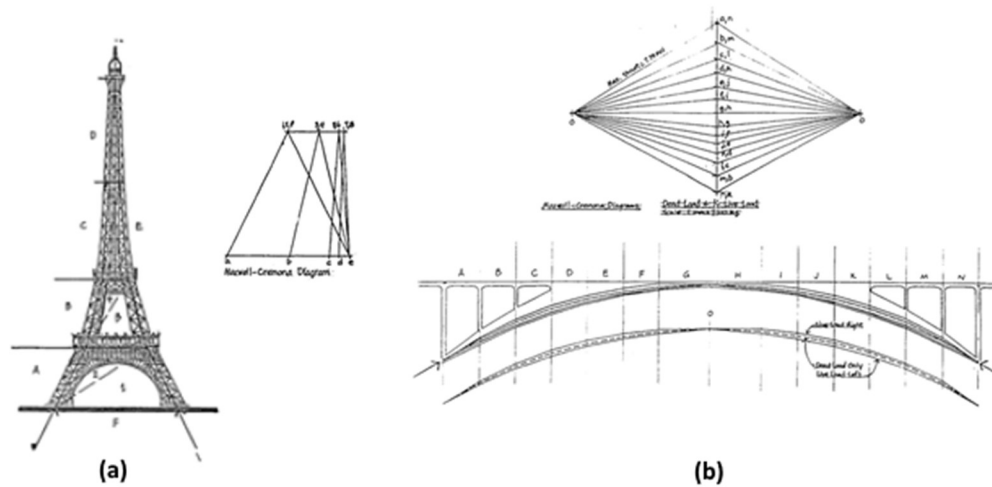


Figure 4. (a) Eiffel Tower by Gustave Eiffel and **(b)** Salginatobel Bridge by Robert Maillart [1]

After the development of numerical approaches in the 20th century, graphical methods became less popular. However, numerical methods inhibit structural engineers from gaining an intuitive understanding of how a structure's form influences its forces and vice versa. There exists more reliance on concentrating narrowly on prescriptive design procedures such that the process of synthesizing structural form is being lost [1]. As explained by Allen [1] the application of graphic statics provides a way of visualizing and understanding trussed structures that is more revealing than common numerical methods. Currently, graphic statics is being used in the optimization of structures [7]. The geometry of structures can be optimized to create efficient systems that reduce material volume. Structural optimization can be achieved by applying principles of graphic statics thereby providing the structural engineer with insight into the distribution of forces in a structure. Additionally, by using design variables in the force domain,

an optimized structural form can be determined that ensures equilibrium as described by Beghini et al [7]. Van Mele and Block [30] explain that the combination of the form and force diagrams provides an intuitive evaluation of structural behavior, performance, and efficiency. By understanding the load paths of a structure, and how those paths change as the geometry is changed, graphic statics allows a structural engineer to find an optimal form that transfers load most efficiently. Graphic statics has influenced the shape of modern structures [2].

Truss structures can be graphically analyzed through the formation of an accurate and to-scale form and force diagrams. The form diagram of a truss is an accurate representation of its geometry, form, and length of each member. For a certain configuration of nodes and polygons that compose the form diagram, there exists reciprocal nodes and polygons that make up the forces developed in the structure called a force polygon [6]. An analysis using graphic statics results in a single diagram that clearly presents the total load path of a structure [6]. The force diagram represents all the internal forces acting within the truss. As stated by Beghini et al. [7], "For a given connectivity of nodes, graphic statics provides all of the information needed to determine the total load path of the structure." This insight has the possibility of aiding a structural engineer in designing a structure against progressive collapse by seeing the alternate load paths changes as a structure is damaged or loses members.

2.5 Structural Complexity Metrics

Structural complexity, as defined by De Biagi [13], is the measure of a structural system's various load paths under an arbitrary loading scheme. Complex structures are frequently related to the number of nodes and elements making up the system. Larger systems, composed of many elements and joints, tend to require more intensive calculations and their behavior is not easy to understand. A complex structure loses important aspects of its structural behavior as members are

damaged or removed [12]. It is possible to quantify structural complexity through two parameters: normalized structural complexity index and average deformation work [14].

The normalized structural complexity index, NSCI, quantifies the efficacy of the load paths that exist within a structural system. Individual load paths are analyzed through fundamental structures. De Biagi [12] defines a fundamental truss structure as a collection of nodes spanned by elements to form a statically determinate structure. A fundamental structure is a subset of a reference structure. The term reference structure is used to refer to the initial statically indeterminate structure. Structures that are neither the reference or fundamental structures will be called intermediate structures. Depending on the degree of static indeterminacy of the reference structure, intermediate structures may not exist or have an inherited degree of static indeterminacy less than the reference structure's degree. It is also notable that not all statically determinate structures are stable or fall within the definition fundamental structures. In Figure 5 (b), a mechanism is formed, which makes the structure unstable. With the applied load a “scissor-like” action would occur and the truss would fail. Furthermore, De Biagi [15] remarks that the elements in a fundamental structure must “span all the nodes.” For example, the truss structure in Figure 5 (c), the dashed member would be removed in a complexity analysis. Although the remaining structure would be stable, the node would be free thus violating the definition. The cases that either are unstable or violate the definition are not included in the analysis.

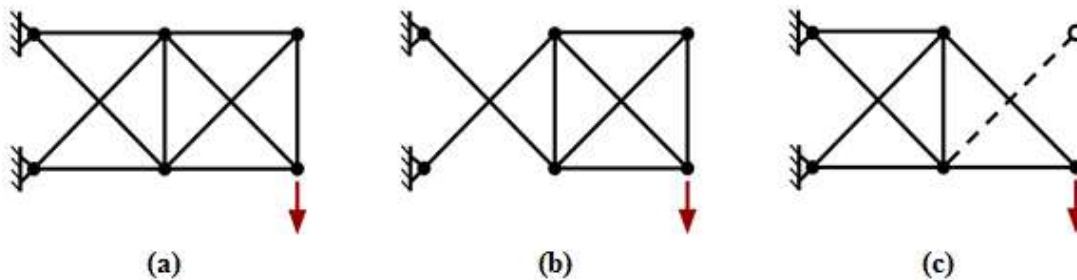


Figure 5. (a) Two-bay reference truss structure (b) Unstable fundamental truss structure (c) Truss structure that violates the definition of fundamental structures

In describing a structure's complexity, the NSCI parameters ranges from 0 to 1. A structure with a NSCI close to 0 is considered simple. In general, an NSCI near 0 means that there is a particular load path, "prevailing-resisting mechanism," that the structure relies on. If the NSCI is close to 1, the structure is referred to as complex. A complex structure is desired because many load paths contribute equally to the overall structural system behavior. De Biagi [15] summarizes, "the NSCI indirectly evaluates the uniformity in efficiency of the load paths across the structural scheme, an increase of complexity implies a better redistribution under damage." Damage can be applied to the structural system through member removal or by reducing the stiffness of the elements. For a truss system, a simple method for damaging truss members is by reducing their cross-sectional areas, which might occur in real-life due to corrosion.

To understand the behavior of a structure that has been damaged the average deformation work, M-value, can be computed. The M-value is a measure of the average response of a truss structure as a single arbitrary element is removed [15]. By relating both the NSCI and M-value parameters, De Biagi [15] showed that as the M-value decreases the NSCI tends to 1. Therefore, it is preferred that a structure have a relatively low M-value, which will result in a complex structure. In general, the two parameters measure robustness of a structural system through an evaluation of the increment of deformation work [15]. By implementing both parameters in an analysis, the efficacy of load paths within a structure and the response of the system to random damage or removal can be captured.

CHAPTER 3: METHODOLOGY

3.1 Application of Graphic Statics and Complexity Metrics

Graphic statics was applied to several statically indeterminate truss structures. In addition, the truss structures analyzed were subjected to damage through member removal, and their corresponding force polygons were drawn. The application of the method allowed for the internal forces of a statically indeterminate truss structures to be visualized as they undergo progressive collapse. Furthermore, the complexity analysis presented by De Biagi [15] for a statically indeterminate truss structure was replicated. Scripts and functions using MATLAB were written in order to create both De Biagi's analyses: progressive damage and parametric analyses. The analyses were then run on a truss model proposed and analyzed by De Biagi and the generated results were compared. Following the confirmation, the analysis methods were applied to two additional truss structures in order to provide further insight into the structural complexity parameters implemented. The truss structures analyzed with complexity metrics were also analyzed with graphic statics. The results of these two methods were then compared.

3.2 Graphic Statics

To find the internal forces of a statically determinate structural system several methods can be employed. In recent decades, numerical techniques are more commonly used as a solution method for finding internal forces in a truss structure. Typical numerical methods include method of joints and method of sections. Using equilibrium relationships, each joint of the truss is analyzed using method of joints. When applying method of sections, a theoretical cut is applied through the truss and the severed members are similarly analyzed using equilibrium relationships. These methods require the use of arithmetic, algebra, and basic trigonometry. Alternatively, by applying

graphical methods, the forces generated in a structure can be visualized through a force polygon. A summarized comparison between numerical and graphical methods is depicted in Figure 6.

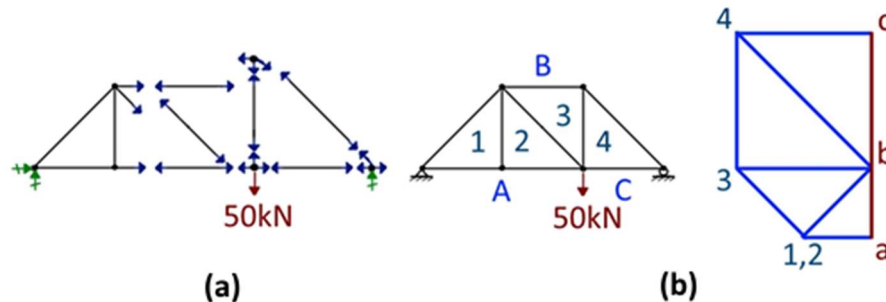


Figure 6. (a) Numerical methods of sections and joints compared to **(b)** a graphic statics analysis of a Pratt truss

Through the application of graphic statics, a statically determinate truss's internal forces can be determined without the use of mathematical computations. All that is required is several drafting instruments and a sharp pencil, which can be used to draft the reciprocal force polygon of a truss structure. Detailed procedures on drawing force polygons are described in the text *Shaping Structures* and *Form and Forces* by [1][3]. To draw the force polygon, the topology of the truss being analyzed should be drawn accurately and to scale. This scaled drawing is called the form or space diagram. Figure 6 (b) illustrates a summary of graphic statics applied to a simplified Pratt truss structure. The applied loading and truss's dimensions were selected arbitrarily. After all applied loads are added to the form diagram, the drawing is labeled per Bow's notation [9]. To label the form diagram according to Bow's notation (interval notation) capital letters are written between external forces and reactions. Capital letters are assigned to spaces in a clockwise manner around the truss; a particular starting point is not required. Numbers are assigned to the internal spaces formed by the truss members. The assignment of the letters and numbers can be used to name members and joints. Members are named by the letters and numbers adjacent to their location. Joint names consist of the series of letters and numbers that surround it written in order

by reading in a clockwise manner. For example in Figure 7, using Bow's notation the left most diagonal member is named {B-1} and the joint with the load directly applied to it is called {A-2-3-4-C}. For the remainder of this thesis, Bow's notation will be denoted by letters and numbers in braces. Bow's notation can also be applied to the force polygon. Lower case letters are used when labeling the force polygon.

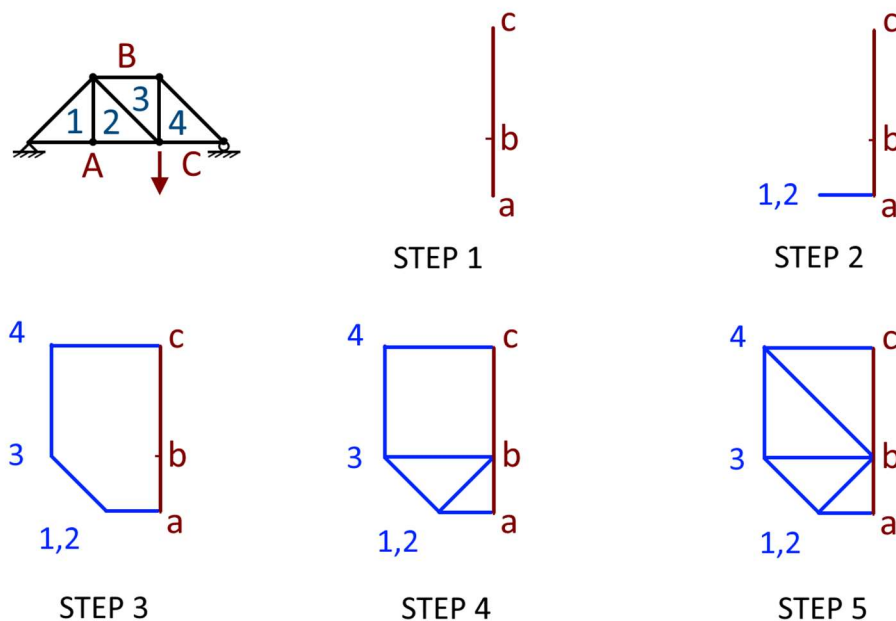


Figure 7. Force polygon construction for a simplified Pratt truss

With the form diagram drawn and fully labeled, the force polygon can be constructed. The force polygon is drawn to a convenient scale relative to the magnitude of applied and internal forces. The load line is drawn first, which consists of the externally applied loads and corresponding reactions. Lower case letters are assigned to the load line in accordance with the capital letters marking the exterior spaced. A process is subsequently followed where each nodal location is drawn on the load line in relation to form diagram. The internal forces are represented by lines drawn in parallel and directly transferred from the form diagram. Each transferred line that intersects with another line is labeled by a number on the force diagram. The magnitude of the

internal member forces can be found by measuring the force diagram directly. Each geometric shape constructed from a joint is a local force polygon. The formation of all the individual force polygons mapped to the load line form a global force polygon. Each line in the force polygon has a directional sense and can be used to understand the character of each member, if it is in compression or tension. The use of interval notation allows for the character to be determined. By reading the name of the joint in a clockwise sequence and concurrently following the path traced on the force polygon, the directional sense of the force with respect to the members is determined. If the force is moving away from the joint it is in tension, otherwise it is in compression. A structure's equilibrium can be confirmed if the last line segment on the force diagram connects with the appropriate point accurately forming closed polygons. Zalewski and Allen [1] explain that if the final line drawn does not reach its intended destination by 2 or 3% then the error is formed from an accumulation of drafting errors, otherwise a significant error has been made.

3.2.1 Force Polygons Relation to Progressive Collapse

Graphic statics is optimal for observing a structure's load path. In particular, the visualization of loads is beneficial to evaluate the form of a structure. As explained by Beghini et al. [7] reciprocal diagrams allow a structural engineer to determine the load paths that a structure exhibits at varying geometries and connectivity. The purpose of the research is to explore and analyze the effects progressive collapse has on the force diagram of a statically indeterminate truss structure. For a truss structure to experience progressive collapse, it must have some degree of redundancy. A statically indeterminate truss exhibits a level of redundancy through its additional members which create several load paths. A statically determinate truss only has a single load path, and therefore the removal of a single member will cause collapse.

An issue arises when applying graphic statics to an indeterminate truss structure. Since there are more members than equilibrium relationships, multiple load paths exist, and the internal forces cannot be determined graphically. Hudson and Squire [19] remark that it is not possible to draw a force polygon as there are more unknown forces found at a given joint in a truss with redundant members than there are equilibrium equations. The amount of axial force can vary depending on the stress relationship the members generate. The use of equilibrium relations cannot alone be used to determine the internal force distribution in an indeterminate truss. The application of constitutive and compatibility relationships is needed to solve for the internal forces. Although, if the axial forces are determined by other means, an accurate force polygon can be drawn illustrating the internal forces and the overall system equilibrium. Gerhardt [18] explains that graphic analyses can be presented as a means of illustrating analytically determined results. From this determination, the use of graphic statics can be applied to a statically indeterminate system for visualization purposes. To draw the force polygon of an indeterminate structure the internal forces must be determined through other methods, and since computer-aided analysis programs exist, the determination of the forces is not difficult.

In this research, a first-order elastic analysis was performed on all the indeterminate truss topologies investigated using OpenSees or MATLAB. OpenSees is open-source software that allows users to develop computational models and simulations of structural systems [22]. MATLAB is a programming language platform that allows users to compute, visualize, and analyze numerical information. MATLAB also enables the integration of other programs such as OpenSees [21]. OpenSees could be used to analyze, through non-linear analysis, the internal forces and reactions of a truss subjected to progressive collapse. Once analyzed, the information was used to draft the force polygon by hand or transferred for post-processing in MATLAB to create form

and force diagrams. Furthermore, when interested in understanding the effects of progressive damage, damage can be applied to truss members by reducing their member axial stiffness. For the application of graphic statics, the desired damage was achieved by removing a member. The general graphical analysis implemented is summarized by the flowchart presented in Figure 8.

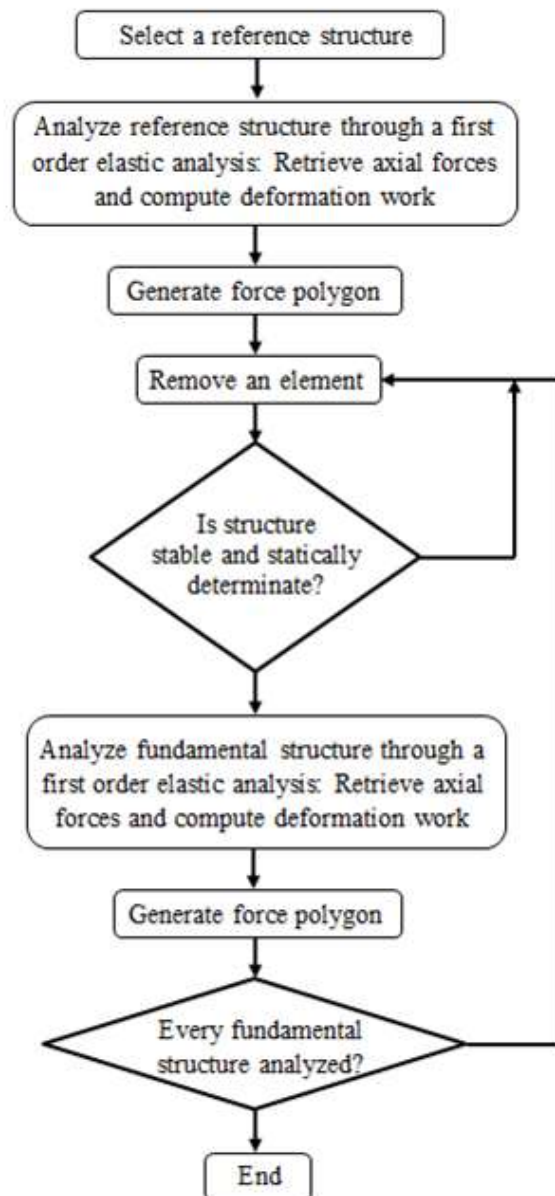


Figure 8. Flow diagram for analyzing the removal of elements in a structure through graphic statics

By applying computer programs, an automated method of producing force polygons for multiple trusses subject to a variety of loading schemes and damage or member removal is provided. Van Mele and Block [30] summarize that the combination of modern technologies and graphic statics can create drawings that enable the visualization of the relation between form and forces in response to manipulations of structural members. Through this visualization and relation between form and forces, a clear understanding of internal load paths and redistribution of forces through a variety of truss topologies can be investigated. Furthermore, force polygons can be generated for rather complicated truss topologies as seen in Figure 9.

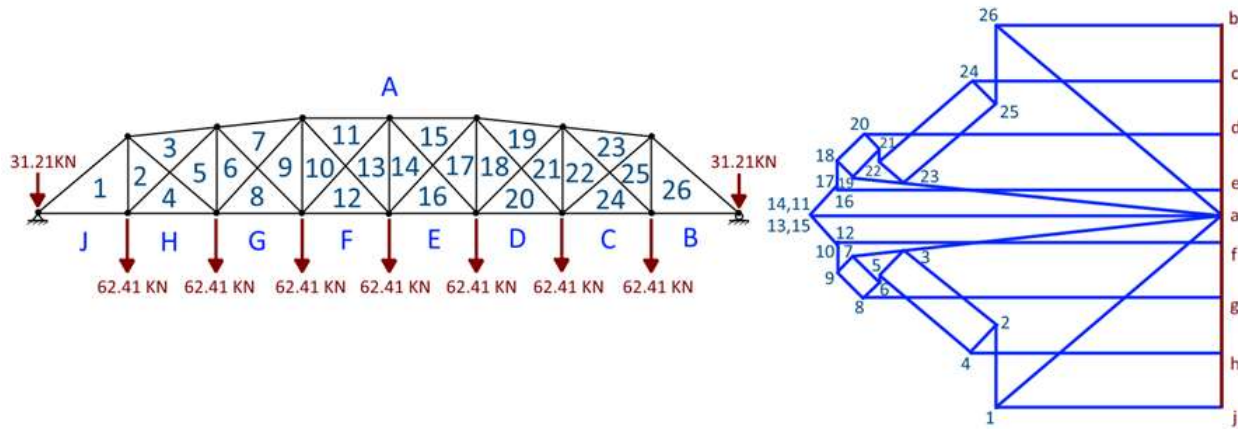


Figure 9. A through-truss steel bridge [23] analyzed to demonstrate the application of graphic statics to a relatively complicated indeterminate truss structure

Throughout this research, graphic statics was applied to several statically indeterminate truss structures to examine how their axial forces redistribute during progressive collapse. The force polygons presented in this thesis were all drafted by hand. Furthermore, the research presented in this thesis aided in the development of a MATLAB script that generates force polygons through topology input.

3.3 Complexity Metrics

A structure's complexity can be quantified through the NSCI parameter [12]. The NSCI is determined by normalizing the ratio between deformation work produced in a reference structure and the deformation work of all of its fundamental structures. The deformation work is equivalent to the work performed by externally applied forces. Deformation work, W , is computed through Clapeyron's Theorem [20]:

$$W = \frac{1}{2}(F)(\Delta) \quad (\text{Eqn. 3})$$

where W is the deformation work, F is the applied load, and Δ is the displacement at the applied load. The deformation work is the potential energy (total strain energy) developed by a linear elastic structure and can be quantified by the applied load and the displacement it produces. A truss structure's deformation work can be calculated through its total strain energy, U :

$$W = U = \sum_{i=1}^r \left(\frac{1}{2} \right) \left(\frac{P^2 L}{AE} \right) \quad (\text{Eqn. 4})$$

where r is the total number of elements that make up a truss and for each individual element P is the axial force developed, L is the length, A is the cross-sectional area, and E is the elastic modulus. The summation of each member's strain energy equals the deformation work produced by the system. The ratio between the deformation work of the reference structure and a single fundamental structure is called the performance ratio, ψ , and is computed as:

$$\psi = \frac{W_{ref.}}{W_{fund.}} \quad (\text{Eqn. 5})$$

where $W_{ref.}$ is the deformation work developed in the reference structure and $W_{fund.}$ is the deformation work developed by the fundamental structures [15]. The ratio describes the effectiveness of a fundamental structure's load path compared to the original statically indeterminate truss (reference structure). Each fundamental structure represents a possible load

path. The deformation work developed by the fundamental structure will always be larger than that of the reference structure. Intuitively, a fundamental structure produces a larger deformation work because it has fewer members to resist the applied loads. The performance ratio ψ ranges from 0 to 1. If the ratio is close to 1, the load path identified through the fundamental structure is considered to be effective. In contrast, the load path is not effective if the ratio is close to 0. If the performance ratio tends to 0, the deformation work produced by the fundamental structure is much greater than that of the reference structure.

In his research on structural complexity, De Biagi [12] discusses entropy and the application of graph theory to measure the behavior of a structure and identify load paths, respectively. The reader is referred to De Biagi [14] for an introduction to information entropy and graph theory. The relevant metric that came from De Biagi's study is the structural complexity index, SCI. By relating deformation work of all fundamental structures to the deformation work of the reference structure, a measure of a structure's complexity is obtained. The SCI is expressed as:

$$SCI = -\sum_{i=1}^n \left(\frac{\psi_i}{\sum_{j=1}^n \psi_j} \log \frac{\psi_i}{\sum_{j=1}^n \psi_j} \right) \quad (\text{Eqn. 6})$$

where n is the number of fundamental structures and ψ_i is the performance ratio of the i -th fundamental structure. The logarithm comes from entropy measures [15]. The deformation work determines the SCI value, but it is also affected by the number of nodes and elements making up the structure. Therefore, the SCI can be normalized in order to compare the complexities of different structures. The normalized structure complexity index, NSCI, can compare complexities of structures that have different areas and number of elements. In order to determine the NSCI, the SCI is normalized as follows:

$$NSCI = \frac{SCI}{\log n} \quad (\text{Eqn. 7})$$

where the $\log n$ represents the maximum possible value the SCI can obtain. The SCI's maximum value is representative of each fundamental structure's load path, having a performance ratio of exactly 1. Similar to the performance ratio, the NSCI parameter ranges between 0 and 1. A structural system is considered complex if the NSCI approaches 1. Otherwise, if the NSCI tends to 0 it is referred to as a simple structure. Although the effectiveness of a structure's load paths can be quantified by determining a structure's NSCI parameter, the NSCI does not account for damage.

To deal with damage, De Biagi [15] explored a structure's average deformation work, M-value. The M-value captures a structure's behavior as it is damaged. Like the NSCI parameter, the M-value is a function of deformation work, but it is related to the total number of elements that make up the structure's topology. Damage is applied to the truss structures by affecting the axial stiffness of its elements through:

$$K_{i,\xi} = K_o(1 - \xi) \quad (\text{Eqn. 8})$$

where $K_{i,\xi}$ is the axial stiffness of the i -th damaged element, K_o is the undamaged axial stiffness, and ξ is a damage parameter. The damage parameter, ξ , ranges from 0 to 1; where 0 means no damage is applied and a ξ of 1 represents full element removal. By dividing the increment of deformation work created from a single fundamental structure by the reference structure's deformation work the increment of elastic energy, Ω_i , in the i th element can be determined:

$$\Omega_i = \frac{W_{i,\xi} - W_o}{W_o} \quad (\text{Eqn. 9})$$

where $W_{i,\xi}$ the deformation work generated by the i -th element at a damage level ξ and W_o is the deformation work produced by the reference structure. The Ω_i metric captures the variation of deformation work as the i -th member is subjected to different levels of damage. The M-value

describes the average effect a single element has on a structural system if removed. By determining the deformation work produced by member removal, the M-value can be computed as:

$$M = \frac{1}{r} \sum_{i=1}^r \frac{W_{i,\xi=1} - W_0}{W_0} \quad (\text{Eqn. 10})$$

where r is the total number of elements. Large M-values are undesirable, which correspond to large deformations. The relationship between NSCI and the M-value can be determined through a parametric analysis. As demonstrated by De Biagi [15] larger M-values correspond to low NSCI values and vice versa. A robust structure will have a NSCI value close to 1 and small a M-value. This combination relates to multiple load paths being effective in keeping the displacement close to the reference structures. De Biagi [15] comments that as the NSCI parameter increases it is apparent that the impact of random damage has on a structure is less severe. To demonstrate this relation, De Biagi [15] conducted two analyses on a statically indeterminate truss structure: a progressive damage and parametric analysis. These analyses were replicated by writing scripts in MATLAB. The analyses quantifying load path efficacy and member contribution relate to the issues explored by this research and add numerical value to the graphical analysis investigated.

3.3.1 Progressive Damage Analysis

Through a progressive damage analysis the increment of deformation work, Ω , is compared with the amount of damage applied through the damage parameter, ξ . An element's contribution is apparent through the amount of deformation work it generates as damage is increased. If an element compared to the others has an increasing Ω value as the damage applied increases then that element can be viewed as having a greater contribution to the overall system. The MATLAB routine implemented to plot the increment of deformation work, Ω , with respect to the damage parameter is presented in the flowchart in Figure 10, see Appendix B for MATLAB code.

In addition to the increment of deformation work plotted with respect to the increasing damage parameter levels, the axial forces developed in each member can also be compared. The relevant cases are the axial forces in the undamaged structure elements, the maximum axial forces amongst all damage levels, and the minimum axial forces developed amongst all damage levels with respect to each individual element. The noted axial forces plotted with respect to the element arrangement allows for easy identification of where the highest and lowest axial forces are developed in relation to their location in the structural system.

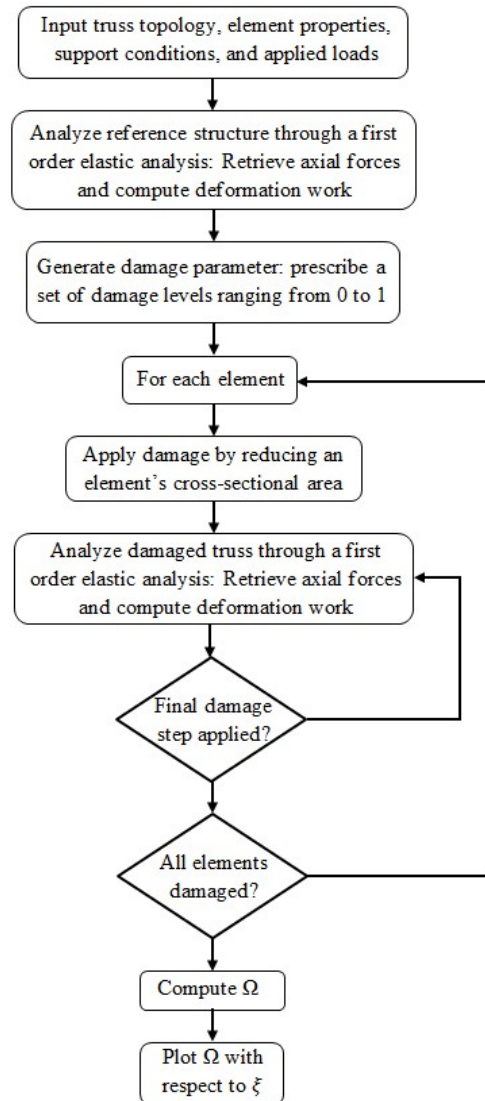


Figure 10. Flow diagram illustrating the progressive damage analysis MATLAB routine

3.3.2 Parametric Analysis

A parametric analysis results in the ability to assess a structure's capability to distribute loads (NSCI parameter) related to the behavior of the damaged structure (M-value). Embedded within the parametric analysis, structural stability needs to be considered for the analyzed fundamental structures. As noted previously, not all statically determinate structures fall within the definition of fundamental structures defined by De Biagi [15]. In order to remove unstable structures from the analysis an additional script was written in MATLAB that calculated the condition number of the truss structures stiffness matrix with respect to free degrees of freedom. The stiffness matrix for a truss structure is a collection of element stiffness coefficients that connote the force required to produce a unit deformation. The degrees of freedom related to element ends are a set of independent displacements that specify completely the displaced position of the system. The free degrees of freedom are those defining element ends that are not supported by any boundary conditions (i.e. pins or rollers). By combining these particular matrix elements of the stiffness matrix and calculating the condition number, stability can be determined. If the matrix is ill-conditioned then the structure is unstable, but if the matrix is well-conditioned then the structure is stable. Following a stability check, a list of stable fundamental structures can be created to be analyzed. The parametric analysis is illustrated in the flowchart shown in Figure 11, see Appendix C for written MATLAB code.

In the evaluation of NSCI and M, multiple structures were analyzed using the same truss topology but applying different areas to the elements. As recommended by De Biagi [13] the cross-sectional area of one element was made constant at $1 \times 10^{-3} \text{ m}^2$ for scaling the properties of structural complexity. The remaining elements were assigned random areas using the MATLAB

command “rand.” The random areas are independent, identically distributed with a uniform distribution between 0 and $2 \times 10^{-2} \text{ m}^2$.

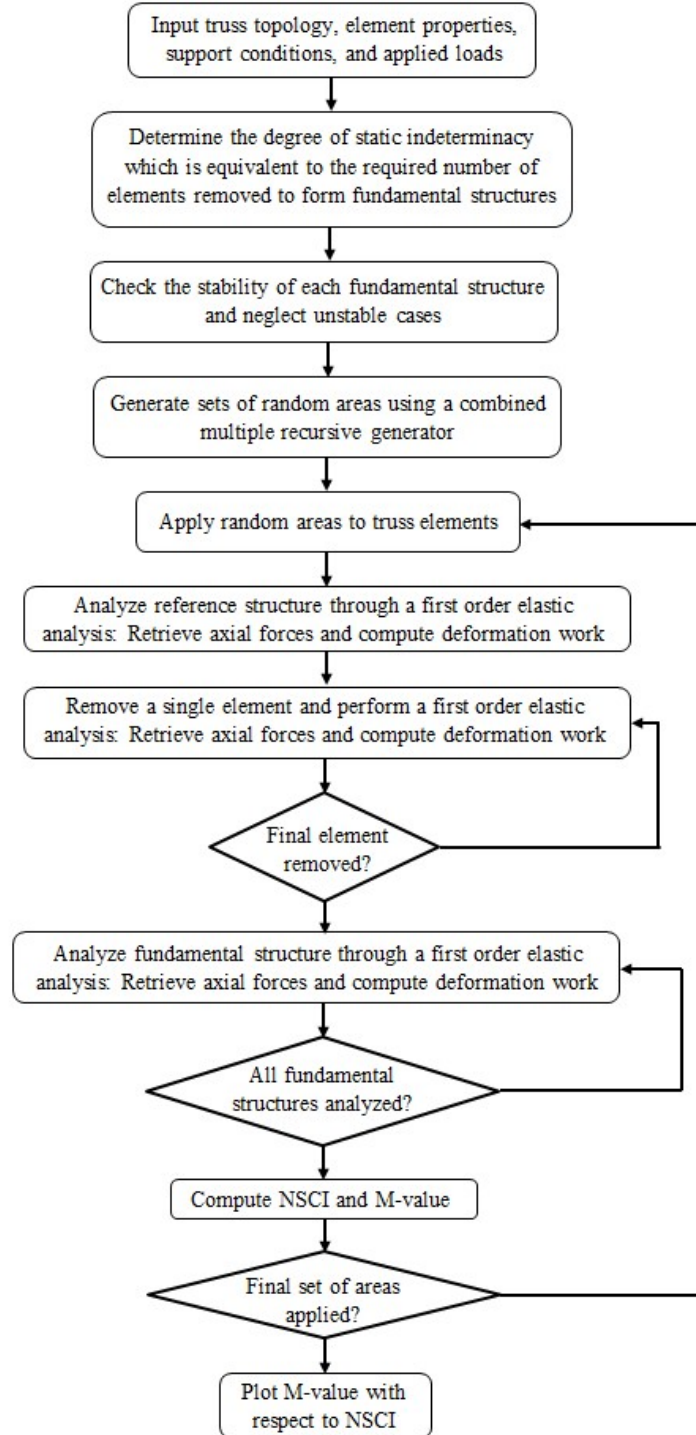


Figure 11. Flow diagram for parametric analysis MATLAB routine

3.4 Statically Indeterminate Truss Structures Analyzed

The structures investigated in this research are described in this section. Each structure possessed a reasonable amount of fundamental structures that allowed for their force polygons to be drawn by hand. Additionally, each structure analyzed had some degree of static indeterminacy. Although indeterminate, the truss topologies chosen were relatively simple in order to understand their behavior by visual inspection. In Figures 12 to 15 a brief description of the structures used is presented.

3.4.1 Pratt Truss

This truss was subjected to progressive collapse through member removal. The Pratt truss has one degree of static indeterminacy as determined by static indeterminacy equation described previously (where $M=10$, $R=3$, $J=6$). There are 10 elements with an arbitrarily selected 50 kN force applied to the node shown in Figure 12. The external supports are a pin and a roller. The internal nodes were modeled as joints, and all the nodal locations span a grid of 2 m spacing. Force polygons were drawn for the reference structures and each of its fundamental structures. For the reference structure, the forces were determined through a first-order elastic analysis using MATLAB. The subsequent force polygons drawn for the fundamental structures utilized the graphic statics analysis method. The resulting axial forces were confirmed with a first-order elastic analysis run in MATLAB.

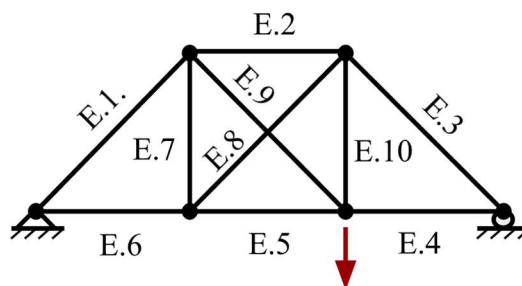


Figure 12. A simplified statically indeterminate Pratt truss

3.4.2 Three-bay Cantilever Truss

De Biagi [15] analyzed the relationship between the NSCI and M-value by applying both a progressive damage and parametric analysis on the statically indeterminate three-bay truss illustrated in Figure 13. The three-bay truss is three degrees statically indeterminate ($M=15$, $J=8$, $R=4$). The nodes span a grid of 2 m spacing, and the supports are represented by two pins. An arbitrarily selected 50 kN force was applied to the node shown in Figure 13. The elastic modulus of each element is 210 GPa and the reference cross-sectional area was set at $1 \times 10^{-3} \text{ m}^2$. Note that the reference area assigned to element 1 was held constant for scaling purposes. The area corresponds to an 88.9 x 3.6 EN 10210 structural hollow tubular section. The truss was analyzed by both the parametric and progressive damage analyses written in MATLAB. Results from both analyses were compared to those presented by De Biagi [15] to confirm the scripts' accuracy as discussed in Section 4.2.

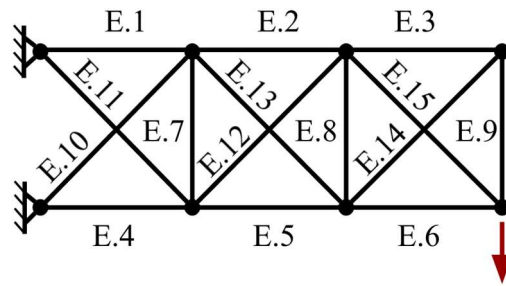


Figure 13. Statically indeterminate three-bay truss presented by De Biagi [15]

3.4.3 Single-Bay Cantilever Truss

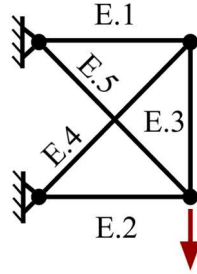


Figure 14. Statically indeterminate single-bay truss

Directly based on the cantilever truss shown in Figure 13, a single bay truss structure was analyzed with both MATLAB routines. The truss is one degree statically indeterminate ($M= 5$, $R= 4$, $J= 4$) and has 50 kN load applied at the node shown in Figure 14. The material properties, nodal spacing, and support conditions are the same as the three-bay cantilever truss. Furthermore, the same random areas were assigned to the elements for each analysis. The simplicity of the truss topology also allowed for force polygons to be drafted by hand. The force polygons were compared with the results obtained in the parametric and progressive damage analyses.

3.4.4 Two-Bay Cantilever Truss

The two-bay cantilever truss shown in Figure 15 is an extension of the single bay truss. This truss is two degrees statically indeterminate ($M= 10$, $R= 4$, $J= 6$). Element properties, support conditions, and nodal spacing remained the same. The 50 kN load is applied to the node as shown in Figure 15. Like the previous trusses, this two-bay truss was analyzed by the scripts written in MATLAB. Force polygons for the reference, intermediate, and fundamental structures were drawn and related to the parametric and progressive damage analyses.

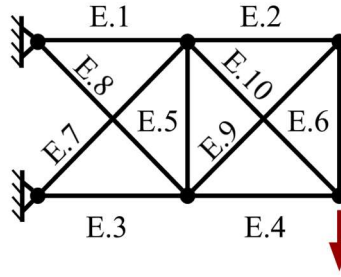
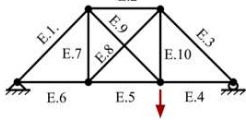
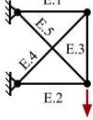
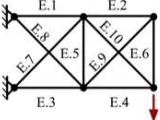
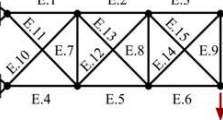


Figure 15. Statically indeterminate 2-bay truss

3.4.5 Degree of Static Indeterminacy and Fundamental Structures

A summary of each analyzed truss structure's degree of static indeterminacy, number of statically determinate structures created from the initial indeterminate system, and the total number of fundamental structures is presented in Table 1. The change in the amount of fundamental structures does not correspond the degree of static indeterminacy of the reference structure. If the geometry of the truss is changed and the degree of static indeterminacy is kept the same, the amount of fundamental structures can change as seen with the Pratt and single-bay trusses. Furthermore, if the degree of static indeterminacy is increased the amount of fundamental structures will increase. As the degree of static indeterminacy increases the number of fundamental structures increases at a greater rate, emphasizing the need for efficient computational approaches.

Table 1. Statically indeterminate truss structure characteristics and the amount of statically determinate and fundamental structures corresponding to each reference structure

Truss Structure				
Degree of Static Indeterminacy	1	1	2	3
Number of Statically Determinate Structures	10	5	45	455
Number of Fundamental Structures	6	5	29	169

CHAPTER 4: RESULTS

4.1 Pratt Truss

As described in Figure 12, a simplified Pratt truss was examined using graphic statics as shown in Figure 16. The force polygon adjacent to the reference structure in Figure 16(a) was drawn using forces determined through a first-order elastic analysis. Since the force polygon resulted in a closed shape, the truss, under the applied load, is in equilibrium. In addition, fundamental truss structures and their corresponding force polygons were drawn and are shown in Figure 16(b) to (g). Note that the reaction forces at the supports are drawn in red while the element axial forces are drawn in blue. These colors will be used consistently throughout the remainder of the thesis to denote the different types of structural forces.

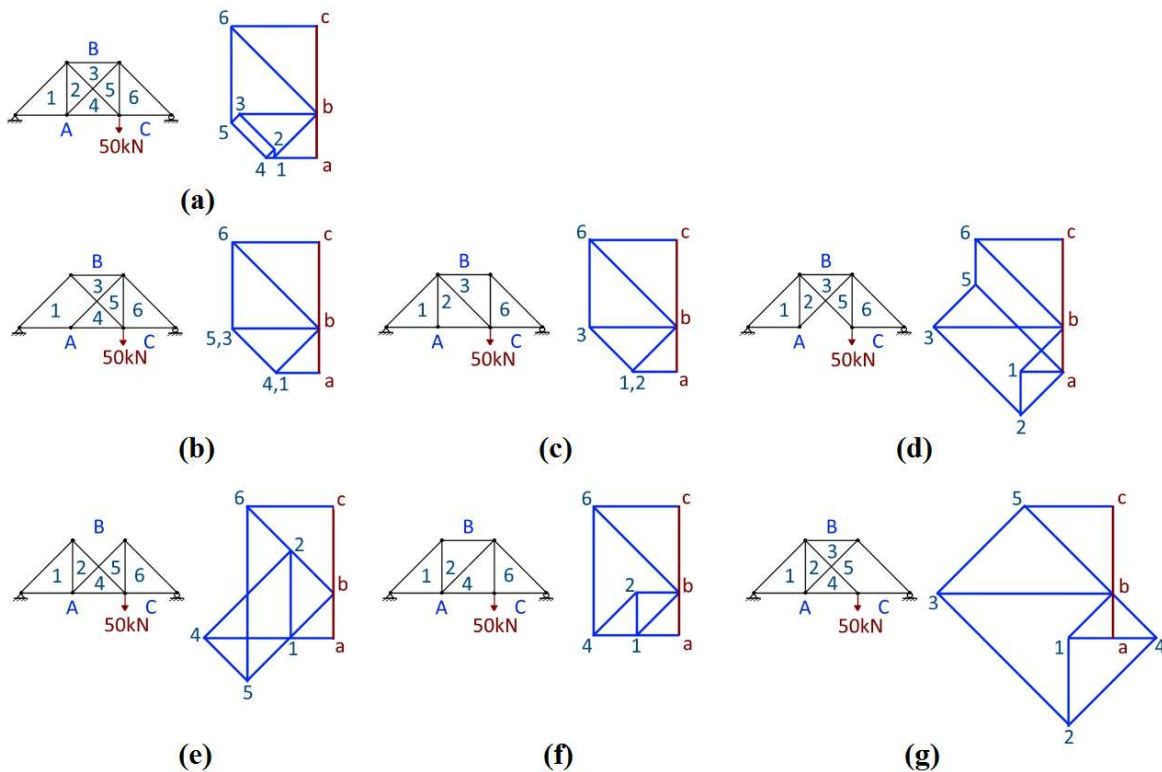


Figure 16. Collection of the (a) reference and (b)-(g) fundamental structures along with their corresponding force polygons of a Pratt truss

It is notable that the force polygons of the fundamental structures vary from the reference structure's force polygon. Although the size and shape of the fundamental force polygons change, they are still closed, which shows that equilibrium is still satisfied. If another element were removed from the fundamental structure, the whole truss would collapse. Therefore, the topologies and the corresponding force polygons presented in Figure 16(b) to (g) can be considered as “last-resort” structures. Through visual inspection, force polygons in Figure 16(b), (c), and (f) are similar in “size” to the force polygon in Figure 16(a). Force polygons in Figure 16(b) and (c) have the exact same shape and size. The reason both force polygons are equivalent is due to the generation of zero force members. Zero force members are elements that do not carry any load.

By not changing significantly, it is shown that the removal of elements {1-2}, {2-4}, and {2-3} does not drastically change the distribution of axial forces within the structure. In addition, the character of the members, whether the members are in tension or compression, remains the same, apart from a few elements becoming zero force members. In Figure 16(d), (e), and (g) the force polygons notably change in shape and size. These changes correspond to a large force redistribution produced from the removal elements {A-4}, {B-3}, or {5-C}. Additionally, several members change from being in tension to compression, and no zero force members are present.

The sizeable force polygon variations in Figure 16(d), (e), and (g) show that the elements removed contribute more to the structural system than the elements removed in Figure 16(b), (c), and (f). The removal of elements in Figure 16(b), (c), and (f) barely changed the reference structure's force polygon. The remaining elements were capable of carrying the load that the removed members held without a significant increase to their own axial forces. These fundamental structures are efficient load paths. With respect to Figure 16(d), (e), and (g), the variation in their force polygons expresses that the members removed carried a significant amount of the load

applied and removing them caused a significant redistribution of forces to adjoining members. Such a redistribution could result in a failure of the adjoining elements leading to progressive collapse. In particular, the force polygon in Figure 16(g) is the largest of all the fundamental structures' force polygons. The removal of element {5-6} corresponds to the force polygon in Figure 16(g). Hence, element {5-6} plays a significant role in supporting the applied force. Intuitively, the element's significance can be understood since the vertically downward load is applied directly at the node connected to that element. By understanding the flow of forces through the structure, the downward load would be resisted in large part by the vertical member. Without that member, the other members must compensate and support more load than they originally were intended to. In considering designing against progressive collapse, a structural engineer would want to design the members that cause the largest force polygon variation to be more robust so as not to fail. By increasing the member's capacity, the failure load paths are forced to those members that create the least amount of variation in the force polygons, and thus the lowest redistribution of forces.

To verify that the observations made from the force polygons, the strain energy of each element and total deformation work were calculated. The strain energies and deformation work for the reference and each fundamental structure are presented in Table 2. The element numbers presented correspond to those shown in Figure 12.

Table 2. Strain energies and deformation work determined from the Pratt truss

Strain Energy (kN-m)		<i>Fundamental Structures (element removed)</i>					
<i>Bow's Notation</i>		{B-3}	{A-4}	{1-2}	{4-2} or {3-5}	{2-3} or {5-4}	{5-6}
<i>Element</i>	<i>Reference Structure</i>	E.2	E.5	E.7	E.8	E.9	E.10
E.1	1.35E-02	1.35E-02	1.35E-02	1.35E-02	1.35E-02	1.35E-02	1.35E-02
E.2	1.57E-02	--	4.31E-02	1.92E-02	1.92E-02	4.79E-03	7.66E-02
E.3	5.42E-02	5.42E-02	5.42E-02	5.42E-02	5.42E-02	5.42E-02	5.42E-02
E.4	1.92E-02	1.92E-02	1.92E-02	1.92E-02	1.92E-02	1.92E-02	1.92E-02
E.5	6.77E-03	4.31E-02	--	4.79E-03	4.79E-03	1.92E-02	4.79E-03
E.6	4.79E-03	4.79E-03	4.79E-03	4.79E-03	4.79E-03	4.79E-03	4.79E-03
E.7	1.72E-04	1.92E-02	4.79E-03	--	0	4.79E-03	1.92E-02
E.8	4.86E-04	5.42E-02	1.35E-02	0	--	1.35E-02	5.42E-02
E.9	8.90E-03	1.35E-02	5.42E-02	1.35E-02	1.35E-02	--	1.22E-01
E.10	2.30E-02	7.66E-02	4.79E-03	1.92E-02	1.92E-02	4.31E-02	--
Deformation Work (kN-m)		<i>Fundamental Structures (element removed)</i>					
<i>Reference Structure</i>		E.2	E.5	E.7	E.8	E.9	E.10
0.147		0.298	0.212	0.148	0.148	0.177	0.368
<i>Figure 16</i>	(a)	(e)	(d)	(b)	(c)	(f)	(g)

From the strain energies, it is shown that the removal of element {1-2} or {4-2} results in the same values. Thus, relating that the force polygons in Figure 16(b) and (c) to the deformation work produced with the removal of elements seven or eight. The removal of these two elements results in the same load path. Removal of element {5-6} results in the largest deformation work. The large deformation work is undesirable because it corresponds to a large displacement. Removal of element {5-6} was also noted to cause the largest variation in the force polygons. Therefore, if a force polygon significantly varies from its reference structure's force polygon, then it can be associated with generating the largest deformation work. The information conveyed through an examination of the force polygons can aid a structural engineer in quickly understanding the behavior of a structure and the relative importance each member has in supporting the applied forces.

4.2 Verification of De Biagi Analyses

In order to verify the complexity analysis code developed for this thesis, the analysis was applied to the same 3-bay statically indeterminate truss, Figure 13, analyzed by De Biagi [15]. The

complexity analysis included both the progressive damage and parametric analyses. The resulting graphs, Figures 17 to 19, were compared to those presented by De Biagi [15]. In general, the plots either matched or had minor variations that were produced from small changes in the coded analysis routine.

The maximum and minimum axial forces generated through the application of damage on one element are presented in Figure 17.

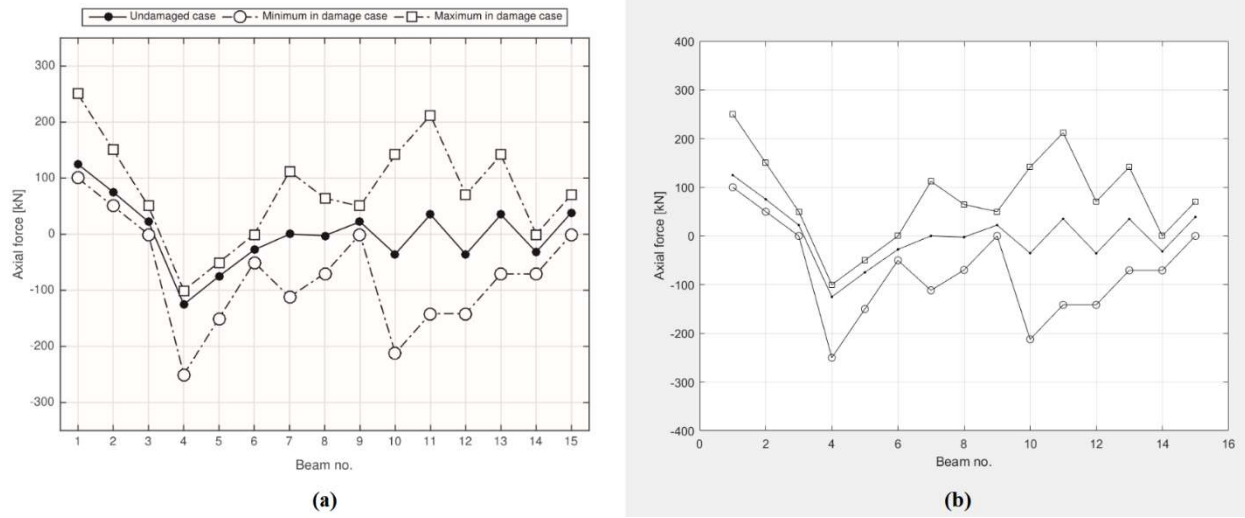


Figure 17. Comparison of axial forces in truss elements pre- and post- damage from (a) De Biagi [15] results and (b) MATLAB script results

The results generated from the MATLAB script written for this thesis agree with those produced by De Biagi [15]. Like De Biagi, Figure 17(b) was generated using the progressive damage analysis.

Furthermore, the increment of deformation work was plotted against the damage parameter using the progressive damage analysis, Figure 18.

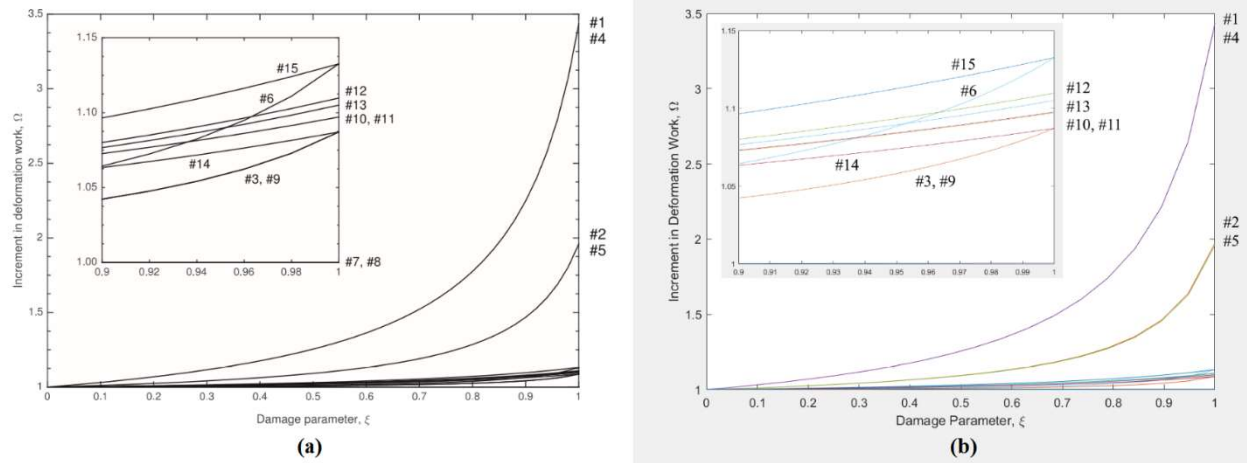


Figure 18. Comparison of the increment of deformation work for progressive damage from **(a)** De Biagi [15] results and **(b)** MATLAB script results

Both plots consistently illustrate that the increment of deformation work, Ω , increases non-linearly as the damage parameter, ξ , increases. As damage is applied to elements 1, 4, 2, and 5 increases, the amount of deformation work progressively increases. The remaining elements do not generate significant amounts of deformation work. Damage to elements 7 and 8 result in no deformation work. The plots compared in Figures 17 and 18 were generated using the progressive damage analysis.

In Figure 19, the plots compared were generated using the parametric analysis.

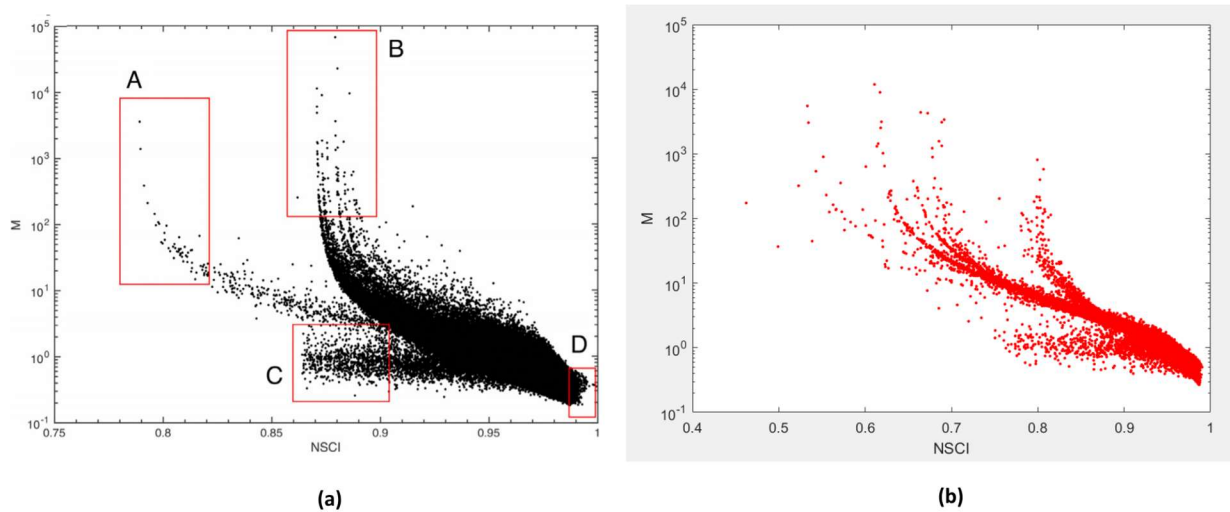


Figure 19. Comparison of parametric analyses from **(a)** De Biagi [15] results and **(b)** MATLAB script results

De Biagi analyzed 2×10^5 different sample sets of random element areas, while the analysis in Figure 19(b) only analyzed 1×10^5 different samples. The distribution and trends of the results presented in Figure 19(b) approximates those generated by De Biagi [15] in Figure 19(a). For example, both plots contain asymptotic tails at low NSCI values and high M-values. The analysis performed does not reproduce the exact numerical values of De Biagi [15]. The reasons for the differences are still being investigated. Many of the conclusions drawn from the parametric analysis are based on the relative magnitudes of the NSCI and M-value.

4.3 Single-Bay Cantilever Truss

The truss presented in Figure 14 was analyzed for progressive collapse using both graphic statics and the complexity analyses. The reference structure's force polygon, Figure 20(a), demonstrates how each member contributes in supporting the applied load. By removing a single member, two possible load paths are created. The force polygons associated with each fundamental structure is displayed in Figure 20(b) and (c).

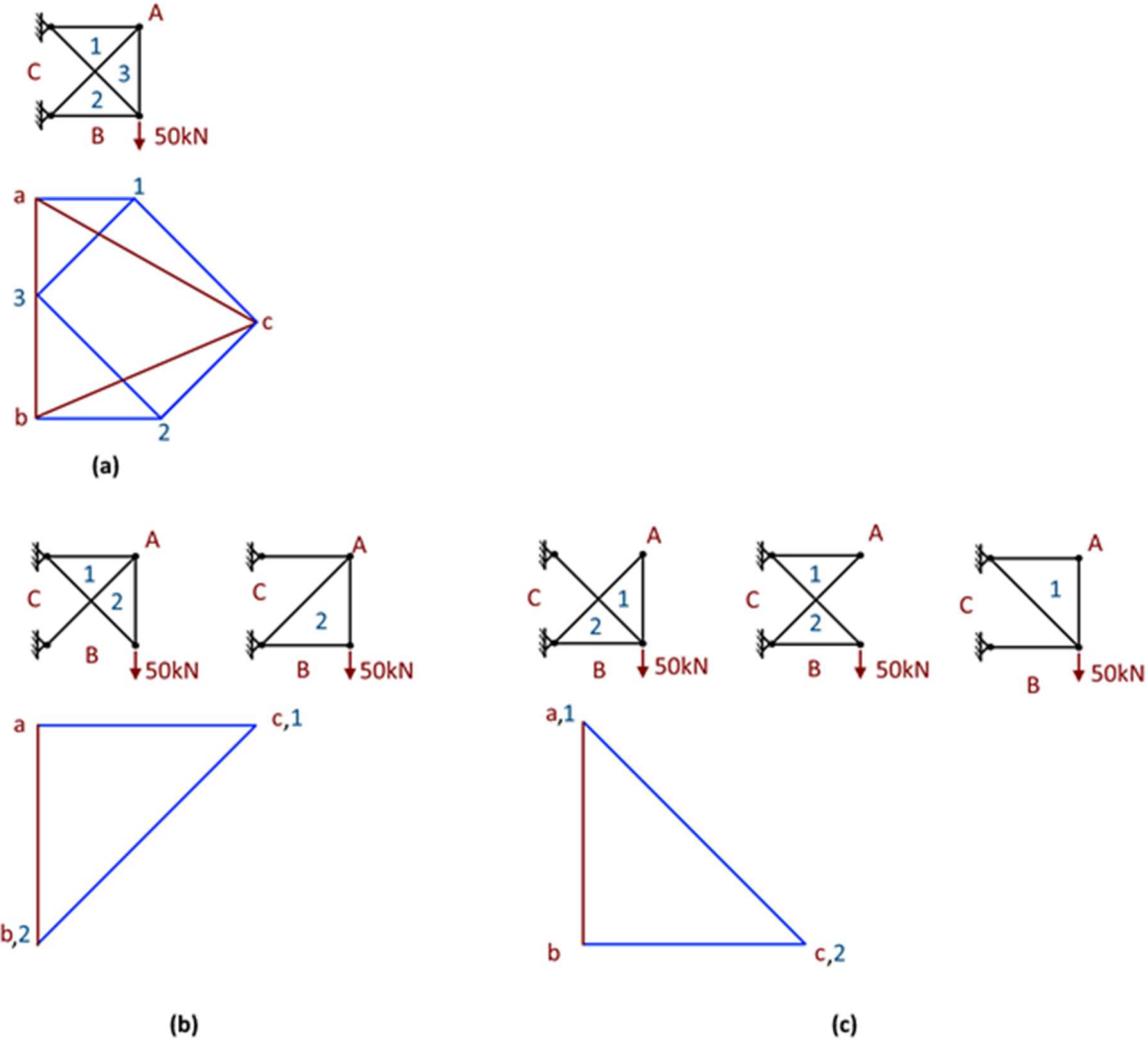


Figure 20. (a) Force polygon of reference structure (b) & (c) force polygons of fundamental structures

Although five fundamental structures are presented in Figure 20(b) and (c), only two force polygons are associated with them. The removal of members {B-2} or {C-1} result in the force polygon in Figure 20(b). The removal of elements {A-1}, {A-3}, or {C-2} results in the force polygon in Figure 20(c). Multiple fundamental structures correspond to a single force polygon because of elements becoming zero force members. The loads supported by element {B-2} or {C-1} are transferred to {A-1} and {C-2}, causing either element {C-2} or {C-1} to become ineffective. A similar case is presented through the removal of elements {A-1}, {A-3}, or {C-2}.

The loads supported by these elements are transferred to {C-1} and {B-2}, and elements {A-3} or {A-1} do not support any loads. In both cases, the force polygons are similar in size, but their individual shape reflects which load path is active. The force polygons take the form of a triangle, and the hypotenuse of each triangle shows which diagonal element is engaged. Like the Pratt truss, the strain energies and total deformation work were computed and are presented in Table 3.

Table 3. Strain energies and deformation work determined from the single-bay truss

Strain Energy (kN-m)		<i>Fundamental Structures (element removed)</i>				
<i>Bow's Notation</i>		{A-1}	{B-2}	{A-3}	{1-3} or {2-C}	{C-1} or {3-2}
<i>Element</i>	<i>Reference Structure</i>	E.1	E.2	E.3	E.4	E.5
E.1	2.41E-06	--	1.23E-05	0	0.00E+00	1.23E-05
E.2	3.84E-06	1.23E-05	--	1.23E-05	1.23E-05	0
E.3	2.41E-06	0	1.23E-05	--	0	1.23E-05
E.4	6.82E-06	1.40935E-36	3.49E-05	0	--	3.49E-05
E.5	1.09E-05	3.49E-05	0	3.49E-05	3.49E-05	--
Deformation Work (kN-m)		<i>Fundamental Structures (element removed)</i>				
	<i>Reference Structure</i>	E.1	E.2	E.3	E.4	E.5
	2.63E-05	4.72E-05	5.96E-05	4.72E-05	4.72E-05	5.96E-05
<i>Figure 20</i>	(a)	(c)	(b)	(c)	(c)	(b)

The existence of zero force members is shown by observing that there are several strain energies equal to zero. Furthermore, the removal of element {A-1}, {A-3}, or {1-3} results in the same element strain energies and total deformation work. Similarly, equivalent element strain energies and total deformation work are produced by removing elements {B-2} or {C-1}. Therefore, the strain energies and deformation correspond to the force polygons in demonstrating that there are two possible load paths when a single element is removed from the truss.

In addition to applying graphic statics, a parametric analysis was conducted on the single bay truss. The results of the analysis are presented in Figure 21.

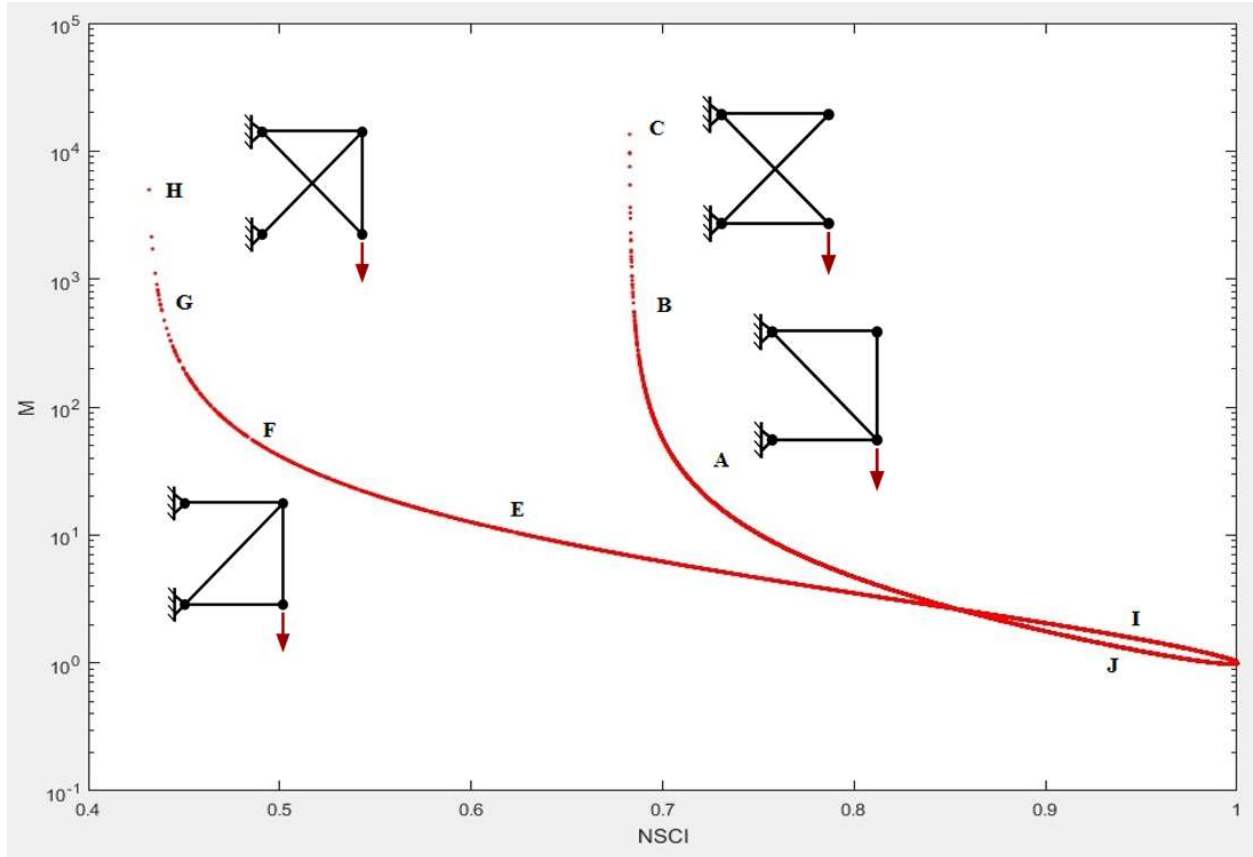


Figure 21. Results of a parametric analysis for a single bay cantilever truss

The areas used in the analysis were randomly generated, and the two curves shown in Figure 21 demonstrate that the NSCI and M-value parameters are perfectly correlated. The correlation observed in Figure 21 can be explained by recognizing that due to zero force members there are only two distinct fundamental structures. Compatibility for a truss with two members connected at a single node and with random member areas, results in member deformations which are perfectly correlated.

Furthermore, the tails can be related to the force polygons previously observed in Figure 20. From the force polygons and strain energies, there are exactly two possible load paths. Even though five possible fundamental structures exist, the applied load can only be distributed throughout the structure through two load paths. By inspecting the points generated in Figure 21, the tails can be related to specific fundamental structures. Each point represents a single structure,

and Table 4 presents the areas that correspond to a representative sample of points taken from Figure 21.

Table 4. Element areas of specific structures selected from Figure 21

NSCI	7.2E-01	6.8E-01	6.8E-01	8.0E-01	6.2E-01	4.9E-01	4.4E-01	4.3E-01	9.5E-01	9.5E-01
M	1.9E+01	6.5E+02	1.4E+04	5.0E+00	1.1E+01	5.5E+01	4.8E+02	5.0E+03	1.5E+00	1.3E+00
ELEMENT	AREAS									
1	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03
2	9.2E-03	4.2E-03	3.0E-03	1.6E-02	1.7E-02	1.3E-02	1.4E-02	7.6E-08	3.3E-04	1.5E-03
3	1.3E-02	1.2E-06	1.9E-08	5.0E-03	3.1E-03	3.3E-03	8.4E-03	1.9E-02	6.2E-03	1.2E-02
4	1.6E-04	1.3E-02	1.1E-02	1.4E-03	3.8E-03	1.9E-02	1.7E-02	5.3E-03	8.5E-03	2.7E-03
5	9.8E-03	1.1E-02	2.3E-03	1.4E-02	7.7E-05	2.1E-05	2.8E-06	1.3E-02	8.1E-03	1.5E-02
LOCATION	A	B	C	D	E	F	G	H	I	J

**The capital letters in the row LOCATION correspond to the the letters in Figure 21*

The removal of elements {B-2} and {C-1} corresponds to the tail in Figure 21 has a vertical asymptote near an NSCI value of 0.42. The tail that asymptotes close to an NSCI value of 0.68 reflects the removal of elements {A-1}, {A-3}, or {C-1}. The determination of which fundamental structure applies to each tail was made by relating the smallest areas in Table 4 to the elements removed. Elements with small cross-sectional areas cannot support large amounts of load. In Table 4, the smallest cross-sectional areas are highlighted in yellow. Although there is not a notable difference between force polygons (b) and (c), the smaller deformation work generated by the fundamental structures with elements {A-1}, {A-3}, or {1-3} removed are desirable but probably not much preferable. Similarly, the structures associated to the with a larger NSCI and a low M-value are preferred. In general, there are two possible load paths as shown by the force polygons, and the parametric analysis demonstrates that one load path is preferred over the other. Although in this case, all members are of equal importance as seen through deformation work.

4.4 Two-Bay Cantilever Truss

To increase the number of load paths and variability in the analyses conducted, a more complicated truss was considered. The truss presented in Figure 15 was analyzed using both graphic statics and the complexity analyses. To form a fundamental structure two elements must

be removed because the two-bay truss is two degrees statically indeterminate. Therefore, single element removal results in the formation of an intermediate statically indeterminate structure. The reference structure and all possible intermediate structures along with their corresponding force polygons are presented in Figure 22.

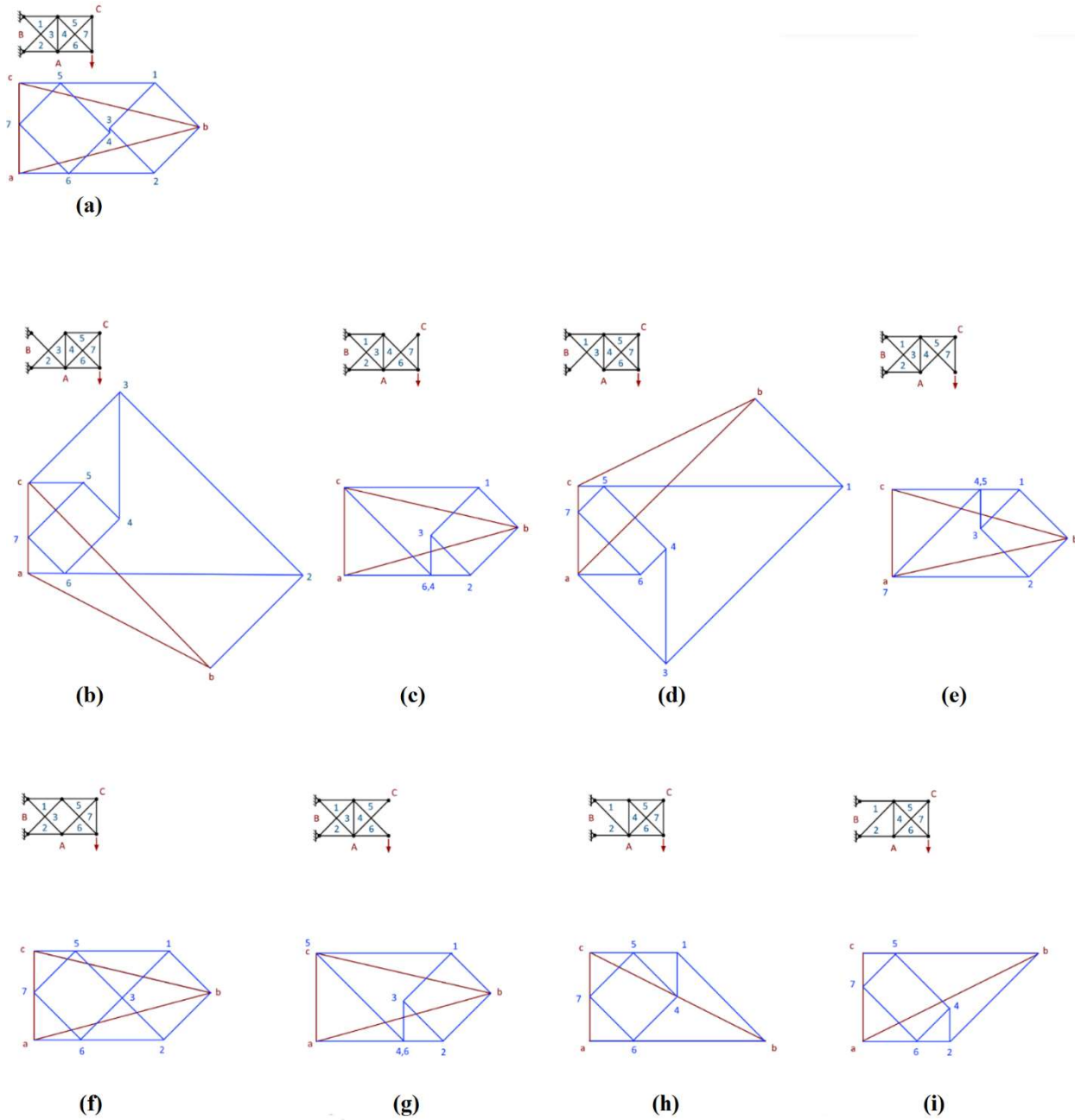


Figure 22. (a) Reference structure of 2-bay truss (b) - (k) intermediate structures of 2-bay truss

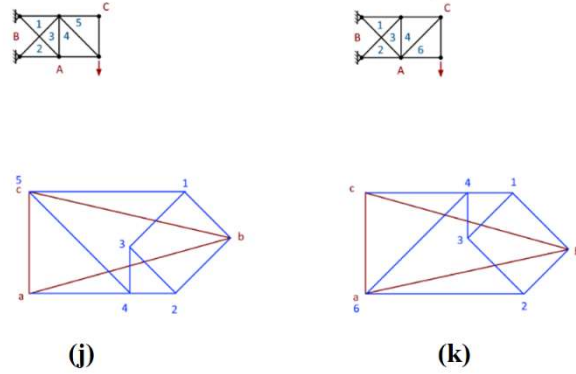


Figure 22. (continued)

Through visual inspection, element contribution can be estimated from the intermediate truss structures' force polygons. The intermediate structures associated with force polygons in Figure 22(b) and (d) show a large redistribution of forces with the removal of the respective member. In particular to these force polygons, the removal of elements {C-1} and {A-2} cause a large force redistribution, which can be seen by the drastic change in shape and size of their force polygons with respect to the reference structure. Intuitively this would be expected since the elements removed are those closest to the supports. Members near the supports typically contribute more to the structure's stability since they are the last link in transferring the forces to the foundations. In addition, force polygons in Figure 22(g), (j), and (c) are identical. Also, force polygons in Figure 22(e) and (k) are identical due to zero force members. The force polygon in Figure 22(f) most resembles the reference polygon. In part, this is due to the fact that element {3-4} did not support a large amount of forces to begin with. Its removal had little effect on the overall distribution of forces.

The strain energy and deformation work produced by the reference and each intermediate structure is presented in Table 5. The large deformation work produced through element's {C-1} and {A-2} removal confirms the conclusions drawn from visually inspecting the force polygons.

Table 5. Strain energies and deformation work determined from the two-bay truss for the reference and each intermediate truss structure

Strain Energy (kN-m)		Intermediate Structures (element removed)									
Bow's Notation		[C-1]	[C-5]	[A-2]	[A-6]	[3-4]	[C-7]	{1-3} or {2-B}	{B-1} or {3-2}	{6-4} or {5-7}	{4-5} or {7-6}
Element	Reference Structure	E.1	E.2	E.3	E.4	E.5	E.6	E.7	E.8	E.9	E.10
E.1	1.96E-02	--	2.09E-02	7.76E-02	1.79E-02	2.02E-02	2.09E-02	8.62E-03	3.45E-02	2.09E-02	1.79E-02
E.2	1.73E-03	3.14E-03	--	7.40E-04	8.62E-03	1.90E-03	0	2.16E-03	1.35E-03	0	8.62E-03
E.3	1.92E-02	7.76E-02	1.79E-02	--	2.09E-02	1.86E-02	1.79E-02	3.45E-02	8.62E-03	1.79E-02	2.09E-02
E.4	2.63E-03	1.35E-03	8.62E-03	4.31E-03	--	2.43E-03	8.62E-03	2.16E-03	3.14E-03	8.62E-03	6.96E-34
E.5	1.85E-05	1.68E-02	1.69E-03	1.44E-02	1.69E-03	--	1.69E-03	2.16E-03	1.35E-03	1.69E-03	1.69E-03
E.6	1.73E-03	3.14E-03	1.00E-31	7.40E-04	8.62E-03	1.90E-03	--	2.16E-03	1.35E-03	1.11E-32	8.62E-03
E.7	6.24E-03	2.44E-02	7.59E-03	9.75E-02	4.77E-03	6.87E-03	7.59E-03	--	2.44E-02	7.59E-03	4.77E-03
E.8	5.95E-03	9.75E-02	4.77E-03	2.44E-02	7.59E-03	5.37E-03	4.77E-03	2.44E-02	--	4.77E-03	7.59E-03
E.9	4.88E-03	8.88E-03	0	2.09E-03	2.44E-02	5.37E-03	0	6.10E-03	3.83E-03	--	2.44E-02
E.10	7.44E-03	3.83E-03	2.44E-02	1.22E-02	0	6.87E-03	2.44E-02	6.10E-03	8.88E-03	2.44E-02	--
Deformation Work (kN-m)		Intermediate Structures (element removed)									
Reference Structure		E.1	E.2	E.3	E.4	E.5	E.6	E.7	E.8	E.9	E.10
6.94E-02		2.37E-01	8.59E-02	2.34E-01	9.45E-02	6.95E-02	8.59E-02	8.83E-02	8.74E-02	8.59E-02	9.45E-02
Figure 22	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)

To progress towards collapse, a second element was removed, thus producing 29 fundamental structures. All possible fundamental structures are presented in Figure 23. Note Figure 23 spans several pages.

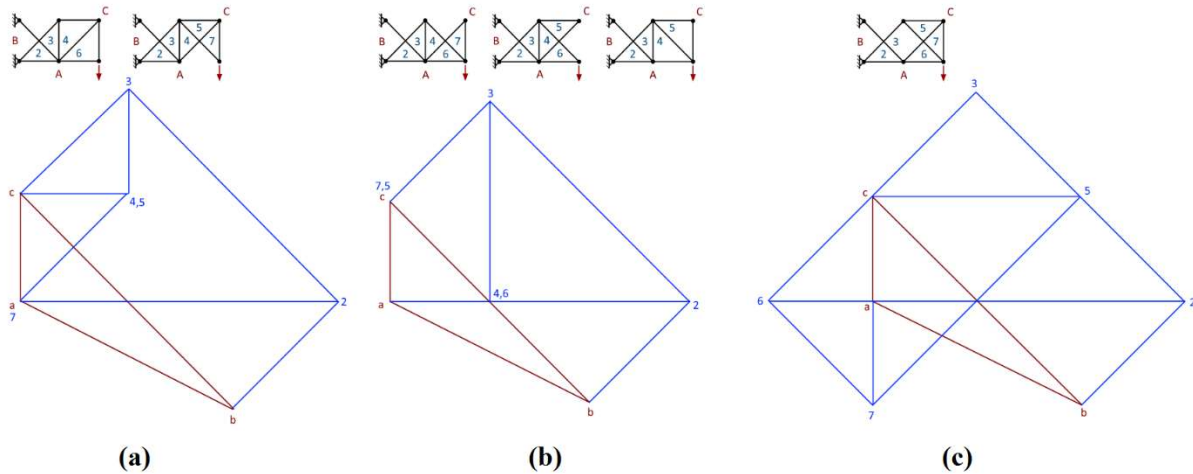
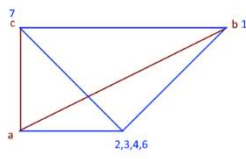
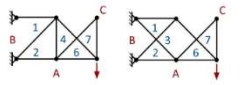
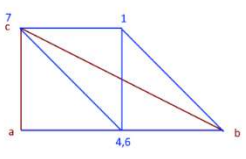
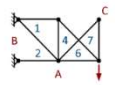


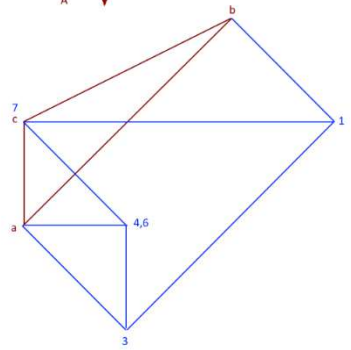
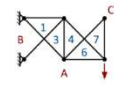
Figure 23. Two-bay fundamental structures and their corresponding force polygon



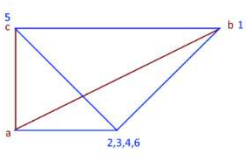
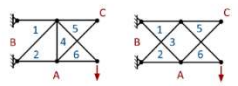
(d)



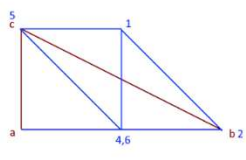
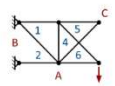
(e)



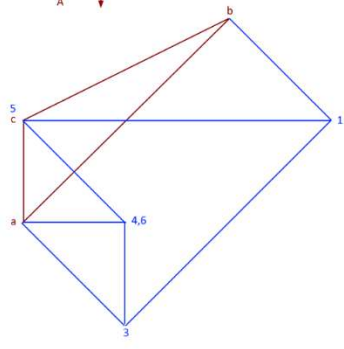
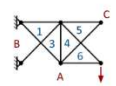
(f)



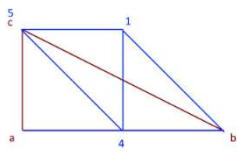
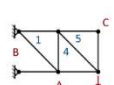
(g)



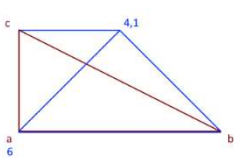
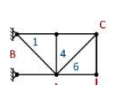
(h)



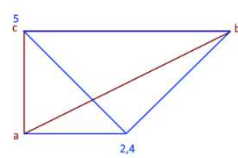
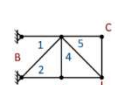
(i)



(j)



(k)



(l)

Figure 23. (continued)

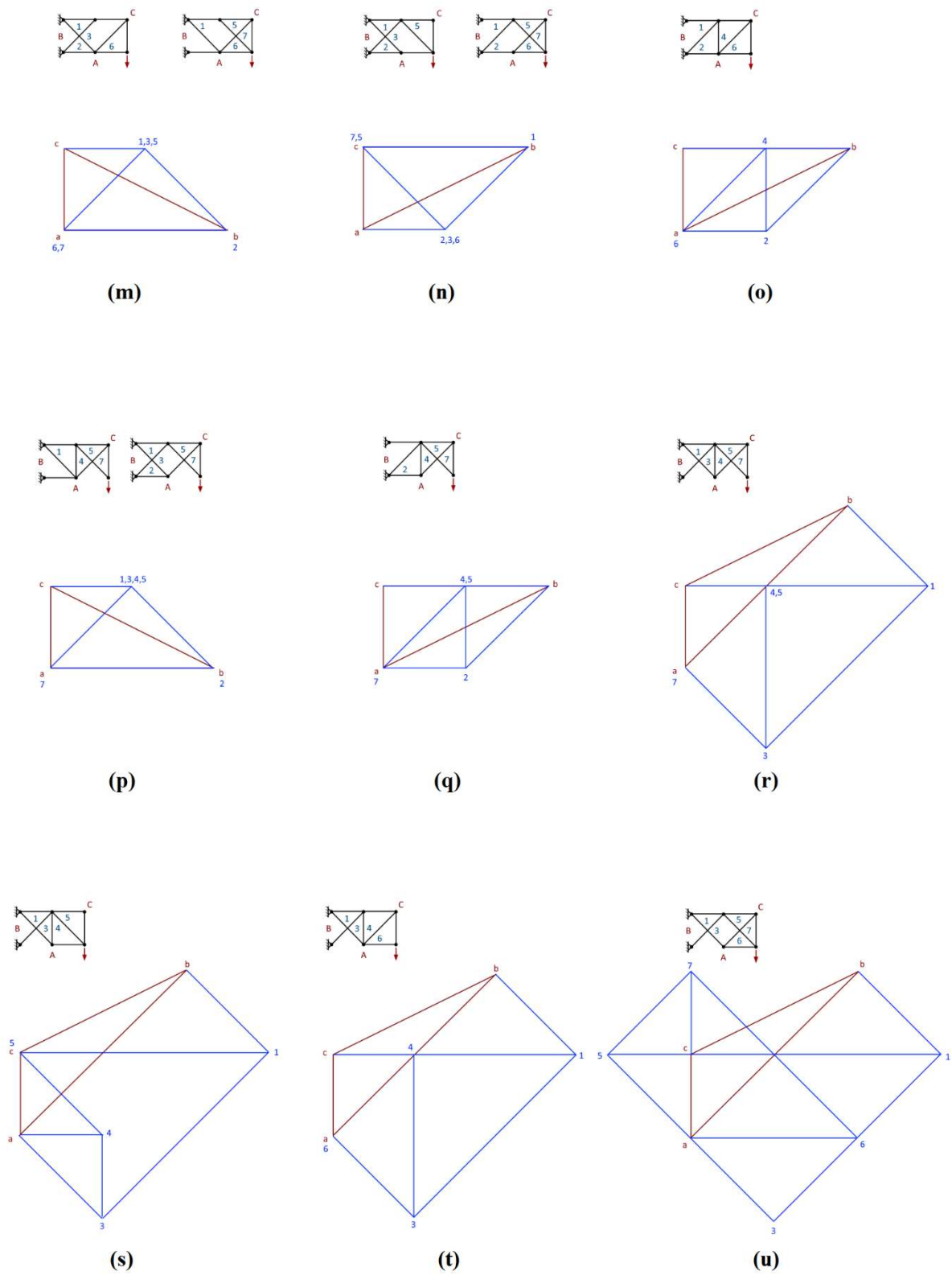


Figure 23. (continued)

The largest force polygons correspond to the fundamental structures that either have elements {C-1} or {A-2} removed. Although the removal of either element {C-1} or {A-2} alone produces an increase in the force polygon's shape and size, when coupled with the removal of element {3-4} they form the largest force polygons when compared to all fundamental structures. Through visual inspection, the force polygons with the least variation in size and shape are shown in Figure 23(d), (e), (g), (h), (j), (k), (l), (m), (n), (o), (p), and (q). The fundamental structures that correspond to these force polygons have a wide range of removed elements, but do not include elements {C-1} or {A-2}.

The deformation work and corresponding element strain energies for each fundamental structure is presented in Table 6.

Table 6. Strain energies and deformation work for all fundamental two-bay truss structures

Strain Energy (kN-m)		Fundamental Structures (elements removed)								
Bow's Notation		{C-1} & {C-5}	{C-1} & {A-6}	{C-1} & {3-4}	{C-1} & {C-7}	{C-1} & {6-4}	{C-1} & {4-5}	{C-5} & {A-2}	{C-5} & {3-4}	{C-5} & {1-3}
Element	Reference Structure	E.1 & E.2	E.1 & E.4	E.1 & E.5	E.1 & E.6	E.1 & E.9	E.1 & E.10	E.2 & E.3	E.2 & E.5	E.2 & E.7
E.1	1.96E-02	--	--	--	--	--	--	7.76E-02	3.45E-02	8.62E-03
E.2	1.73E-03	--	8.62E-03	3.45E-02	4.46E-32	0	8.62E-03	--	--	--
E.3	1.92E-02	7.76E-02	7.76E-02	7.76E-02	7.76E-02	7.76E-02	7.76E-02	--	8.62E-03	3.45E-02
E.4	2.63E-03	8.62E-03	--	8.62E-03	8.62E-03	8.62E-03	2.79E-33	8.62E-03	8.62E-03	8.62E-03
E.4	1.85E-05	3.45E-02	8.62E-03	--	3.45E-02	3.45E-02	8.62E-03	8.62E-03	--	8.62E-03
E.6	1.73E-03	7.13E-31	8.62E-03	3.45E-02	--	1.78E-31	8.62E-03	0	4.01E-31	4.46E-32
E.7	6.24E-03	2.44E-02	2.44E-02	2.44E-02	2.44E-02	2.44E-02	2.44E-02	9.75E-02	2.44E-02	--
E.8	5.95E-03	9.75E-02	9.75E-02	9.75E-02	9.75E-02	9.75E-02	9.75E-02	2.44E-02	4.99E-33	2.44E-02
E.9	4.88E-03	6.30E-32	2.44E-02	9.75E-02	6.30E-32	--	2.44E-02	6.30E-32	0	0
E.10	7.44E-03	2.44E-02	0	2.44E-02	2.44E-02	2.44E-02	--	2.44E-02	2.44E-02	2.44E-02
Deformation Work (kN-m)		Fundamental Structures (elements removed)								
Reference Structure		E.1 & E.2	E.1 & E.4	E.1 & E.5	E.1 & E.6	E.1 & E.9	E.1 & E.10	E.2 & E.3	E.2 & E.5	E.2 & E.7
Figure 23	6.94E-02	2.67E-01	2.50E-01	3.99E-01	2.67E-01	2.67E-01	2.50E-01	2.41E-01	1.00E-01	1.09E-01
		(b)	(a)	(c)	(b)	(b)	(a)	(f)	(d)	(e)

Table 6. (continued)

Strain Energy (kN-m)		Fundamental Structures (elements removed)								
Bow's Notation		{C-5} & {B-1}	{A-2} & {A-6}	{A-2} & {3-4}	{A-2} & {C-7}	{A-2} & {6-4}	{A-2} & {4-5}	{A-6} & {3-4}	{A-6} & {1-3}	{A-6} & {B-1}
Element	Reference Structure	E.2 & E.8	E.3 & E.4	E.3 & E.5	E.3 & E.6	E.3 & E.9	E.3 & E.10	E.4 & E.5	E.4 & E.7	E.4 & E.8
E.1	3.45E-02	7.76E-02	7.76E-02	7.76E-02	7.76E-02	7.76E-02	7.76E-02	8.62E-03	8.62E-03	3.45E-02
E.2	--	8.62E-03	8.62E-03	2.79E-33	0	8.62E-03	8.62E-03	8.62E-03	8.62E-03	6.27E-33
E.3	8.62E-03	--	--	--	--	--	--	3.45E-02	3.45E-02	8.62E-03
E.4	8.62E-03	--	3.45E-02	8.62E-03	8.62E-03	0	--	--	--	8.62E-03
E.4	0	3.45E-02	--	8.62E-03	8.62E-03	3.45E-02	--	2.79E-33	8.62E-03	--
E.6	4.46E-32	8.62E-03	8.62E-03	--	1.78E-31	8.62E-03	8.62E-03	8.62E-03	8.62E-03	--
E.7	2.44E-02	9.75E-02	9.75E-02	9.75E-02	9.75E-02	9.75E-02	6.16E-33	--	2.44E-02	2.44E-02
E.8	--	2.44E-02	2.44E-02	2.44E-02	2.44E-02	2.44E-02	2.44E-02	2.44E-02	--	5.54E-34
E.9	0	2.44E-02	2.44E-02	6.30E-32	--	2.44E-02	2.44E-02	2.44E-02	2.44E-02	3.94E-33
E.10	2.44E-02	6.30E-32	9.75E-02	2.44E-02	2.44E-02	--	1.58E-32	1.58E-32	0	2.44E-02
Deformation Work (kN-m)		Fundamental Structures (elements removed)								
Reference Structure		E.2 & E.8	E.3 & E.4	E.3 & E.5	E.3 & E.6	E.3 & E.9	E.3 & E.10	E.4 & E.5	E.4 & E.7	E.4 & E.8
Figure 23	1.00E-01	2.76E-01	3.73E-01	2.41E-01	2.41E-01	2.76E-01	1.09E-01	1.09E-01	1.18E-01	1.00E-01
		(d)	(r)	(u)	(i)	(s)	(t)	(p)	(p)	(q)

Table 6. (continued)

Strain Energy (kN-m)				Fundamental Structures (elements removed)						
Bow's Notation	{3-4} & {1-3}	{3-4} & {B-1}	{3-4} & {6-4}	{3-4} & {4-5}	{C-7} & {1-3}	{C-7} & {B-1}	{1-3} & {6-4}	{1-3} & {4-5}	{B-1} & {6-4}	{B-1} & {4-5}
Element	E.5 & E.7	E.5 & E.8	E.5 & E.9	E.5 & E.10	E.6 & E.7	E.6 & E.8	E.7 & E.9	E.7 & E.10	E.8 & E.9	E.8 & E.10
E.1	8.62E-03	3.45E-02	3.45E-02	8.62E-03	8.62E-03	3.45E-02	8.62E-03	8.62E-03	3.45E-02	3.45E-02
E.2	8.62E-03	0	0	8.62E-03	0	6.96E-34	0	8.62E-03	6.96E-34	8.62E-03
E.3	3.45E-02	8.62E-03	8.62E-03	3.45E-02	3.45E-02	8.62E-03	3.45E-02	3.45E-02	8.62E-03	8.62E-03
E.4	1.11E-32	8.62E-03	8.62E-03	6.96E-34	8.62E-03	8.62E-03	8.62E-03	6.96E-34	8.62E-03	1.57E-33
E.4	--	--	--	--	8.62E-03	0	8.62E-03	2.79E-33	0	8.62E-03
E.6	8.62E-03	0	0.00E+00	8.62E-03	--	--	4.46E-32	8.62E-03	0	8.62E-03
E.7	--	2.44E-02	2.44E-02	0	--	2.44E-02	--	--	2.44E-02	2.44E-02
E.8	2.44E-02	--	0	2.44E-02	2.44E-02	--	2.44E-02	2.44E-02	--	--
E.9	2.44E-02	0	--	2.44E-02	3.94E-33	3.94E-33	--	2.44E-02	--	2.44E-02
E.10	0	2.44E-02	2.44E-02	--	2.44E-02	2.44E-02	2.44E-02	--	2.44E-02	--
Deformation Work (kN-m)				Fundamental Structures (elements removed)						
	E.5 & E.7	E.5 & E.8	E.5 & E.9	E.5 & E.10	E.6 & E.7	E.6 & E.8	E.7 & E.9	E.7 & E.10	E.8 & E.9	E.8 & E.10
	1.09E-01	1.00E-01	1.00E-01	1.09E-01	1.09E-01	1.00E-01	1.09E-01	1.09E-01	1.00E-01	1.18E-01
Figure 23	(m)	(n)	(n)	(m)	(h)	(g)	(j)	(k)	(l)	(o)

As described through the visual inspection of the force polygons presented in Figure 23, the removal of both elements {C-1} and {3-4} and elements {A-2} and {3-4} have the largest effect on the two-bay truss. Their removal not only produces a large redistribution of forces, but also generates a large deformation work. The removal of element {C-1} with either element {C-5}, {A-6}, {C-7}, {6-4}, {4-5}, or {A-2} also generates a large deformation work. The corresponding force polygons also increase in size and change shape. Similar increases in deformation work and changes in force polygons are observed by removing element {A-2} with either element {A-6}, {C-7}, {6-4}, or {4-5}.

Like the single bay truss, a parametric analysis was conducted on the two-bay truss. The results of the parametric analysis are presented in Figure 24.

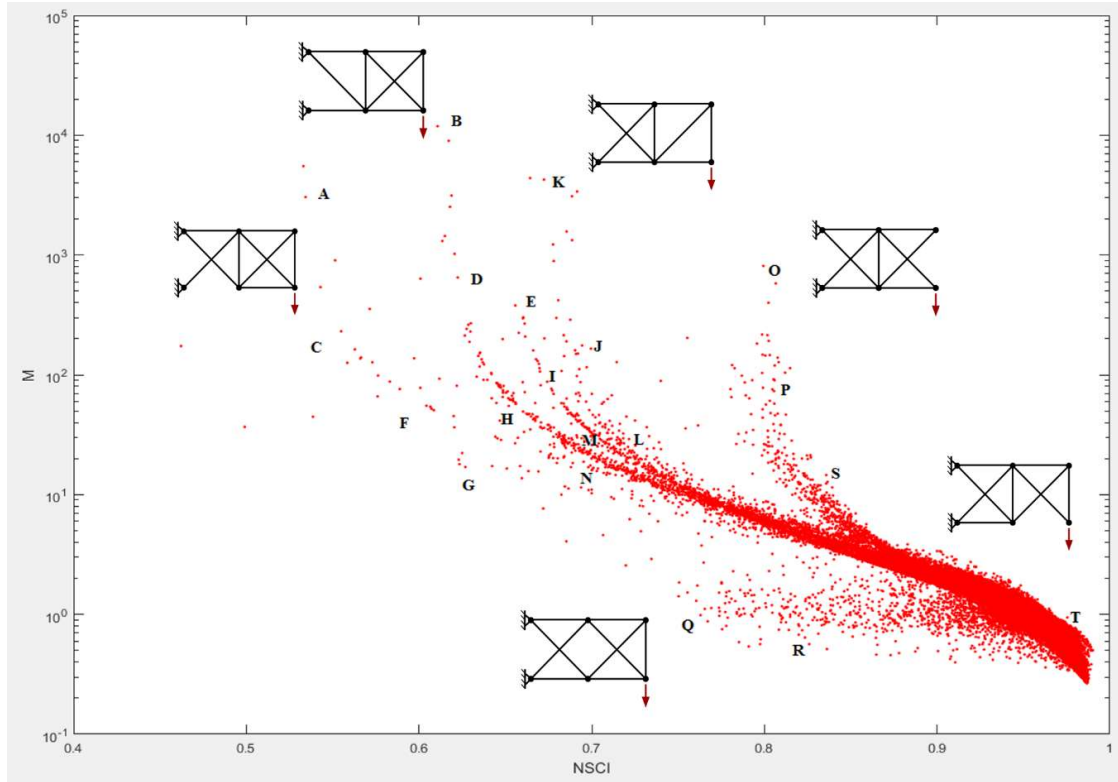


Figure 24. Results of a parametric analysis for a two-bay cantilever truss

Unlike the single bay cantilever truss, the two-bay truss' NSCI and M-value parameters are not perfectly correlated. It is apparent by the large number of fundamental structures that many load paths exist. In this case, the generation of random areas produces scatter in the NSCI and M-value parameters. Although, several trends do exist. Representative values were investigated and presented in Table 7.

Table 7. Element areas of specific structures selected from Figure 25

NSCI	5.3E-01	6.1E-01	5.7E-01	6.2E-01	6.6E-01	6.1E-01	6.3E-01	6.6E-01	6.8E-01	6.9E-01
M	3.0E+03	1.2E+04	1.4E+02	6.5E+02	3.8E+02	5.1E+01	2.2E+01	4.9E+01	6.9E+01	1.5E+02
ELEMENT	AREAS									
1	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03
2	6.5E-03	1.8E-02	2.3E-03	1.3E-02	3.3E-03	1.8E-02	1.5E-02	2.0E-02	1.8E-02	1.5E-02
3	3.8E-08	9.7E-03	7.8E-03	1.4E-02	7.1E-03	1.7E-06	1.7E-02	8.4E-03	4.3E-03	4.7E-03
4	1.1E-02	8.0E-03	8.4E-03	8.6E-04	1.6E-02	1.3E-02	3.4E-05	3.0E-03	1.2E-02	1.7E-02
5	4.5E-03	1.8E-02	1.5E-02	1.5E-02	2.3E-03	1.1E-02	5.7E-03	1.0E-02	1.5E-02	5.0E-03
6	1.6E-02	1.1E-03	1.9E-02	2.0E-02	4.3E-04	1.5E-02	2.8E-03	1.6E-02	3.9E-03	1.8E-02
7	4.9E-03	7.6E-08	4.5E-06	1.1E-06	1.5E-02	1.6E-02	4.9E-05	1.8E-05	1.5E-02	1.7E-02
8	8.9E-03	1.9E-02	4.2E-03	4.8E-03	8.8E-07	4.6E-04	1.8E-02	1.7E-02	5.3E-06	1.7E-02
9	9.2E-03	5.3E-03	1.6E-02	3.3E-03	6.1E-03	1.1E-02	1.4E-02	9.7E-03	2.5E-03	7.2E-03
10	1.5E-02	1.3E-02	1.3E-04	1.8E-02	3.2E-03	2.0E-02	5.9E-03	4.8E-03	1.6E-02	2.3E-06
LOCATION	A	B	C	D	E	F	G	H	I	J

Table 7. (continued)

NSCI	6.7E-01	6.7E-01	7.0E-01	7.0E-01	8.0E-01	8.0E-01	6.6E-01	8.3E-01	8.4E-01	9.8E-01
M	4.3E+03	4.3E+03	3.6E+01	2.3E+01	8.1E+02	4.4E+01	4.9E+01	1.1E+00	1.5E+01	9.4E-01
ELEMENT	AREAS									
1	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03	1.0E-03
2	1.0E-02	6.4E-03	1.2E-02	1.7E-02	1.9E-02	2.3E-06	3.0E-03	1.9E-04	2.0E-06	1.3E-04
3	1.1E-02	1.1E-02	1.4E-02	9.8E-03	6.8E-03	1.6E-02	6.9E-03	1.5E-03	7.7E-03	1.5E-02
4	4.1E-03	3.7E-06	5.2E-03	1.5E-02	1.5E-02	1.0E-02	1.7E-02	9.2E-03	2.0E-04	4.5E-05
5	5.0E-03	5.6E-03	2.3E-03	5.3E-04	1.9E-02	1.3E-02	9.0E-07	1.4E-05	1.3E-02	1.6E-02
6	1.8E-02	1.8E-03	1.2E-02	2.6E-03	9.1E-08	1.7E-02	5.2E-03	1.2E-02	1.5E-02	1.7E-02
7	5.4E-03	4.6E-03	5.8E-03	3.0E-05	1.1E-02	1.1E-02	4.3E-03	1.0E-02	9.5E-03	1.7E-02
8	9.9E-03	1.3E-02	9.6E-06	5.4E-03	1.0E-02	1.0E-02	1.6E-02	8.2E-03	7.9E-03	2.7E-04
9	8.2E-03	9.6E-03	1.5E-02	6.1E-03	6.9E-03	5.0E-03	1.2E-02	7.5E-03	1.6E-02	4.8E-04
10	1.0E-07	5.0E-04	7.6E-03	1.5E-02	3.5E-03	6.1E-03	1.6E-02	1.2E-02	9.1E-03	1.6E-02
LOCATION	K	L	M	N	O	P	Q	R	S	T

By examining the areas, the effectiveness of various load paths can be determined. In Table 7, the least cross-sectional areas are highlighted in yellow. Points A, C, and F correspond to elements {3-4} or {C-7} being removed. As shown in Figure 22, the removal of element {3-4} causes the largest variation in the force polygon. Similarly, when element {3-4} is assigned a relatively small area, it results in a high M-value and a low NSCI, which is undesirable. A large force polygon can be associated with a high M-value and low NSCI parameter. Additionally, the tail marked by B, D, H, and N corresponds to a structure where element {1-3} has a small area. The tail marked by E, I, and M corresponds to element {B-1} being assigned a small area. Both these tails have high M and low NSCI values, and are associated with members close to the supports. In contrast, points with low M and high NSCI values are associated with members far from the supports. Therefore, it can be concluded from both the parametric analysis and graphic statics observations that the members closest to the supports contribute the most to the structural system, if lost, these members will cause the greatest force redistribution throughout the structural system.

CHAPTER 5: CONCLUSIONS

5.1 Summary of Results

The results presented in this thesis explore the application of graphic statics applied to the progressive collapse of statically indeterminate structures. Through visual inspection, the force polygons of fundamental structures provide insight on member contribution and overall system performance. Graphic statics can aid a structural engineer in designing a structure against progressive collapse by relaying how forces are redistributed through variations in a force polygon. If the force polygon of a fundamental structure significantly changes shape and size, it provides a warning to the structural engineer that the removed member carries a significant amount of load and its removal will cause other members to have to carry greater additional loads. Through force polygons, a structural engineer can understand the behavior of a structure and the flow of forces within it.

Furthermore, member contribution and load path effectiveness were examined through complexity metrics. Quantitative relationships can be drawn between force polygons, deformation work, and complexity metrics. In particular to the parametric analysis employed, the NSCI and M-vale metrics can be associated with specific force polygons that represent either desirable or undesirable failure paths. Ultimately, both analyses allow a structural engineer to determine which members contribute the most to the structural system. Additionally, the MATLAB code written will be used to further study force redistribution and member contribution through complexity metrics applied to truss structures subject to member removal and damage. The creation of both the progressive damage and parametric analyses aided the author in the overall investigation and will continue to influence future explorations. The tools that were developed in this thesis will enable

the continued exploration in quantifying the effects of progressive collapse through force polygons.

5.2 Recommendation for Future Work

The results presented in this thesis should be further studied to see if a metric can be determined from or applied to the force polygons from graphic statics. A potential relationship between a force polygon's area and perimeter with its related axial forces and strain energy may exist. Also, the tools developed in this research should be applied to more complicated truss topologies. With respect to the application of graphic statics to the progressive collapse of statically indeterminate truss structures, several alternative analysis methods could be applied and examined through a graphical analysis. For example, the applied load could be increased to yield truss members. Instead of applying a first-order elastic analysis with member removal, an inelastic analysis is applied where members form plastic hinges, which causes their axial force capacity to remain constant as the increased load is then transferred to adjoining members. Throughout the analysis force polygons could be drawn each time a plastic hinge is formed. In addition, the code developed to draw force polygons should be incorporated into the complexity analysis. Therefore, as the complexity analysis is conducted on a truss structure, the force polygons that represent certain load paths are generated concurrently. Together these analyses will provide a way to visualize load paths and a measure of how effective the load paths are. Furthermore, mathematical theory, such as graph theory, should be investigated and included in the exploration using graphic statics to quantify the effects of progressive collapse.

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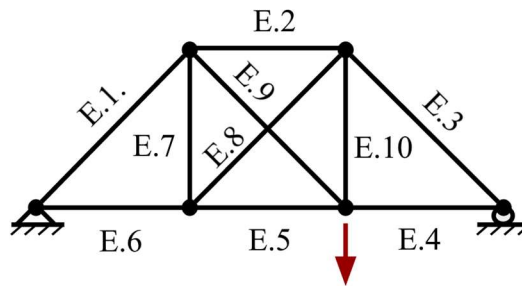
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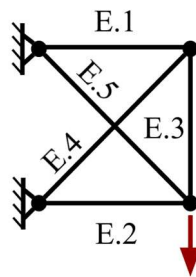
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Appendix A: MATLAB Truss Topology Input Files

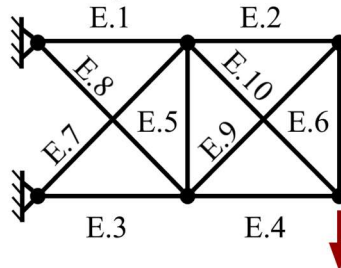
Pratt Truss



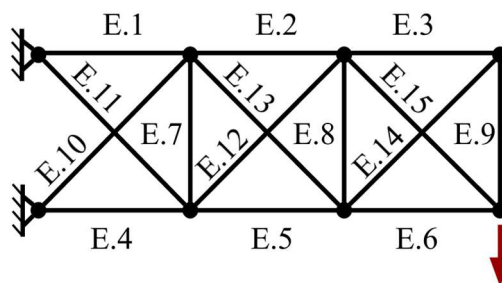
Single-Bay Cantilever Truss



Two-Bay Cantilever Truss



Three-Bay Cantilever Truss



Pratt Truss Topology Input File

```
% MEO_Pratt_Truss
% Author: Maximilian Ororbias

% Pratt truss topology
%
%           4
%       5o-----6o
%       /  |  \9 /  |  \
%       /  |  \|  |  \
%       7/  |5 /\  |6  \|10
%       /  | /8 \  |  \
% PIN 1o-----2o-----3o-----4oROLLER
%           1           2           3

%Structural Topology and Property User-Defined Input

nnodes = 6;           %Number of nodes
nele = 10;            %Number of elements
Emodulus = 210000;    %Youngs Modulus
Poisson = 0.3;        %Poissons ratio
Shear = Inf;          %Shear Area
Inertia = 0;          %Moment of Inertia
force = -50;          %Applied force
fnode = 3;            %Node at which force is applied

%Youngs Modulus
E = Emodulus*ones(nele,1);
%Moment of Inertia
Izz = Inertia*ones(nele,1);
% Poissons Ratio
v = Poisson*ones(nele,1);
%Shear Area
Ayy = Shear*ones(nele,1);

%Nodal Coordinates
coord = [...
    0    0;...
    2    0;...
    4    0;...
    6    0;...
    2    2;...
    4    2;...
];

% fixity = [Support condition of nodes' six d.o.f. (free=NaN, fixed=0, val
for prescribed displacement)]
fixity = [...
    0    0    0;...
    NaN NaN 0;...
    NaN NaN 0;...
    NaN 0 0;...
    NaN NaN 0;...
    NaN NaN 0;...
];
```

```

];

% concen = [concentrated force or moment on nodes' three d.o.f.]
concen = [...
    0    0    0;...
    0    0    0;...
    0   -50   0;...
    0    0    0;...
    0    0    0;...
    0    0    0;...
];

%Element Connectivity
ends = [...
    1    2    0    0   Inf   Inf   Inf   Inf;...
    2    3    0    0   Inf   Inf   Inf   Inf;...
    3    4    0    0   Inf   Inf   Inf   Inf;...
    5    6    0    0   Inf   Inf   Inf   Inf;...
    2    5    0    0   Inf   Inf   Inf   Inf;...
    3    6    0    0   Inf   Inf   Inf   Inf;...
    5    1    0    0   Inf   Inf   Inf   Inf;...
    2    6    0    0   Inf   Inf   Inf   Inf;...
    5    3    0    0   Inf   Inf   Inf   Inf;...
    6    4    0    0   Inf   Inf   Inf   Inf;...
];

%element i's unit web vector
webdir = [...
    0    1.0000    0;...
    0    1.0000    0;...
    0    1.0000    0;...
    0    1.0000    0;...
   -1.0000    0    0;...
   -1.0000    0    0;...
   -0.7071    0.7071    0;...
   -0.7071    0.7071    0;...
    0.7071    0.7071    0;...
    0.7071    0.7071    0;...
];

```

Single-Bay Cantilever Truss Topology Input File

```
% MEO_1Panel_Truss
% Author: Maximilian Ororbia

% 1 panel truss topology
%      3
% PIN 1o----2o
%      \ 5/ |
%      \ /  |
%      /\  |3
%      /4 \ |
% PIN 3o----4o
%      2

%Structural Topology and Property User-Defined Input

nnodes = 4;           %Number of nodes
nele = 5;             %Number of elements
Emodulus = 210000;    %Youngs Modulus
Poisson = 0.3;        %Poissons ratio
Shear = Inf;          %Shear Area
Inertia = 0;          %Moment of Inertia
force = -50;          %Applied force
fnode = 4;            %Node at which force is applied

%Youngs Modulus
E = Emodulus*ones(nele,1);
%Moment of Inertia
Izz = Inertia*ones(nele,1);
% Poissons Ratio
v = Poisson*ones(nele,1);
%Shear Area
Ayy = Shear*ones(nele,1);

%Nodal Coordinates
coord = [...
    0    2;...
    2    2;...
    0    0;...
    2    0;...
];

% fixity = [Support condition of nodes' six d.o.f. (free=NaN, fixed=0, val
for prescribed displacement)]
fixity = [...
    0    0    0;...
    NaN NaN  0;...
    0    0    0;...
    NaN NaN  0;...
];
```

```

% concen = [concentrated force or moment on nodes' three d.o.f.]
concen = [...
    0    0    0;...
    0    0    0;...
    0    0    0;...
    0   -50    0;...
];

%Element Connectivity
ends = [...
    1    2    0    0   Inf   Inf   Inf   Inf;...
    3    4    0    0   Inf   Inf   Inf   Inf;...
    4    2    0    0   Inf   Inf   Inf   Inf;...
    3    2    0    0   Inf   Inf   Inf   Inf;...
    4    1    0    0   Inf   Inf   Inf   Inf;...
];

%element i's unit web vector
webdir = [...
    0    1.0000    0;...
    0    1.0000    0;...
   -1.0000    0    0;...
   -0.7071    0.7071    0;...
    0.7071    0.7071    0;...
];

```


Two-Bay Cantilever Truss Topology Input File

```
% MEO_2Panel_Truss
% Author: Maximilian Ororbia

% 2 panel truss topology
%      1      2
% PIN 1o-----2o-----3o
%      \ 8/ | \10/ |
%      \ /  | \ /  |
%      /\  |5 /\  |6
%      /7 \ | /9 \ |
% PIN 4o-----5o-----6o
%      3      4

%Structural Topology and Property User-Defined Input

nnodes = 6;           %Number of nodes
nele = 10;            %Number of elements
Emodulus = 210000;    %Youngs Modulus
Poisson = 0.3;        %Poissons ratio
Shear = Inf;          %Shear Area
Inertia = 0;          %Moment of Inertia
force = -50;          %Applied force
fnode = 6;            %Node at which force is applied

%Youngs Modulus
E = Emodulus*ones(nele,1);
%Moment of Inertia
Izz = Inertia*ones(nele,1);
% Poissons Ratio
v = Poisson*ones(nele,1);
%Shear Area
Ayy = Shear*ones(nele,1);

%Nodal Coordinates
coord = [...
    0      2;...
    2      2;...
    4      2;...
    0      0;...
    2      0;...
    4      0;...
];

% fixity = [Support condition of nodes' six d.o.f. (free=NaN, fixed=0, val
for prescribed displacement)]
fixity = [...
    0      0      0;...
    NaN    NaN    0;...
    NaN    NaN    0;...
    0      0      0;...
    NaN    NaN    0;...
    NaN    NaN    0;...
];
```

```

];

% concen = [concentrated force or moment on nodes' three d.o.f.]
concen = [...
    0    0    0;...
    0    0    0;...
    0    0    0;...
    0    0    0;...
    0    0    0;...
    0   -50    0;...
];

%Element Connectivity
ends = [...
    1    2    0    0   Inf   Inf   Inf   Inf;...
    2    3    0    0   Inf   Inf   Inf   Inf;...
    4    5    0    0   Inf   Inf   Inf   Inf;...
    5    6    0    0   Inf   Inf   Inf   Inf;...
    5    2    0    0   Inf   Inf   Inf   Inf;...
    6    3    0    0   Inf   Inf   Inf   Inf;...
    4    2    0    0   Inf   Inf   Inf   Inf;...
    5    1    0    0   Inf   Inf   Inf   Inf;...
    5    3    0    0   Inf   Inf   Inf   Inf;...
    2    6    0    0   Inf   Inf   Inf   Inf;...
];

%element i's unit web vector
webdir = [...
    0    1.0000    0;...
    0    1.0000    0;...
    0    1.0000    0;...
    0    1.0000    0;...
   -1.0000    0    0;...
   -1.0000    0    0;...
   -0.7071    0.7071    0;...
    0.7071    0.7071    0;...
   -0.7071    0.7071    0;...
    0.7071    0.7071    0;...
];

```

Three-Bay Cantilever Truss Topology Input File

```
% MEO_3Panel_Truss
% Author: Maximilian Ororbis

% 3 panel truss topology
%      1      2      3
% PIN 1o-----2o-----3o-----4o
%      \|11/ | \|13/ | \|15/ |
%      \|   | \|   | \|   |
%      /\  |7 /\  |8 /\  |9
%      /10\ | /12\ | /14\ |
% PIN 5o-----6o-----7o-----8o
%      4      5      6

%Structural Topology and Property User-Defined Input

nnodes = 8;           %Number of nodes
nele = 15;            %Number of elements
Emodulus = 210000;    %Youngs Modulus
Poisson = 0.3;        %Poissons ratio
Shear = Inf;          %Shear Area
Inertia = 0;          %Moment of Inertia
force = -50;          %Applied force
fnode = 8;            %Node at which force is applied

%Youngs Modulus
E = Emodulus*ones(nele,1);
%Moment of Inertia
Izz = Inertia*ones(nele,1);
% Poissons Ratio
v = Poisson*ones(nele,1);
%Shear Area
Ayy = Shear*ones(nele,1);

%Nodal Coordinates
coord = [...
    0      2;... %NODE 1
    2      2;... %NODE 2
    4      2;... %NODE 3
    6      2;... %NODE 4
    0      0;... %NODE 5
    2      0;... %NODE 6
    4      0;... %NODE 7
    6      0;... %NODE 8
];

% fixity = [Support condition of nodes' six d.o.f. (free=NaN, fixed=0, val
for prescribed displacement)]
fixity = [...
    0      0      0;... %NODE 1
    NaN    NaN    0;... %NODE 2
    NaN    NaN    0;... %NODE 3
    NaN    NaN    0;... %NODE 4
```

```

0      0      0;... %NODE 5
NaN    NaN    0;... %NODE 6
NaN    NaN    0;... %NODE 7
NaN    NaN    0;... %NODE 8
];

% concen = [concentrated force or moment on nodes' three d.o.f.]
concen = [...
0      0      0;... %NODE 1
0      0      0;... %NODE 2
0      0      0;... %NODE 3
0      0      0;... %NODE 4
0      0      0;... %NODE 5
0      0      0;... %NODE 6
0      0      0;... %NODE 7
0     -50     0;... %NODE 8
];

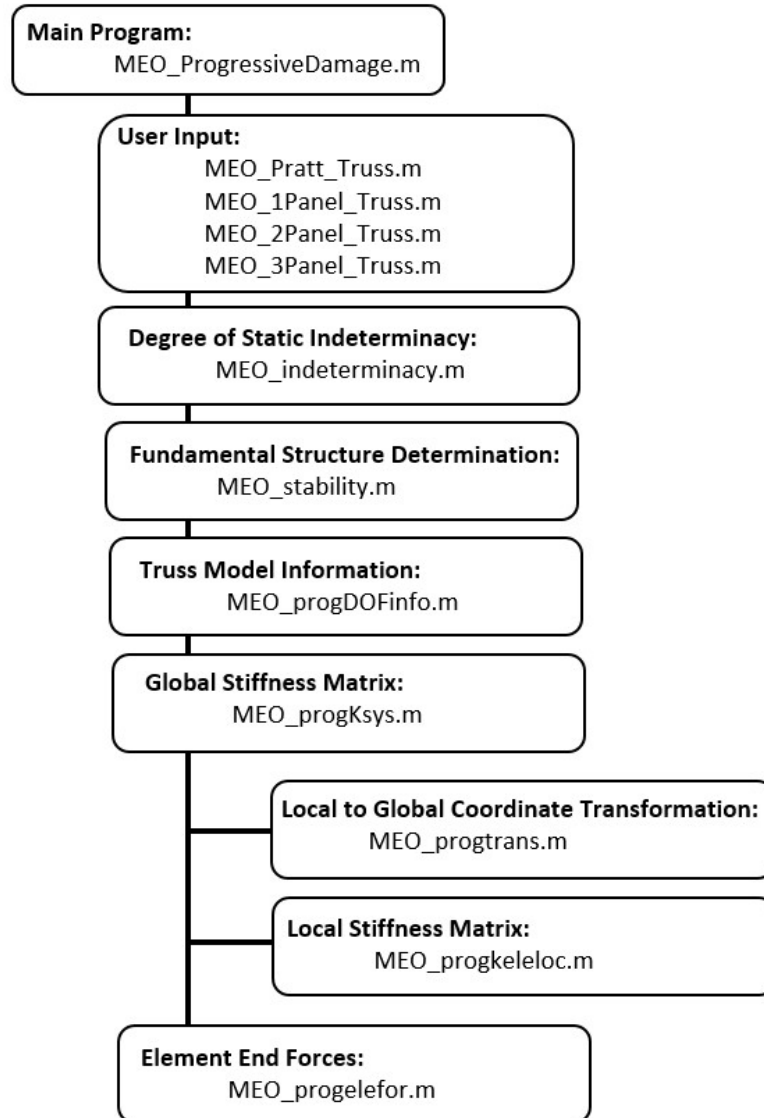
%Element Connectivity
ends = [...

1      2      0      0    Inf    Inf    Inf    Inf;... %ELE 1
2      3      0      0    Inf    Inf    Inf    Inf;... %ELE 2
3      4      0      0    Inf    Inf    Inf    Inf;... %ELE 3
5      6      0      0    Inf    Inf    Inf    Inf;... %ELE 4
6      7      0      0    Inf    Inf    Inf    Inf;... %ELE 5
7      8      0      0    Inf    Inf    Inf    Inf;... %ELE 6
6      2      0      0    Inf    Inf    Inf    Inf;... %ELE 7
7      3      0      0    Inf    Inf    Inf    Inf;... %ELE 8
8      4      0      0    Inf    Inf    Inf    Inf;... %ELE 9
5      2      0      0    Inf    Inf    Inf    Inf;... %ELE 10
6      1      0      0    Inf    Inf    Inf    Inf;... %ELE 11
6      3      0      0    Inf    Inf    Inf    Inf;... %ELE 12
7      2      0      0    Inf    Inf    Inf    Inf;... %ELE 13
7      4      0      0    Inf    Inf    Inf    Inf;... %ELE 14
8      3      0      0    Inf    Inf    Inf    Inf;... %ELE 15
];

%element i's unit web vector
webdir = [...
0      1.0000      0;...
0      1.0000      0;...
0      1.0000      0;...
0      1.0000      0;...
0      1.0000      0;...
0      1.0000      0;...
-1.0000      0      0;...
-1.0000      0      0;...
-1.0000      0      0;...
-0.7071      0.7071      0;...
0.7071      0.7071      0;...
-0.7071      0.7071      0;...
0.7071      0.7071      0;...
-0.7071      0.7071      0;...
0.7071      0.7071      0;...
];

```

Appendix B: MATLAB Scripts for Progressive Damage Analysis



Progressive Damage Analysis Main Routine

```
% MEO_ProgressiceDamage_3P
% Author: Maximilian Ororbias

% Progressive Collapse Analysis
% Purpose: Analyzes a truss' reference and fundamental structures for
% stabiliy by simulating member removal. Members are 'removed' by setting
% their areas equal to zero.

clear; close all

%Start time
tic

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- LOAD TRUSS TOPOLOGY DEFINED INPUT -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%User Note: Change amount area assignments where noted
% run MEO_Pratt_Truss.m
% run MEO_1Panel_Truss.m
% run MEO_2Panel_Truss.m
run MEO_3Panel_Truss.m

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- END OF USER INPUT -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- DEGREE OF STATIC INDETERMINACY -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Determines the number of elements needed to be removed to generate a
% statically determinate system
[rmv] = MEO_indeterminacy(nele, nnodes, fixity);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- END OF INDETERMINACY -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- MAXTAN2 STABILITY CHECK -----%%
%%----- CHECK EACH FUNDAMENTAL STRUCTURE -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%NEED TO CHANGE THE AMOUNT OF AREAS TESTED IN THE STABILTIY CODE
[fslist, nfs, qq] = MEO_stability(nele, rmv, nnodes, fixity, concen, ends,
coord, webdir, Izz, E, v, Ayy);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- END STABILITY ANALYSIS -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- PROGRESSIVE DAMAGE ANALYSIS -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%% ---- ANALYZE REFERENCE STRUCTURE ---- %%%

% Assign elements to fixed area across all samples
refA = 0.9654e-3; %Reference Area as defined by De Biagi
Aref = refA*ones(nele,1);

% Model information
[f_dof,s_dof,global_disps,global_concens] =
MEO_progDOFinfo(nnodes,fixity,concen);

% AREA INPUT WITHIN THIS FUNCTION
A = Aref;

[K_system] = MEO_progKsys(nnodes,nele,ends,coord,webdir,A,Izz,E,v,Ayy);

% Organization
Kff = K_system(f_dof,f_dof);
Pf = global_concens(f_dof);
Kfs = K_system(f_dof,s_dof);
Ksf = K_system(s_dof,f_dof);
Kss = K_system(s_dof,s_dof);
Ps = global_concens(s_dof);
% Supportted degrees of freedom
Deltas = global_disps(s_dof);

%%% -- Displacements -- %%%
Deltaf = Kff\Pf;
global_disps(f_dof) = Deltaf;
DEFL = reshape(global_disps,3,nnodes)';

% Just vertical displacements
Y_FSDisp = DEFL(:,2)';

% Deformation Work
% Reference Structure
RS_Defwork = (Y_FSDisp(:,fnode)*force)/2;

%%% -- Axial Forces -- %%%

Deltaf = Kff\(Pf-Kfs*Deltas);

global_disps(f_dof) = Deltaf;

Ps = Ksf*Deltaf + Kss*Deltas - Ps;
global_concens(s_dof) = Ps;
global_concens(f_dof) = 0;

```

```

% Element end forces
[ELE_FOR] =
MEO_progelefor(global_disps, nele, ends, coord, webdir, A, Izz, E, v, Ayy);

% Axial Force
RS_Ax1F = ELE_FOR(:, 4);

%%% ---- PROGRESSIVE DAMAGE ANALYSIS ---- %%%

% Damage parameter
% Number of damage levels
ndam=1000;
% Create vector of damage levels
% Ranges from 0 (no damage) to 1 (element removal)
xi=linspace(0,1,ndam);

% Preallocation
Omega = zeros(nele, ndam);
Ax1F = zeros(nele, ndam, nele);

% Loop over each of members
for i=1:nele

    % Loop over each damage level
    for j=1:ndam

        % Calculate reduced area of i-th element
        Ared = Aref(i,1)*(1-xi(1,j));

        % Assign areas to a vector (one reduced, others nominal)
        Adam = Aref;
        Adam(i,1) = Ared;

        % Model information
        [f_dof, s_dof, global_disps, global_concens] =
MEO_progDOFinfo(nnodes, fixity, concen);

        % AREA INPUT WITHIN THIS FUNCTION
        [K_system] =
MEO_progKsys(nnodes, nele, ends, coord, webdir, Adam, Izz, E, v, Ayy);

        % Organization
        Kff = K_system(f_dof, f_dof);
        Pf = global_concens(f_dof);
        Kfs = K_system(f_dof, s_dof);
        Ksf = K_system(s_dof, f_dof);
        Kss = K_system(s_dof, s_dof);
        Ps = global_concens(s_dof);
        % Supportted degrees of freedom
        Deltas = global_disps(s_dof);

        % Displacements
        Deltaf = Kff\Pf;
        global_disps(f_dof) = Deltaf;

```



```

DEFL = reshape(global_disps,3,nnodes)';

% Just vertical displacements
Y_1EDisp = DEFL(:,2)';

% Deformation Work
% One element removed
OE_Defwork = (Y_1EDisp(:,fnode)*force)/2;

Omega(i,j) = (OE_Defwork - RS_Defwork)/RS_Defwork+1;

% AXIAL FORCES

Deltaf = Kff\(Pf-Kfs*Deltas);

global_disps(f_dof) = Deltaf;
Ps = Ksf*Deltaf + Kss*Deltas - Ps; %+ FEA_s;
global_concens(s_dof) = Ps;
global_concens(f_dof) = 0;

% Element end forces
[ELE_FOR] =
MEO_progelefor(global_disps,nele,ends,coord,webdir,Adam,Izz,E,v,Ayy);

% Axial Force
AxlF(:,j,i) = ELE_FOR(:,4);

end
end

% PLOTS

% Omega and xi values
figure(1)
plot(xi,Omega)
xlabel('Damage Parameter, \xi')
ylabel('Increment in Deformation Work, \Omega')
% xlim([.9 1])
% ylim([1 1.15])

%
figure(2)

% Organize and collect element axial forces
% Preallocation
MAX = zeros(1,nele);
MIN = zeros(1,nele);

for i=1:nele

    Elem_F = zeros(ndam,nele);
    % Formulate matrix w/ axial forces generated in each element
    Elem_F(:, :) = AxlF(i, :, :);

```

```

% Elm_F = (Element No., Damage Step)
Elem_F = Elem_F';
% Remove removed member's non-existent force (Max dam. = no element)
Elem_F(i,ndam) = NaN;
% Remove element being damaged from being considered
Elem_F((i),:) = [];

% Maximum Axial Forces MAX = (:, Element No.)
MAX(1,i) = max(max(Elem_F));
% Minimum Axial Forces MIN = (:, Element Order Axial Force)
MIN(1,i) = min(min(Elem_F));
clear Elem_F

end

% Reference structure's axial forces
plot(1:nele,RS_Ax1F,'-k.')
hold on
% Maximum axial forces
plot(1:nele,MAX,'-ks')
hold on
% Minimum axial forces
plot(1:nele,MIN,'-ko')
grid on
xlabel('Beam no.')
ylabel('Axial force [kN]')
% ylim([-400 400])
% xlim([0 16])

```

Degree of Static Indeterminacy Determination

```
function [rmv] = MEO_indeterminacy(nele, nnodes, fixity)
% Author: Maximilian Ororbias

% The function determines the degree of static indeterminacy for a truss
% structure given the number of elements, nodes, and reactions. The output
% is in the form of rmv which is the amount of members/elements needed to
% be removed to form a fundamental structure (statically determinate)

%Quantify the number of reactions (supported degrees of freedom)
Rctns = sum(sum(fixity(:,1:2)==0));
%Degree of static indeterminacy => Elements + Reactions = 2 * #Nodes
Ind_deg = nele + Rctns - 2 * nnodes;
%Number of elements needed to be removed
rmv = Ind_deg;

end
```

Fundamental Structures Generation

```
function [fslist, nfs, qq] = MEO_stability(nele, rmv, nnodes, fixity, concen,
ends, coord, webdir, Izz, E, v, Ayy)
% Author: Maximilian Ororbia

% Determines stable fundamental structures from a user input reference
% structure
%OUTPUT:    fslist    nfs    qq
% [fslist, nfs, qq] = MEO_Stability(nele, rmv, nnodes, fixity, concen,
% ends, coord, webdir, Izz, E, v, Ayy)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- MAXTAN2 STABILITY CHECK -----%%
%%----- CHECK EACH FUNDAMENTAL STRUCTURE -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% All possible combinations to remove elements
% remele = combntns(1:NoEle,Rmv);
remele = nchoosek(1:nele, rmv);
[mem1,mem2] = size(remele);
combos = zeros(mem1+1,mem2);
[ma,mb] = size(combos);

for i = 1:ma-1
    combos(i+1,:) = remele(i,:);
end

%PREALLOCATION
%Stable and unstable structures: 1 = stable // 0 = unstable
stability = zeros(ma,2);

%TWO MEMBER REMOVAL
for i=1:ma
    if i == 1
        A = 10*ones(nele,1);
    else
        A = 10*ones(nele,1);
        ele_A1 = combos(i,1);
        ele_A2 = combos(i,2);
        ele_A3 = combos(i,3);
%ADDED/REMOVE FOR No. OF ELEMENTS REMOVED
        Azero = 0;
        A(ele_A1) = Azero;
        A(ele_A2) = Azero;
        A(ele_A3) = Azero;
%ADDED/REMOVE FOR No. OF ELEMENTS REMOVED
    end

    [f_dof] = MEO_progDOFinfo(nnodes, fixity, concen);

    [K_system] = MEO_progKsys(nnodes, nele, ends, coord, webdir, A, Izz, E, v, Ayy);
```

```

%Organization
Kff = K_system(f_dof,f_dof);

% STABILITY CHECK
stblty = rcond(Kff);
% Check if structure is stable or unstable
% i
rcond(Kff);
if rcond(Kff) < 1e-16 || rcond(Kff) > 1e4 %CHANGE RCOND IF NECESSARY
    AFLAG = 0; % Denotes an ill-conditioned coefficient matrix
else
    AFLAG = 1; %Denotes an well-conditioned coefficient matrix
end

if AFLAG == 0
    stability(i,1) = 0;
    stability(i,2) = stblty;
elseif AFLAG == 1
    stability(i,1) = 1;
    stability(i,2) = stblty;
end
end

clear f_dof s_dof global_disps global_concens K_system

% ORGANIZATION OF COLLECTED DATA
% Combine possible combinations and stabilitiy (w/ rcond value) matrices
% ( element rem., element rem., stable(1)/unstable(0), rcond(kff) )
str = horzcat(combos,stability);
%Collection of stable structures (rmw+1 => column # with stabilitiy)
s_str = str(str(:,rmv+1)==1,:);
%Collection of unstable structur
u_str = str(str(:,rmv+1)==0,:);

%Amount of stable structures
[q,w] = size(s_str);
%Preallocation to fundamental structures list %USED FOR PARA. ANALYSIS
fslist = s_str(2:end,1);
fslist(:,2) = s_str(2:end,2);
fslist(:,3) = s_str(2:end,3);
%ADDED/REMOVE FOR No. OF ELEMENTS REMOVED
%Amount of fundamental structures determined by fslist
[qq,zz]=size(fslist);

% Number of fundamental structures (Equivalent to qq)
nfs=q-1; % q == Number of stable fundamental structures including reference

end

```

Development of Model Information

```
function [f_dof,s_dof,global_disps,global_concens] =
MEO_progDOFinfo(nnodes,fixity,concen)
% Author: Maximilian Ororbia

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Purpose: Generates and returns vectors that extract information from
% user input. This function provides necessary information to run a
% 2-dimensional 1st-order elastic analysis as defined by user through
% mastan2 input.
%
% MEO_DOF_information(nnodes,fixity,concen)
%
% Dictionary of Variables
%
% Input Information:
%
% * nnodes          == total number of nodes
%
% * fixity(i,1:3) == prescribed displacements for node i's 3 d.o.f.
%                   fixity(i,1) = prescribed disp. in global X direction
%                   fixity(i,2) = prescribed disp. in global Y direction
%                   fixity(i,3) = prescribed rotation about global Z axis
%                   if fixity(i,j) = NaN then, j-degree of freedom of node i is
%                               free.
%
% * concen(i,1:3) == concentrated loads for node i's 3 d.o.f.
%                  concen(i,1) = force in global X direction
%                  concen(i,2) = force in global Y direction
%                  concen(i,3) = moment about global Z axis
%
% Output Information:
%
% * fdof = vector of free degree of freedom numbers
% * sdof = vector of supported d.o.f. numbers
% * glo_disp = vector of global displacements
% * glo_P = vector of applied global concentrated loads
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Vector of applied global concentrated loads {P}
global_concens = reshape(concen',3*nnodes,1); %nnodes *accounts for 3 dof
%Vector of global displacements {D}
global_disps = reshape(fixity',3*nnodes,1);
%Vector of free degree of freedom numbers
f_dof = find(isnan(global_disps));
%Vector of supported d.o.f. numbers
s_dof = find(~isnan(global_disps));

end
```

Global Stiffness Matrix Assembly

```
function [K_system] =
MEO_progKsys(nnodes, nele, ends, coord, webdir, A, Izz, E, v, Ayy)
% Author: Maximilian Ororbia

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Purpose: Generates a global stiffness matrix and the corresponding fixed
%end actions that also reference the global coordinate system.
%
% MEO_K_system(nnodes, nele, ends, coord, webdir, A, Izz, E, Ayy, Wt, v,
w, thermal)
%
% Dictionary of Variables
%
%   Input Information:
%
%   * nnodes           == total number of nodes
%   * coord(i,1:2)     == node i's coordinates
%                       coord(i,1) = X coordinate
%                       coord(i,2) = Y coordinate
%   * nele             == total number of elements
%   * ends(i,1:8)      == element i's nodal information
%                       ends(i,1) = start node #
%                       ends(i,2) = finish node #
%   * A(i)             == element i's cross sectional area
%   * Izz(i)           == element i's moment of inertia about its local z-
z axis
%   * E(i)             == element i's material elastic modulus, Young's
Modulus
%   * webdir(i,1:3)    == element i's unit web vector. This is a unit
vector
%                       that defines the element's local y-y axis with
%                       respect
%                       to the global coordinate system. It is based only
on the
%                       structures undeformed geometry.
%                       webdir(i,1) = x component of element's unit web
vector
%                       webdir(i,2) = y component of element's unit web
vector
%                       webdir(i,3) = z component of element's unit web
vector
%                       (which will always be zero for 2D
analysis)
%   * w(i,1:2)         == element i's uniform load which references its
%                       local coordinate system
%                       w(i,1) = x component of uniform load
%                       w(i,2) = y component of uniform load
%
%   * Wt(i)            == element i's material weight density
%   * truss            == flag to indicate if structure is a truss or not
%                       truss = 0   System is not a truss
%                       truss = 1   System is a truss
%
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Preallocation of both the system stiffness and fixed end actions matrices
K_system = zeros(3*nnodes, 3*nnodes);
%FEA_system = zeros(3*nnodes, 1);

%Begin assembling stiffness matrix by examining individual elements
for element = 1:nele

    %Start node (i):
    start_node = ends(element, 1);
    %End node (j):
    end_node = ends(element, 2);

    % Call previously developed element length and transformation matrix
    ni_coord = coord(start_node, 1:2); %Start node coordinates
    nj_coord = coord(end_node, 1:2);   %End node coordinates
    web_dir = webdir(element, :);
    [L, GAM]=MEO_progtrans(ni_coord, nj_coord, web_dir);

    %Call functions:
    % Local Stiffness Matrix
    [kele_local] = MEO_progkeleloc(L,A(element),Izz(element),E(element), ...
        ends(element,5),ends(element,6),v(element),Ayy(element));

    %Transform local stiffness matrix for the element from referencing
    %local coordinates to global coordinates
    kele_global = GAM' * kele_local * GAM; % 6x6
    % (the transpose of GAMMA times local stiffness times GAMMA)

    %Formation of K_system by compiling components from the previously
    %developed element global stiffness matrix
    K_system(3*start_node-2:3*start_node, 3*start_node-2:3*start_node) = ...
        K_system(3*start_node-2:3*start_node, 3*start_node-2:3*start_node)
+...
        kele_global(1:3, 1:3);

    K_system(3*end_node-2:3*end_node, 3*start_node-2:3*start_node) = ...
        K_system(3*end_node-2:3*end_node, 3*start_node-2:3*start_node) +...
        kele_global(4:6, 1:3);

    K_system(3*start_node-2:3*start_node, 3*end_node-2:3*end_node) = ...
        K_system(3*start_node-2:3*start_node, 3*end_node-2:3*end_node) +...
        kele_global(1:3, 4:6);

    K_system(3*end_node-2:3*end_node, 3*end_node-2:3*end_node) = ...
        K_system(3*end_node-2:3*end_node, 3*end_node-2:3*end_node) +...
        kele_global(4:6, 4:6);

end
end

```


Local to Global Coordinate System Transformation

```
function [L, GAM]=MEO_progtrans(ni_coord, nj_coord, web_dir)
% Author: Maximilian Ororbis

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Purpose: Transforms an element's local coordinates to global coordinates
%           and calculates length of element given user coordinates
%
%           [L, GAM]=MEO_transform(ni_coord, nj_coord, web_dir)
%
% Dictionary of Variables
%
%   Input Information:
%
%       * Start node, i, coordinates:
%           ni_coord == [xi yi]
%       * End node, j, coordinates:
%           nj_coord == [xj yj]
%
%       Unit vector that defines the element's local y-y axis with
%       respect to the global coordinate system. It is based only on
%       the structures undeformed geometry:
%
%       * web_dir(i,1:3) == [x component, y component, z component]
%
%   Output Information:
%       L == Element length
%       GAM = Global to local transformation matrix GAMMA (6x6 matrix)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Preallocation of the GAMMA transformation matrix
GAM = zeros(6,6); % 6x6

%Determine length of member using given coordinates
delta1 = nj_coord(1)-ni_coord(1); %x distance between nodes
delta2 = nj_coord(2)-ni_coord(2); %y distance between nodes
%Length (pythagorean theorem)
L = sqrt((delta1)^2+(delta2)^2);

%Assemblage of GAMMA transformation matrix
%Component of GAMMA referencing local x-coordinate system
GAM(1,1:2) = [(delta1)/L (delta2)/L];
GAM(4,4:5) = GAM(1,1:2);
%Component of GAMMA referencing local y-coordinate system
GAM(2,1:3) = web_dir;
GAM(5,4:6) = GAM(2,1:3);
%Component of GAMMA referenicng local z-coordinate system
GAM(3,1:3) = cross(GAM(1,1:3),GAM(2,1:3)); %cross --> cross product
GAM(6,4:6) = GAM(3,1:3);

end
```

Local Stiffness Matrix Assembly

```
function [kele_local] = MEO_progkeleloc(L,A,Izz,E,K1,K2,v,Ayy)
% Author: Maximilian Ororbias

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Purpose: Function generates a 6x6 element stiffness matrix for a
% two-dimensional frame member or truss structure. Furthermore it accounts
% for the effects that are prevalent to connection properties
%
%       MEO_kele_local(L,A,Izz,E,Snode_flag,Enode_flag,K1,K2,v,Ayy,truss)
%
% Dictionary of Variables
%
% Input Information:
%
%       * L == Member length
%       * A == Member Area
%       * Izz == Moment of Inertia
%       * E == Elastic Modulus
%       * v == Poissons Ratio
%       * Ayy == Shear Area
%       * K1 == Rotational stiffness coefficient for connection at
%               start node, if start node connection is flexible
%       * K2 == Rotational stiffness coefficient for connection at
%               end node, if end node connection is flexible
%       * Snode_Flag == Flag at start node defining connection type
%       * Enode_Flag == Flag at end node defining connection type
%       * Truss == Flag indicating whether system is a truss
%
% Output Information:
%
%       * kele_local == 6x6 element stiffness matrix
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Inclusion of shear deformations
% Shear Modulus:
G = E/(2*v+2);
% Shear conversion factor
B_shr = 1 + (3*E*Izz)/(L^2*G*Ayy);

%Inclusion of connections through the determination of connection stiffness
%at element ends
%Connection constants
alpha1 = K1*L/E/Izz;
alpha2 = K2*L/E/Izz;
if isinf(alpha1), alpha1 = 1e16; end
if isinf(alpha2), alpha2 = 1e16; end
alp = 1/((4*B_shr-3)*alpha1*alpha2+4*B_shr*(alpha1+alpha2)+12);

%Preallocation of an element's local stiffness matrix
kele_local = zeros(6,6); % 6x6
```

```

%Assemble element local stiffness matrix
% Diagonal component of stiffness matrix
kele_local(1,1) = E*A/L;
kele_local(2,2) = alp*12*E*Izz*(alpha1*alpha2+alpha1+alpha2)/L^3;
kele_local(3,3) = alp*4*E*Izz*(alpha1*alpha2*B_shr+3*alpha1)/L;
kele_local(4,4) = kele_local(1,1);
kele_local(5,5) = kele_local(2,2);
kele_local(6,6) = alp*4*E*Izz*(alpha1*alpha2*B_shr+3*alpha2)/L;
% Upper triangular elements of the stiffness matrix
kele_local(1,4) = -1*kele_local(1,1);
kele_local(2,3) = alp*6*E*Izz*(alpha1*alpha2+2*alpha1)/L^2;
kele_local(2,5) = -1*kele_local(2,2);
kele_local(2,6) = alp*6*E*Izz*(alpha1*alpha2+2*alpha2)/L^2;
kele_local(3,5) = -1*kele_local(2,3);
kele_local(3,6) = alp*alpha1*alpha2*(3-2*B_shr)*2*E*Izz/L;
kele_local(5,6) = -1*kele_local(2,6);
% Lower triangular elements of the stiffness matrix (symmetric to upper)
kele_local(4,1) = kele_local(1,4);
kele_local(3,2) = kele_local(2,3);
kele_local(5,2) = kele_local(2,5);
kele_local(6,2) = kele_local(2,6);
kele_local(5,3) = kele_local(3,5);
kele_local(6,3) = kele_local(3,6);
kele_local(6,5) = kele_local(5,6);

end

```

Element End Forces

```
function[ELE_FOR] =
MEO_progelefor(global_disps, nele, ends, coord, webdir, A, Izz, E, v, Ayy)
% Author: Maximilian Ororbia

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Purpose: Calculates all of the element end forces and moments that
% reference each element's local coordinate system
%
%
% Dictionary of Variables
%
%   Input Information:
%   * global_disps == Global Displacement Array
%   * coord(i,1:2) == node i's coordinates
%                     coord(i,1) = X coordinate
%                     coord(i,2) = Y coordinate
%   * nele         == total number of elements
%   * ends(i,1:2)  == element i's nodal information
%                     ends(i,1) = start node #
%                     ends(i,2) = finish node #
%   * A(i)         == element i's cross sectional area
%   * Ayy == Shear Area
%   * v == Poissons Ratio
%   * Izz(i)       == element i's moment of inertia about its local z-
z axis
%   * E(i)         == element i's material elastic modulus, Young's
Modulus
%   * webdir(i,1:3) == element i's unit web vector. This is a unit
vector
%                     that defines the element's local y-y axis with
%                     respect
%                     to the global coordinate system. It is based only
on the
%                     structures undeformed geometry.
%                     webdir(i,1) = x component of element's unit web
vector
%                     webdir(i,2) = y component of element's unit web
vector
%                     webdir(i,3) = z component of element's unit web
vector
%                     (which will always be zero for 2D
analysis)
%   * thermal(i,1:4) == element i's thermal strain effects which
reference its
%                     local coordinate system
%   * w(i,1:2) == element i's uniform load which references its
%                     local coordinate system
%                     w(i,1) = x component of uniform load
%                     w(i,2) = y component of uniform load
%   * Truss == Flag indicating whether system is a truss
%
%   Output Information:
%   * ELE_FOR == element i's internal forces and moments
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Preallocation of the element end force matrix
ELE_FOR = zeros(nele,6);

%Cycle through each element
for element = 1:nele

    %Start node (i)
    start_node = ends(element,1);
    %End node (j)
    end_node = ends(element, 2);

    %Node coordinates
    ni_coord = coord(start_node, 1:2);
    nj_coord = coord(end_node, 1:2);
    web_dir = webdir(element, 1:3);

    %Develop nodal displacement numbering
    ele_disps_global(1:3,1) = global_disps(3*start_node-2:3*start_node);
    ele_disps_global(4:6,1) = global_disps(3*end_node-2:3*end_node);

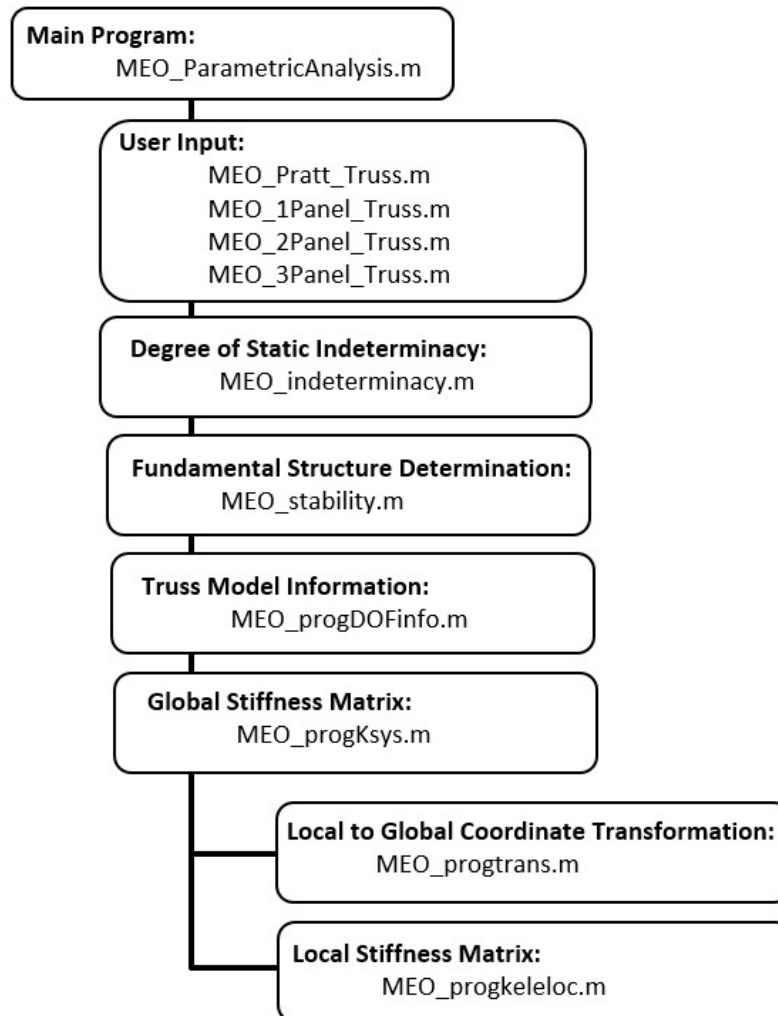
    %Call for functions...
    %Transformation matrix
    [L, GAM]=MEO_progtrans(ni_coord, nj_coord, web_dir);
    %Local stiffness matrix
    [kele_local] = MEO_progkeleloc(L,A(element),Izz(element),E(element),...
        ends(element,5),ends(element,6),v(element),Ayy(element));
    % %      %Lcoal Fixed End Action Vector
    % %      [fea_local] = MEO_FEA(w(element,1),w(element,2),Wt(element),...
    % %
    thermal(element,1),thermal(element,2),thermal(element,3),E(element),...
    % %
    Izz(element),A(element),ends(element,3),ends(element,4),ends(element,5),...
    % %      ends(element,6),v(element),Ayy(element),L,GAM );

    %Generate local displacements
    ele_disp_local = GAM * ele_disps_global;
    %Determine element forces
    ele_forces_local = kele_local * ele_disp_local;% + fea_local;
    %Element end forces/moments
    ELE_FOR(element, 1:6) = ele_forces_local';
end

end

```

Appendix C: MATLAB Scripts for Parametric Analysis



Parametric Analysis Main Routine

```
% MEO_ParametricAnalysis
% Maximilian Ororbias

% Parametric Analysis
% Purpose: Calculates the normalized structural complexity index and the
average
% deformation work of truss structures with a randomly applied set of
% areas.

clear; close all

%Start time
tic

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- LOAD TRUSS TOPOLOGY DEFINED INPUT -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%User Note: Change amount area assignments where noted
% run MEO_Pratt_Truss.m
% run MEO_1Panel_Truss.m
% run MEO_2Panel_Truss.m
run MEO_3Panel_Truss.m

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- END OF USER INPUT -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- DEGREE OF STATIC INDETERMINACY -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Determines the number of elements needed to be removed to generate a
% statically determinate system
[rmv] = MEO_indeterminacy(nele, nnodes, fixity);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- END OF INDETERMINACY -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- MAXTAN2 STABILITY CHECK -----%%
%%----- CHECK EACH FUNDAMENTAL STRUCTURE -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%NEED TO CHANGE THE AMOUNT OF AREAS TESTED IN THE STABILTIY CODE
%CHANGE TO ADD ANOTHER ZERO AREA
[fslist, nfs, qq] = MEO_stability(nele, rmv, nnodes, fixity, concen, ends,
coord, webdir, Izz, E, v, Ayy);
stbl = size(fslist);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- END STABILITY ANALYSIS -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%----- PARAMETRIC ANALYSIS -----%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% PARAMETRIC DAMAGE ANALYSIS

ns = 100; %Number of random areas
%CHANGE FOR DESIRED No. OF SAMPLES

% Set random number generator algorithm and stream
% Generate RVs for other elements
randgen = RandStream('mrg32k3a','Seed',1234);

% Assign element 1 to a fixed area across all samples
refA=1e-3; %Reference Area as defined by De Biagi
Alim=20*refA; %Random Area limit
Arand = rand(randgen,nele-1,ns)*Alim; %m^2

% Create matrix with fixed areas
Aelem(1,1:ns)=refA*ones(1,ns);
Aelem(2:nele,1:ns)=Arand;

% Preallocation of Metrics
M = zeros(1,ns);
NSCI = zeros(1,ns);

% Start loop for each random sample
for j=1:ns

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %% --- ANALYZE REFERENCE STRUCTURED --- %%
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    % Model information
    [f_dof,s_dof,global_disps,global_concens] =
MEO_progDOFinfo(nnodes,fixity,concen);

    % Area input within this function
    A = Aelem(1:nele,j);

    [K_system] = MEO_progKsys(nnodes,nele,ends,coord,webdir,A,Izz,E,v,Ayy);

    % Organization
    Kff = K_system(f_dof,f_dof);
    Pf = global_concens(f_dof);

    % Displacements
    Deltaf = Kff\Pf;
    global_disps(f_dof) = Deltaf;
    DEFL = reshape(global_disps,3,nnodes)';
    % Collect vertical displacements
    %Y_FSDisp(1,:,j) = DEFL(:,2)';
    Y_FSDisp = DEFL(:,2)';

    % Deformation Work of Reference Structure

```



```

RS_Defwork = (Y_FSDisp(:,fnode)*force)/2;

%%% ----- END OF REF. STRT ANALYSIS ----- %%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% --- ANALYZE ONE MEMBER REMOVAL --- %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% CYCLE THROUGH FOR EACH ELEMENT REMOVED

% Prellocation of Displacements
Y_1EDisp = zeros(nele,nnodes);

for k=1:stbl(1)

    % Remove members by making kth member's area zero
    Atemp = Aelem(1:nele,j);
    Atemp(fslist(k),1) = 0;

    % Model information
    [f_dof,s_dof,global_disps,global_concens] =
MEO_progDOFinfo(nnodes,fixity,concen);

    % Area input within this function
    [K_system] =
MEO_progKsys(nnodes,nele,ends,coord,webdir,Atemp,Izz,E,v,Ayy);

    % Organization
    Kff = K_system(f_dof,f_dof);
    Pf = global_concens(f_dof);
    Deltas = global_disps(s_dof);

    % Displacements
    Deltaf = Kff\Pf;
    global_disps(f_dof) = Deltaf;
    DEFL = reshape(global_disps,3,nnodes)';
    % Collect vertical displacements
    Y_1EDisp(k,:) = DEFL(:,2)';

end

% Deformation Work for a single element removed
OE_Defwork(:, :) = (Y_1EDisp(:,fnode)*force)/2;

%%% ----- END OF ONE ELE. REMOVAL ----- %%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% --- ANALYZE FUNDAMENTAL STRUCTURE --- %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Preallocation
Y_2EDisp = zeros(qq,nnodes);

% Start loop for each fundamental stable structure
for k=1:stbl(1)

    % Remove members by making kth member's area zero

```

```

        A2temp = Aelem(1:nele,j);
        A2temp(fslist(k,1),1) = 0;
        A2temp(fslist(k,2),1) = 0; %Add for MEO_2Panel_Truss.m
        A2temp(fslist(k,3),1) = 0; %Add for MEO_3Panel_Truss.m
%CHANGE TO ADD ANOTHER ZERO AREA

        % Model information
        [f_dof,s_dof,global_disps,global_concens] =
MEO_progDOFinfo(nnodes,fixity,concen);

        % Area input within this function
        [K_system] =
MEO_progKsys(nnodes,nele,ends,coord,webdir,A2temp,Izz,E,v,Ayy);

        % Organization
        Kff = K_system(f_dof,f_dof);
        Pf = global_concens(f_dof);
        Deltas = global_disps(s_dof);

        % Displacements
        Deltaf = Kff\Pf;
        global_disps(f_dof) = Deltaf;
        DEFL = reshape(global_disps,3,nnodes)';
        % Collect vertical displacements
        Y_2EDisp(k,:) = DEFL(:,2)';

end

% Deformation Work for each Fundamental Structure
FS_Defwork(:, :) = (Y_2EDisp(:,fnode)*force)/2;

%%% ----- END OF FUNDAMENTAL STRUCTURES ANALYSIS ----- %%%

%%% ----- DETERMINE COMPLEXITY METRICS ----- %%%

% Damage Index (Omega)

% Preallocation
Omega = zeros(nele,ns);

for k=1:stbl(1)%nele %nfs
    Omega(k,:) = (OE_Defwork(k,1) - RS_Defwork)/RS_Defwork;
end

% Average Structural Response, M-value
M(1,j) = (1/nele)*sum(Omega(:,1));

% Psi(i,j) = ratio of deformation work = reference/each fundamental
% Preallocation
psi = zeros(nfs,1);

for k=1:nfs
%
    psi(k,j) = Defwork(1,j)/Defwork(k+1,j);
    psi(k,1) = RS_Defwork/FS_Defwork(k,1);
end

```

```

sumpsi = sum(psi(:,1));

% Structural Complexity Index, SCI & Normalized Structural Complexity
%Index, NSCI

% Preallocation
ratiopsi = zeros(nfs,1);
logpsi = zeros(nfs,1);
multpsi = zeros(nfs,1);

for k=1:nfs
    ratiopsi(k,:) = psi(k,:)/sumpsi(1,:);
    logpsi(k,:) = log2(ratiopsi(k,:));
    multpsi(k,:)= ratiopsi(k,:).*logpsi(k,:);
end

SCI(1,:) = -1*sum(multpsi(:, :));

NSCI(1,j) = SCI(1,:)/log2(nfs);

end

% PLOT COMPLEXITY METRICS

semilogy(NSCI,M, 'r.')
% plot(NSCI,M, 'r.')
xlabel('NSCI')
ylabel('M')

%%% --- END OF PARAMETRIC ANALYSIS --- %%%

% End time
time = toc;

% Save data
save CompMetrics.mat

```