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The Influence of a Weld-Affected Zone on the Compressive and Flexural Strength of Aluminum Members

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**THE INFLUENCE OF A WELD-AFFECTED ZONE
ON THE COMPRESSIVE AND FLEXURAL STRENGTH
OF ALUMINUM MEMBERS**

By

Shengduo Du

A Thesis

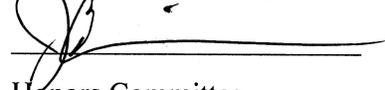
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Approved by:



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5/9/2013

Date Submitted

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Shengduo Du

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Table of Contents

Acknowledgement

Table of Contents

Lists of Tables

List of Figures

Abstract

1 Introduction

1.1 Background and Motivation

1.2 Overview of Weld-Affected Zone

1.3 Research Goals and Approach

1.4 Thesis Outline

2 Literature Review

2.1 AISC Specification

2.2 AA Specification

2.3 Methodology

2.4 Summary

3 Compressive Strength

3.1 Elastic Column Buckling and the Effect of the End Restraint

3.2 Flexural Buckling Strength of Columns

3.3 Comparison of MASTAN Compressive Strength Curves with AISC and AA Curves

4 Flexure Strength

4.1 Lateral-Torsional Buckling of Beams with Uniform Moment

4.2 Lateral-Torsional Buckling of Beams with Moment Gradient

4.3 Comparison of MASTAN Flexural Strength Curves with AISC, AISI and AA Curves

5 Results and Discussions

6 Conclusions and Recommendations

List of References

Appendices

Appendix A Computer Models

Appendix B Compressive Strength

Appendix C Flexure Strength

Lists of Tables

Table 3.1 Comparison of theoretical buckling loads and MASTAN 2 modeling results

Table 3.2 Comparison of elastic buckling loads with varying welding locations

List of Figures

Figure 1.1a. Failure modes of column and beam: Flexure buckling

Figure 1.1b. Failure modes of column and beam: Lateral-torsional

Figure 1.2 Design curve for columns

Figure 1.3 Design curve for beams

Figure 1.4 Weld-affected zone of aluminum member

Figure 2.1 AISC Critical stress curves for compressive members

Figure 2.2 AISC Moment capacity curves for flexural buckling members

Figure 2.3 AA Critical stress curves for compressive members

Figure 2.4 AA Moment capacity curves for flexural buckling members

Figure 3.1 Approximate values of effective length factor, K

Figure 3.2 Second-order $P\delta$ effect in column

Figure 3.3 Minor-axis column curves with imperfections and weld-affected zone at varying locations (MASTAN 2)

Figure 3.4 Minor-axis column curves with imperfections and weld-affected zone at varying locations (FE++)

Figure 3.5 Minor-axis column curves with weld-affected zone at the end

Figure 3.6 Minor-axis column curves with weld-affected zone at the mid-span

Figure 3.7 Minor-axis column curves with weld-affected zone at one-third of the length

Figure 3.8 Minor-axis column curves with weld-affected zone at the one-fourth of the length

Figure 3.9 Comparison of computational and MASTAN2 computer modeling minor-axis column curves with weld-affected zone at varying locations

Figure 3.10 Comparison of computational and FE++ computer modeling minor-axis column curves with weld-affected zone at varying locations

Figure 4.1 Summary of beam curves with uniform moment

Figure 4.2 Beam curves by using MASTAN2 with uniform moment

Figure 4.3 Nominal moment beam curves with $C_b=1.25$

Figure 4.4 Moment diagram ($C_b=1.25$) of welded affected

Figure 4.5 Moment diagram ($C_b=1.25$) of welded affected zone at the right end of the beam

Figure 4.6 Nominal moment beam curves with $C_b=1.67$

Figure 4.7 Moment diagram ($C_b=1.67$) of welded affected zone at the three quarter of the beam

Figure 4.8 Moment diagram ($C_b=1.67$) of welded affected zone at the right end of the beam

Figure 4.9 Nominal moment beam curves with $C_b=2.17$

Figure 4.10 Moment diagram ($C_b=2.17$) of welded affected zone at mid-span of the beam

Figure 4.11 Moment diagram ($C_b=2.17$) of welded affected zone at the three quarter of the beam

Figure 4.12 Nominal moment beam curves with $C_b=2.27$

Figure 4.13 Moment diagram ($C_b=2.27$) of welded affected zone at the one-quarter of the beam

Figure 4.14 Moment diagram ($C_b=2.27$) of welded affected zone at the three-quarter of the beam

Figure 4.15 Comparison of nominal moment beam curves ($C_b=1.0$) generated by MASTAN2 and AISC and AA computational methods

Figure 4.16 Comparison of nominal moment beam curves ($C_b=1.25$) generated by MASTAN2 and AISC and AA computational methods

Figure 4.17 Comparison of nominal moment beam curves ($C_b=1.67$) generated by MASTAN2 and AISC and AA computational methods

Figure 4.18 Comparison of nominal moment beam curves ($C_b=2.17$) generated by MASTAN2 and AISC and AA computational methods

Figure 4.19 Comparison of nominal moment beam curves ($C_b=2.27$) generated by MASTAN2 and AISC and AA computational methods

Figure 5.1 Nominal moment strength ($C_b=1.25$) with varying locations of weld-affected zone

Figure 5.2 Comparison of nominal moment strength curves ($C_b=1.25$) with varying locations of weld-affected zone

ABSTRACT

In structural and mechanical engineering, welding is known to produce a heat affected zone (HAZ), which is also defined in some specifications as a weld-affected zone. The Aluminum Association's (AA) *Specification for Aluminum Structures* conservatively requires that the reduced material yield strength F_y of the weld-affected zone must be assumed to be the yield strength for the entire adjoined member, and this significant reduction has come under recent scrutiny by members of the AA Specification committee.

This thesis presents a computational study on the influence of weld-affected zone on the compressive and flexural strength of aluminum members and the relationship between the longitudinal locations of the weld-affected zone and the buckling strength of beams and columns. This project incorporates the use of two finite element analysis programs: frame analysis software MASTAN2 (2012) and FE++2012. The results of the analysis agree with the prediction that applying reduced material yield strength F_y for the entire adjoined member is conservative.

Analyses of a series of beams and columns of varying length and locations of the weld-affected zone indicate that welding location is a key parameter in determining the compressive and flexural strength. Varying moment gradients applied at the beam models illustrated the different levels of influence of locations of the weld-affected zone in different moment scenarios. By comparing the strength curves of both columns and beams developed by the software, and the specifications of AISC and AA, recommendations for the yield strength of welded aluminum members and its relationship with welding locations are presented.

Chapter 1

Introduction

In structural and mechanical engineering, welding is a fabrication process that is widely used to join materials. This is often done by heating the base material to high temperatures and then fusing the material together through the use of a filler material. Welding is known to produce a heat affected zone (HAZ), which is also called a weld-affected zone. The modulus of elasticity E does not change significantly when welding aluminum structural members together, but the material strength F_y in the weld-affected zone decreases to approximately one-third of its original strength. [1]

1.1 Background and Motivation

A column is usually a vertical structural element that transmits, through compression, the weight of the structure and other loads above it to other structural elements below it. Because a column is a compression member, the main failure mode is flexural buckling (Fig. 1.1(a)). A beam is typically a horizontal structural element that is able to transmit loads to its supports mainly by its bending resistance. The primary failure mode of beam is lateral-torsional buckling (Fig. 1.1(b)).

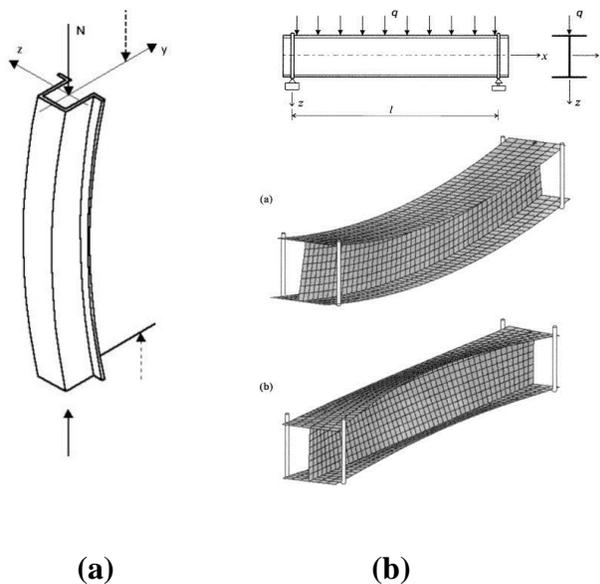


Figure 1.1 Failure modes of column and beam: (a) Flexure buckling (*Katedra za metalnekonstrukcije, 2012*); (b) Lateral-torsional (*Ákos Sapkás, László P. Kollár, 2002*).

The strength design curves of columns and beams appearing in most design specifications worldwide take on the form shown in Figures 1.2 and 1.3.

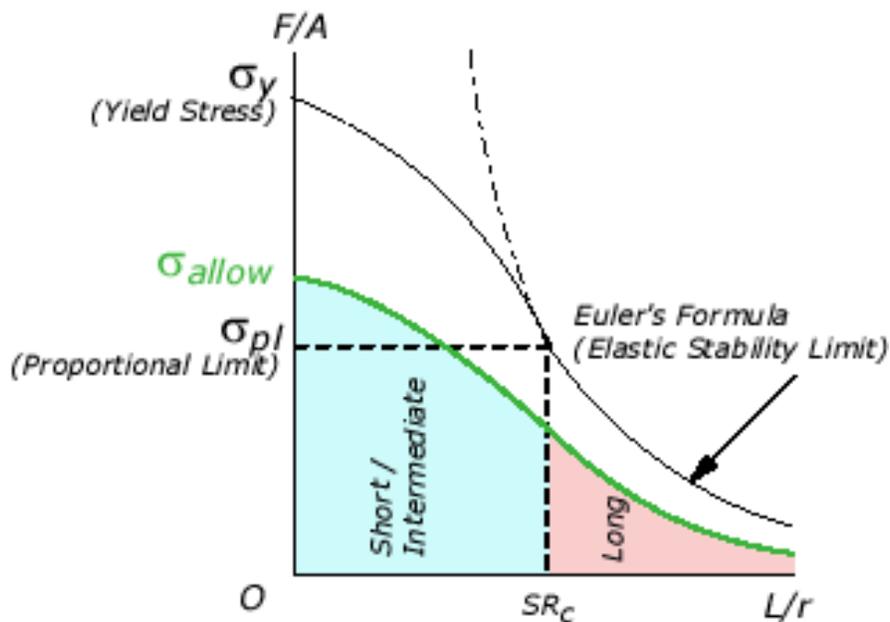


Figure 1.2 Design curve for columns (*AISC, 2012*)

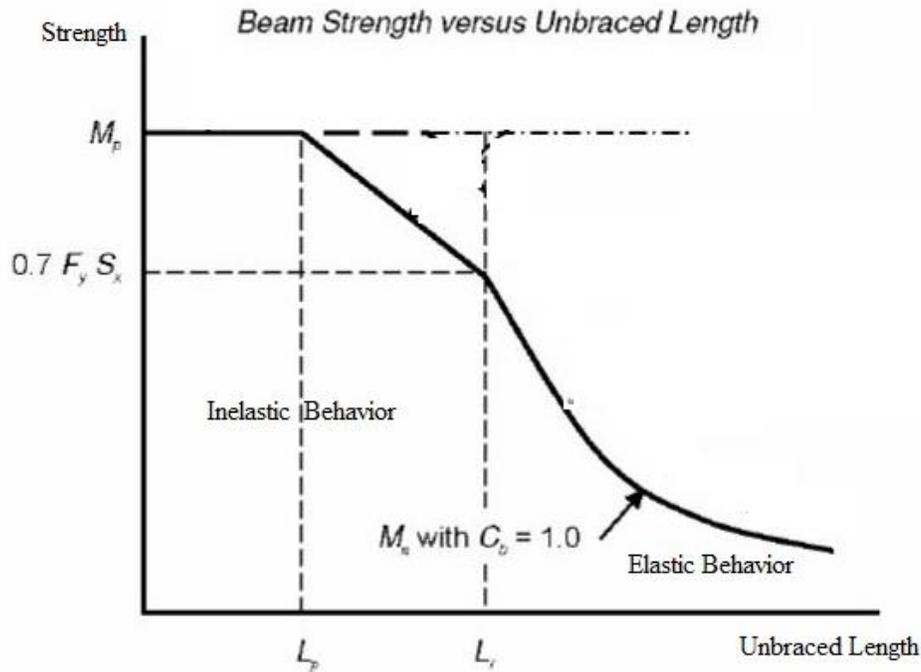


Figure 1. 3 Design curve for beams (*AISC Steel Construction Manual, 14th Ed. Fig F-1, 2011*)

The shapes and trends of these two design curves are very similar. The behavior includes elastic and inelastic regions, which are primarily a function of the unbraced length of the structural member. When the long slender structural member is subjected to load, the member behaves elastically, and thus only the modulus of elasticity E , which is a measure of the elastic stiffness of the material, would affect the strength of the member. On the other hand, as the length of the structural member decreases, the material yield strength F_y also plays an essential role in determining the member's strength because it partially yields before it buckles (inelastic buckling). For both beams and columns, a reduction in either E or F_y will result in much smaller flexural and compression strengths.

1.2 Overview of Weld-Affected Zone

Welding process is known to produce a heat affected zone (HAZ), which is also defined in some specifications as a weld-affected zone (Fig. 1.4). The width of the lateral welding is normally 1% to 2% of the combined length of the two structural members. [2]

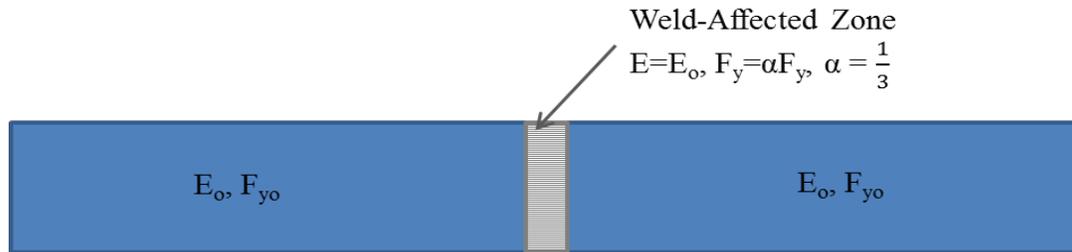


Figure 1.4 Weld-affected zone of aluminum member

Depending on the filler materials used and the temperature of the welding process, the impacts of welding on the base material that surrounds the weld zone can be detrimental. When steel members are welded together using steel filler material, the modulus of elasticity typically remains unchanged for both the members and the weld-affected zone. According to American Institution of Steel Construction, a governing body for the design of steel structures in the United States, the yield strength of the weld filler material must be significantly higher than that of the base material. It is also noted that the heat from the welding process does not significantly change the yield strength of the member base material. As a result, AISC employs the stiffness E , and strength, F_y , of the member base material in defining the member's compressive and flexural strengths. The results, however, are quite different when welding aluminum structural members together. Although the modulus of elasticity E does not change significantly, the material strength F_y in the weld-affected zone decreases to approximately one-third of its original strength (Fig. 1.4). With this in mind, the Aluminum Association's (AA) *Specification for*

Aluminum Structures conservatively requires that this reduced material yield strength, $\frac{1}{3}F_y$, must be assumed for the entire adjoined member when proportioning this member for compression or flexure; this is regardless of the fact that the weld-affected zone only comprises 1% to 2% of the member's length. For short stocky members with a centrally located weld-affected zone, this reduction would seem reasonable. For members with intermediate slenderness and/or a weld-affected zone not near mid-span, this significant reduction has become under recent scrutiny by members of the AA Specification committee. With this in mind, a computational study was developed to investigate the relationship between the longitudinal locations of the weld-affected zone and the buckling strength of aluminum beams and columns.

1.3 Research Goals and Approach

This project incorporates the use of two finite element analysis programs. The less-refined frame analysis software MASTAN2 (2012) was first used to better understand the problem and define a series of beams and columns of varying length and locations of the weld-affected zone. The more refined frame analysis software FE++2012 was then used to assess the results being provided by MASTAN2.

The results include beam and column design curves with the x-axis defining the slenderness ratio, and y-axis the member buckling strength. The results generated from the study will also be compared with the current strength curves provided in the AA Specification. Based on these results, buckling strength curves that account for weld-affected zones, and a new potential design equation that better represents the relationship between the locations of the weld-affected zone and the buckling strength of a structural member are established. The new

buckling strength curves that account for weld-affected zones provide a more accurate and more economical method for designing aluminum structures.

1.4 Thesis Outline

This thesis consists of six chapters, including background description, motivation and objectives in Chapter 1. Chapter 2 illustrates the related information and current design criteria in this area. It includes the AISC and AA design methods. Chapter 3 covers the analysis of the compressive strength of aluminum columns. A reasonable modeling method for a weld-affected zone and the results comparison with AISC and AA specifications are also discussed. Chapter 4 presents the modeling of beams with weld-affected zones and the analysis of the flexural strength by using similar method that was performed for compressive members. Results comparison with AISC and AA specification are presented. Chapter 5 explains the results and discussion of the potential improvement of the current design guidelines of AA and AISC. Finally, summary of research and future research recommendations are presented in Chapter 6.

Chapter 2

Literature Review

According to AISC, the yield strength of the weld filler material must be significantly higher than that of the base material, but the heat provided by the welding process will not significantly decrease the yield strength of the base material. Therefore, AISC uses the stiffness, E , and strength, F_y , of the member base material in defining the member's compressive and flexural strengths. Although the AISC criterion is applied to steel design, the criterion is used in this research when analyzing aluminum as benchmark of comparison. Additionally, the AISC design equations are modified to eliminate the residual stress when applying on aluminum members.

However, welding aluminum structural members will substantially decrease the material strength F_{cy} in the weld-affected zone to approximately one-third of its original strength. To be conservative, AA requires that this reduced material yield strength, F_{cy} , must be applied to the entire adjoined member when proportioning this member for compression or flexure.

2.1 AISC Specification

The compressive behavior of columns include elastic and inelastic behavior, which are primarily determined by the member slenderness, KL/r , where K is the effective length factor, L is the laterally unbraced length of the member and r is the radius of gyration. According to AISC Steel Construction Manual [2] Chapter E Design of Members for Compression, the nominal compressive strength, P_n , shall be determined based on the limit state of flexural buckling:

$$P_n = F_{cr}A_g \quad (\text{E2-1})$$

where the critical stress, F_{cr} is determined as follows:

When $KL/r \leq 4.71 \sqrt{\frac{E}{F_y}}$,

$$F_{cr} = \left[0.658 \frac{F_y}{F_e} \right] F_y \quad (\text{E2-2})$$

When $KL/r > 4.71 \sqrt{\frac{E}{F_y}}$,

$$F_{cr} = 0.877 F_e \quad (\text{E2-3})$$

where the elastic buckling stress F_e is determined as follows:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r} \right)^2} \quad (\text{E2-4})$$

Based on Equation 2-1 to Equation 2-4, Curves in Figure 2.1 below were generated to show the critical stress curves for T6061 Aluminum members:

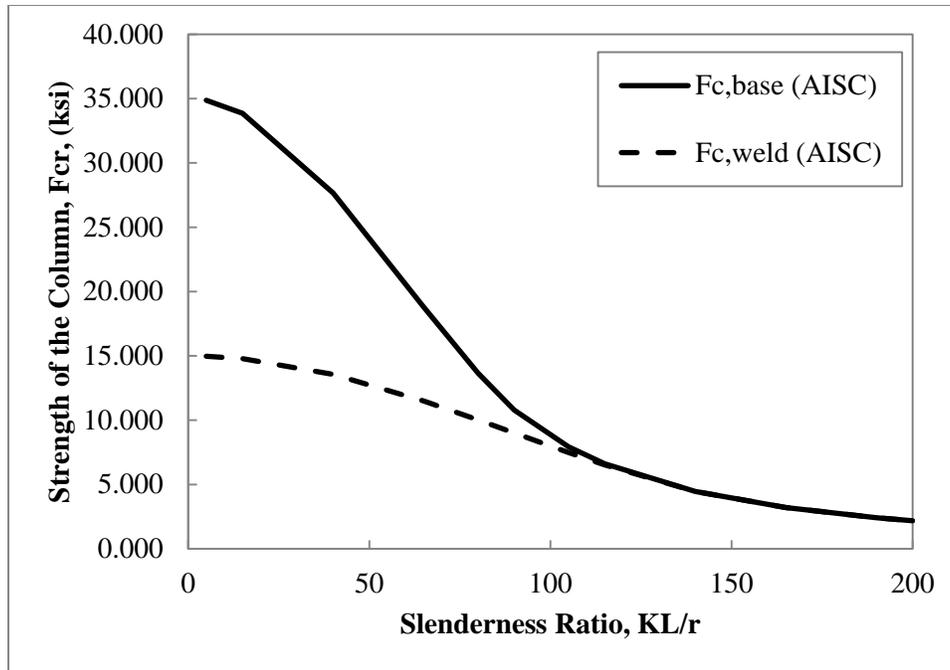


Figure 2.1 AISC Critical stress curves for compressive members

When a long slender structural member is subjected to axial load, the compressive member behaves elastically, and thus only the modulus of elasticity, E , affects the strength of the member, as shown in Equation 2-3. As the length of the compressive members decreases, the material yield strength, F_y , also has an impact on the member's strength due to inelastic buckling. However, because the column transmits the dead load and other loads through compression to other structural elements below it, once the weld-affected zone reaches its strength limit, the entire column fails. Therefore, when designing a column, the material yield strength, F_y , of the weld-affected zone controls the nominal strength of the column, P_n .

The behavior of flexural buckling in beams include yielding, inelastic lateral-torsional buckling and elastic lateral-torsional buckling. The moment capacity is determined by the limiting lengths L_p and L_r that are defined as follows:

$$L_p = r_y \sqrt{\frac{E}{F_y}} \quad (\text{E2-5})$$

$$L_r = 1.95r_{ts} \frac{E}{F_y} \sqrt{\frac{Jc}{S_x h_0} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.67\left(\frac{F_y}{E}\right)^2}} \quad (\text{E2-6})$$

where

$$r_{ts}^2 = \frac{I_y h_0}{2S_x} \quad (\text{E2-7})$$

and the coefficient $c=1$ for doubly symmetric I-shapes.

For all doubly symmetric members and singly symmetric members in single curvature, C_b , the lateral-torsional buckling modification factor for non-uniform moment diagrams when both ends of the segment are braced, accounts for the impact on the flexural strength, and it is determined as follows:

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad (\text{E2-8})$$

where

M_{max} = absolute value of maximum moment in the unbraced segment,

M_A = absolute value of maximum moment at the quarter point of the unbraced segment,

M_B = absolute value of maximum moment at the centerline point of the unbraced segment,

M_C = absolute value of maximum moment at three-quarter point of the unbraced segment.

According to AISC [1] Chapter F Design of Members for Flexure, the nominal flexural strength, M_n , shall be the lower value obtained from the limit states of yielding, plastic moment, and lateral-torsional buckling. The nominal flexural strength M_n is defined as follows:

When $L_b \leq L_p$,

$$M_n = M_p = F_y Z_x \quad (\text{E2-9})$$

where F_y is the specified minimum yield stress of the type of material being used, and Z_x is the plastic section modulus about the x-axis.

When $L_p < L_b \leq L_r$,

$$M_n = C_b \left[M_p - (M_p - F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \quad (\text{E2-10})$$

When $L_b > L_r$,

$$M_n = F_{cr} S_x \leq M_p \quad (\text{E2-11})$$

where

L_b = length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}} \right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_0} \left(\frac{L_b}{r_{ts}} \right)^2} \quad (\text{E2-12})$$

and where

J = torsional constant

S_x = elastic section modulus taken about the x-axis

h_0 = distance between the flange centroids

Based on Equation 2-5 to Equation 2-12, curves in Figure 2.2 were generated to show the moment capacity curves for for T6061 Aluminum members:

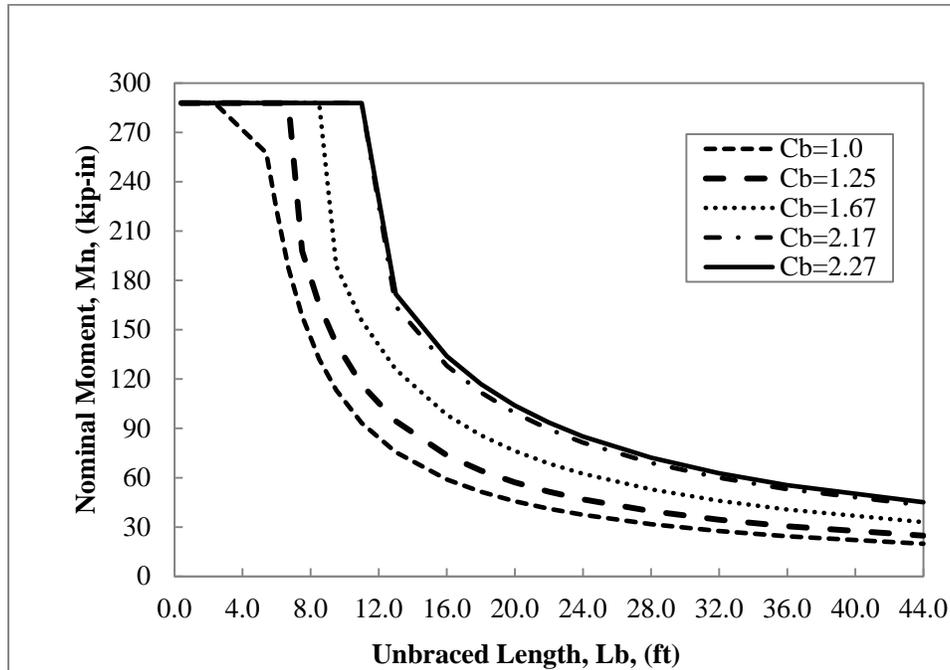


Figure 2.2 AISC Moment capacity curves for flexural buckling members

As shown in the Figure 2.2, when a short beam is subjected to moment, the failure mode is yielding, which is controlled by the yield strength, F_y . As the length of the beam increases, the modulus of elasticity, E , also has an impact on the nominal moment strength and causes inelastic lateral-torsional buckling. When the length of the member reaches the limiting length, L_r , the failure is controlled by elastic lateral-torsional buckling, this is only controlled by the modulus of elasticity, E . When different moment gradient scenarios are applied to the ends of the braced length, the nominal strength curves change. As C_b , the lateral-torsional buckling modification factor increases, the nominal moment increases as shown on the y-axis. However, because beams are horizontal structural elements that are able to transmit loads to their supports mainly by their

bending resistances, the reduced material yield strength, F_y , of the weld-affected zone should not be assumed for the entire adjoined member when proportioning this member for flexure. [1]

2.2 AA Specification

The nominal member buckling strength is described in AA Aluminum Design Manual [1] Chapter E Design of Members for Compression. The nominal member buckling strength of a member supported at both ends with no transverse weld farther than $0.05L$ from the member ends shall be calculated as if there were no welds. The nominal member buckling strength of a member supported at both ends with a transverse weld farther than $0.05L$ from the member ends and a member supported at only one end with a transverse weld at any location shall be calculated as if the entire cross sectional area were weld-affected. The nominal member buckling strength, P_n , is

$$P_n = F_c A_g \quad (\text{E2-13})$$

where the stress corresponding to the uniform compressive strength, F_c is determined as follows:

When $KL/r \leq C_c$,

$$F_c = 0.85 \left(B_c - \frac{D_c KL}{r} \right) \leq F_{cy} \quad (\text{E2-14})$$

When $KL/r > C_c$,

$$F_c = \frac{0.85\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad (\text{E2-15})$$

where B_c , D_c and C_c are the buckling constants, and they are determined as follows:

For weld-affected zones of all tempers (including T6),

$$B_c = F_{cy} \left[1 + \left(\frac{F_{cy}}{1000} \right)^2 \right] \quad (\text{E2-16})$$

$$D_c = \frac{B_c}{20} \left(\frac{6B_c}{E} \right)^{\frac{1}{2}} \quad (\text{E2-17})$$

$$C_c = \frac{2B_c}{3D_c} \quad (\text{E2-18})$$

For temper designations beginning with T6,

$$B_c = F_{cy} \left[1 + \left(\frac{F_{cy}}{2250} \right)^2 \right] \quad (\text{E2-19})$$

$$D_c = \frac{B_c}{10} \left(\frac{B_c}{E} \right)^{\frac{1}{2}} \quad (\text{E2-20})$$

$$C_c = 0.41 \frac{B_c}{D_c} \quad (\text{E2-21})$$

and where F_{cy} is the compressive yield strength.

Based on Equation 2-13 to Equation 2-21, curves in Figure 2.3 were generated to show critical stress curves for compressive members for T6061 Aluminum members:

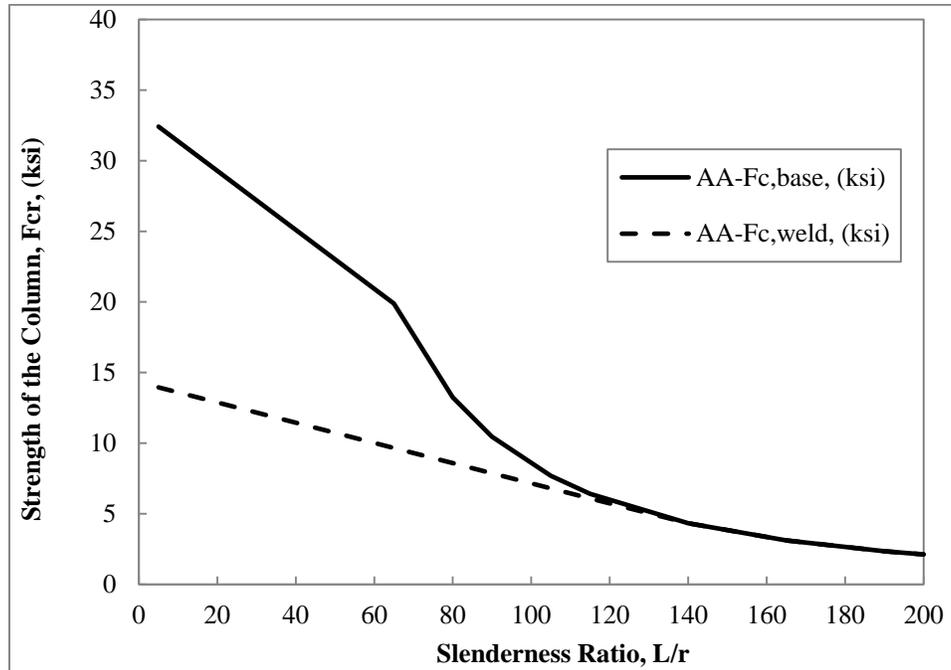


Figure 2.3 AA Critical stress curves for compressive members

Figure 2.3 shows that, similar to *AISC* critical stress curves, the long slender structural member behaves elastically and is only affected by the modulus of elasticity E . As the length of the compressive members decreases, the material yield strength F_y also becomes an important factor in determining the member's strength due to inelastic buckling. However, according to AA, the nominal buckling strength for members with welds shall be calculated as if the entire cross sectional area were weld-affected. Therefore, F_y of the weld-affected zone should be used when proportioning the nominal strength of the column.

The nominal flexural strength is described in AA Aluminum Design Manual [2] Chapter F Design of Members for Flexure. The impact of the weld-affected zone can be ignored when it is within $0.05L$ from the member ends, otherwise such an impact shall be also treated as if the

entire cross-sectional area were weld-affected. The lateral-torsional buckling strength is determined by the one limiting length, S_2 , and the same lateral-torsional buckling modification factor for nonuniform moment that was employed in *AISC* criteria, C_b . The nominal flexural strength M_n is defined as follows:

$$M_n = F_b S_c \quad (\text{E2-22})$$

where, F_b , is the lateral-torsional buckling stress, and S_c is the section modulus on the compression side of the neutral axis. F_b is defined as follows:

when $\frac{L_b}{r\sqrt{C_b}} < S_2$,

$$F_b = B_c - \frac{D_c L_b}{1.2 r_y \sqrt{C_b}} \quad (\text{E2-23})$$

when $\frac{L_b}{r\sqrt{C_b}} \geq S_2$,

$$F_b = \frac{\pi^2}{\left(\frac{L_b}{1.2 r_y \sqrt{C_b}}\right)^2} \quad (\text{E2-23})$$

and where

$$S_2 = 1.2 C_c \quad (\text{E2-23})$$

Based on Equation 2-22 to Equation 2-24, Curves in Figure 2.4 below were generated to show the AA Moment capacity curves for T6061 Aluminum members:

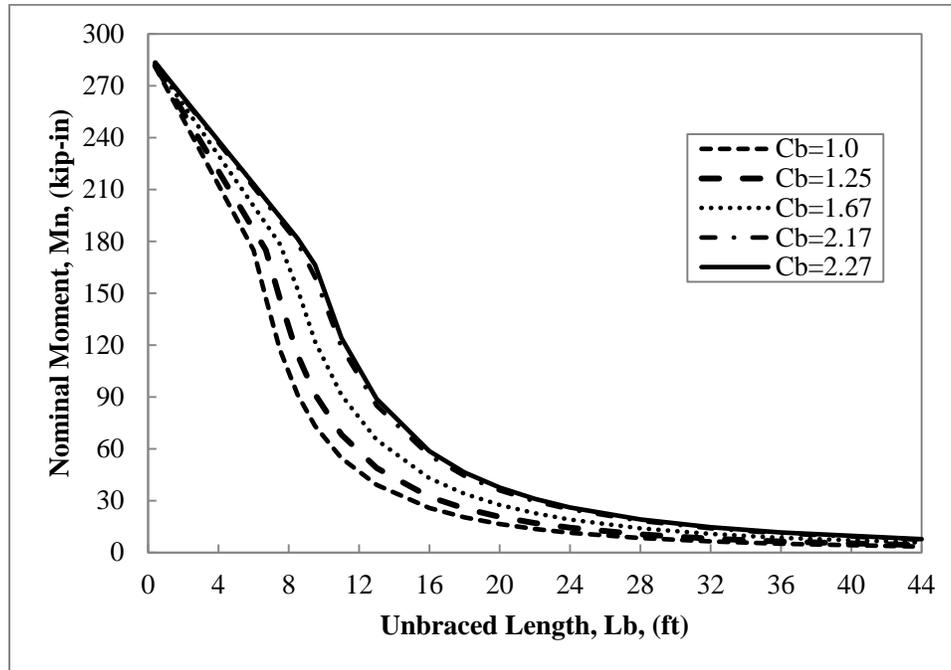


Figure 2.4 AA Moment capacity curves for flexural buckling members

As shown in the Figure 2.4, short members' failure modes are yielding, which is controlled by the compressive yield strength, F_{cy} . As the length of the member increases, the modulus of elasticity, E , is the only parameter that has an impact on the nominal flexural strength and causes elastic lateral-torsional buckling. The nominal strength curves also move vertically in the positive y-direction based on different C_b values, the lateral-torsional buckling modification factor. However, it is skeptical to assume that the reduced material yield strength, F_y , of the weld-affected zone can be attributed to the entire adjoined member when computing the flexural strength.

2.2 Methodology

The MASTAN2 (2012) analysis program used in this study employs the finite element method with the key incremental stiffness equation

$$[K_t]\{d\Delta\}=\{dP\} \quad (E2-24)$$

where $[K_t]$ is a tangent stiffness matrix that reflects the current state of the deformed structure, and $\{d\Delta\}$ contains the displacements corresponding to an increment of loading $\{dP\}$.

In McGuire and Ziemian (2000) and then later in Ziemian and McGuire (2002) attempts are made to improve the ability of the MASTAN2 program to simulate the behavior of systems of small redundancy in which the spread yielding may have a direct impact on limit state behavior, particularly in cases of minor-axis bending. The latter paper, which only deals with in-plane instability, uses the following modified tangent modulus

$$E_{tm} = \tau E_0 \text{ with } \tau = \min \left(\frac{1.0}{(1+p)[1-(p+\alpha_y m_y)]} \right) \quad (E2-22)$$

where the factor α_y is an empirical one, which after calibration with a series of comparative plastic zone analyses was set at 0.65. This value provided good results for typical wide-flange shapes. Meanwhile, $m_y = M_y/M_{py}$, and M_y and M_{py} are the axial yielding load and the major- and minor-axis plastic moments, respectively [11].

FE++ 2012 is an object-oriented application framework, initially developed at Purdue University, has been used and extensively modified for the current research interests. The framework basically is composed of three modules: Main Module, Analysis Module, and Matrix and Utility Library Modules. The main module serves as a parser program.

It reads input files and call necessary routines in FE++ 2012. Furthermore, it creates container lists that are used by the Analysis Module. These lists store information about elements, nodes, loads, or material descriptions. Analysis Module is called by the Main Module to start the analysis. Matrix and Utility Library Modules is an object-oriented matrix library [8] and a utility library because several libraries are utilized during analyses. Users can perform multiple step analyses.

The chosen increment and iterative types adjust the time increment if necessary and all corresponding amplitude curves are updated to the current state accordingly. During setting proper time increment, break times are considered. In the current framework, the “break” in one or more of the amplitude curves occurs at the ends of steps, and at slope discontinuities in the amplitude curves [12].

2.3 Summary

The location of a welding should influence a the change of the strength because there is large possibility that the welding located in the mid-span of the member would cause a greater decrease in strength than a weld located at other locations of the member. Multiple types of computer models were used in this research to investigate the relation between the location of the weld-affected zone and the strength of the structural members.

Different moment gradients also have an impact on the strength of the aluminum members through C_b , so it is highly possible that the moment gradient and the location of the weld-affected zone should have a combined effect on the strength of the welded members. Therefore, computer models are also be used in this study to explore the interaction of the

location of the weld-affected zone and the moment gradient and their combined impact on the strength of the structural members.

As suggested by Randy Kissell [6], an expert specializing in aluminum structures, the I6x4.03 (T6061) shape was the main structural member being used in the research. A 2-inch weld-affected zone was employed in all different lengths of structures for consistency. Partial yielding and imperfections due to manufacturing were also modeled by deflecting the structural member at the mid-span by $L/1000$ in all the computer models, where L is the length of the structural member. Reference to $L/1000$ in columns is provided in "Guide to Stability Design Criteria for Metal Structures" [5] as the acceptable tolerance.

Chapter 3

Compressive Strength

In this chapter, the relationship between elastic column buckling and the effect of the end restraint will be addressed. MASTAN 2 computer modeling results of the flexural buckling strength of column with varying weld-affected zone locations will be discussed and compared to the computational results of AISC and AA specifications.

3.1 Elastic Column Buckling and the Effect of the End Restraint

To verify the theoretical elastic column buckling solutions, a model of six parallel 40-ft long aluminum I6x4.03 (T6061) columns with different end restraint conditions, as show in Figure 3.1 below, were prepared in MASTAN.

Buckled shape of column shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value K	0.65	0.80	1.2	1.0	2.10	2.0
End condition key						

Figure 3.1 Approximate values of effective length factor, K
(AISC Steel Construction Manual, 2012) [2]

These six columns were separated by 10-ft for clarity and the top of each column was not restrained in the vertical direction. Each column was subdivided into 8 elements and the weld-affected zones were subdivided into 2 elements. Columns are subdivided in order to allow MASTAN2 to capture second-order effects and model the column deflection curve more accurately. The second-order effect in a column is shown in Figure 3.2:

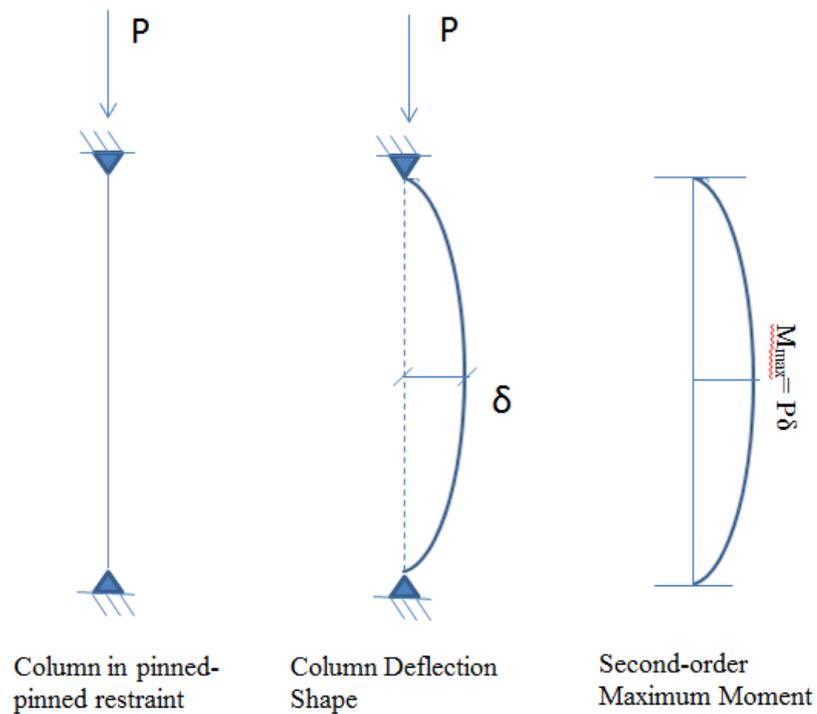


Figure 3.2 Second-order $P\delta$ effect in column

A load of 1-kip was applied to the top of each column before conducting several analyses using the planar frame (2D) elastic critical load method. This method was used to compute the first 10 buckling modes for each column. According to E2-4, the Euler Buckling Equation, P is defined as follows:

$$P = \frac{\pi^2 EI}{(KL)^2} \quad (\text{E3-1})$$

The computational results from E3-1 and the results of MASTAN 2 modeling are shown in Table 3.1,

Table 3.1 Comparison of theoretical buckling loads and MASTAN 2 modeling results

End Restraints	P_{cr} (E3-1) (kip)	P_{cr} (MASTAN) (kip)	% error
Fix-Fix	38.06	38.08	0.051
Fix-Pin	19.42	19.47	0.255
Fix-Roller	9.51	9.51	0.003
Pin-Pin	9.51	9.51	0.003
Fix-Free	2.38	2.38	0.000
Pin-Roller	2.38	2.38	0.000

Table 3-1 shows that the MASTAN 2 modeling results agree well with theoretical buckling loads as the highest percent error is only 0.255%. The lowest buckling loads, both 2.38kip, are fix-free restraint and pin-roller restraint scenarios because the K factors are both 2.0.

The results of computer modeling with varying welding locations on the beam are shown in Table 3.2:

Table 3.2 Comparison of elastic buckling loads with varying welding locations

End-Restraint Description	P_{cr} (MASTAN) (kip)	Weld at End (kip)	Weld at Mid-span (kip)	Weld at One-Third (kip)	Weld at One-Fourth (kip)
Fix-Fix	38.08	38.07	38.07	38.07	39.08
Fix-Pin	19.47	19.47	19.47	19.47	19.47
Fix-Roller	9.51	9.51	9.51	9.51	9.51
Pin-Pin	9.51	9.51	9.51	9.51	9.51
Fix-Free	2.38	2.38	2.38	2.38	2.38
Pin-Roller	2.38	2.38	2.38	2.38	2.38

In Table 3.2, we can see that the elastic buckling loads did not change significantly when the welding location is altered. For fix-fix and fix-pin restraint scenarios, the only change was shown is for. The mid-span welding location model has the lowest buckling load, as seen in Table 3.2, in first and second row in the third column. This is reasonable because of the moment diagram of a typical column in compression.

As in the design of any structural member, the strength of the material must be greater than the stress it experiences. Likewise, the moment experienced by any point along the column height must be less than the moment applied. The moment diagram generated by second-order effect of a column in compression has its maximum at exactly half of the column height and because of this; the strength of anything placed at the mid-span of the column must be greater than the maximum moment along the column height. Similarly, when the weld-affected zone is at the mid-span, the weld-affected zone will also experience the maximum moment of the column. Thus, the maximum moment capacity of the column, without failure, would be limited by strength of the weld-affected zone. However, if the weld-affected zone location is located at somewhere else other than the mid-span, the maximum moment of the column (which is at the mid-span) will not be reached until the weld-affected zone reaches its maximum moment.

3.2 Flexural Buckling Strength of Column

The minor-axis compressive strength of I6x4.03 (T6061) is prepared by using both the Euler Buckling Equation and computational analyses. The column curves are plotted with slenderness ratio L/r as the abscissae, where L is the length of the column and r is the radius of gyration about the minor bending axis. The compressive strength, F_{cr} is defined as follows:

$$F_{cr} = \frac{P_{failure}}{A_g} \quad (E3-2)$$

where A_g is the cross-sectional area of the member. Slenderness ratio values include 5, 15, 40, 65, 80, 90, 105, 115, 140, 165 and 190. This slenderness ratio range is selected to show the inelastic behavior, partial yielding behavior and elastic behavior of the columns. To investigate the influence of the weld-affected zone, a two-inch weld is modeled at the end, mid span, one third of the length and one fourth of the length. The Modulus of Elasticity, E , is 10100 ksi, which is the same for both welding and non-weld area. The yield strength, F_y , is 15ksi and 35ksi for the weld and the non-welding area, respectively. To improve the accuracy of the results, columns are subdivided into eight elements and the welding areas are subdivided into two elements. The impacts of imperfection and partial yielding are not negligible, so all curves include the following cases [4]:

- 1) Computational strength that includes an initial imperfection of $L/1000$, but does not account for partial yielding ran by MASTAN2.
- 2) Computational strength that includes both initial imperfection of $L/1000$ and partial yielding ran by MASTAN2.
- 3) Computational strength that includes both initial imperfection of $L/1000$ ran by FE++.

For each case, the structural members were re-oriented by ninety degrees to investigate minor bending/buckling. A gradually applied load of 100 kips was used for all three cases mentioned above. The load increment size was 0.01 and the maximum number of increments was 1000. The minor-axis column curves with imperfections and the weld-affected zone at varying locations by using MASTAN 2 and FE++ are shown in Figure 3.3 to Figure 3.4, respectively. The notation δ represents imperfections and the notation ρ_y represents partial yielding.

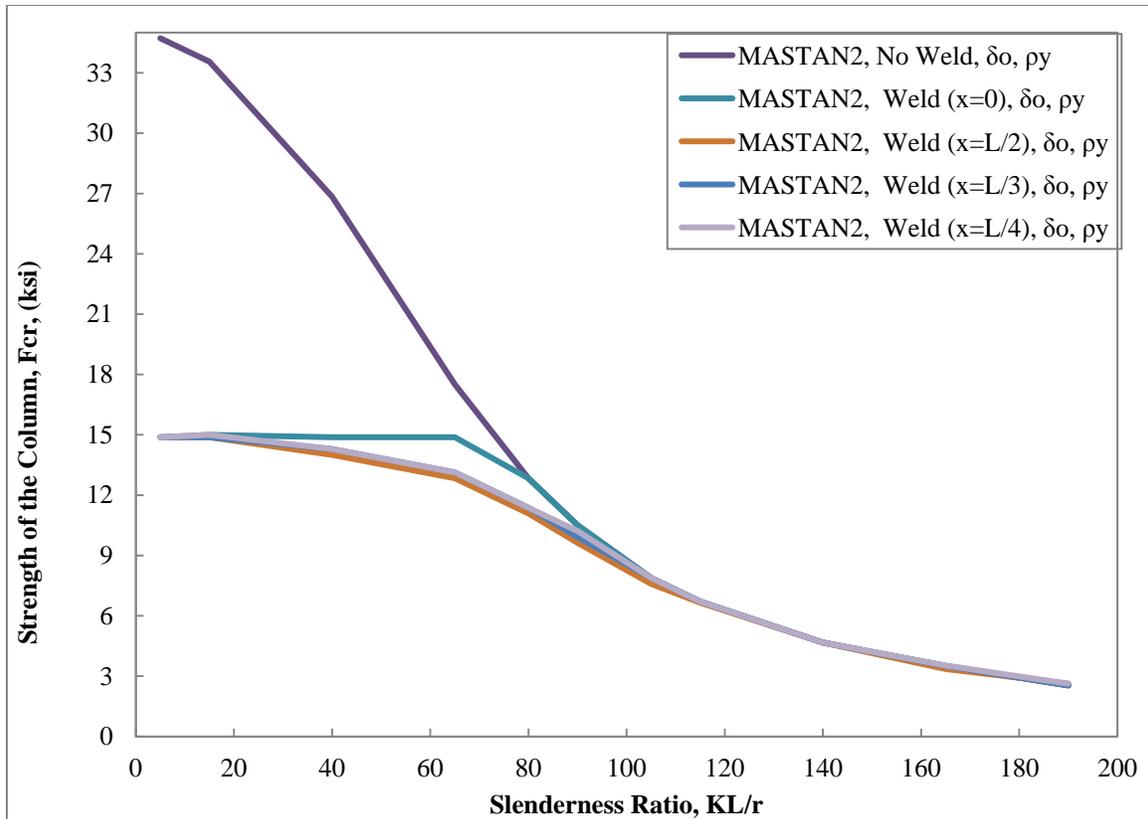


Figure 3.3 Minor-axis column curves with imperfections and weld-affected zone at varying locations (MASTAN 2)

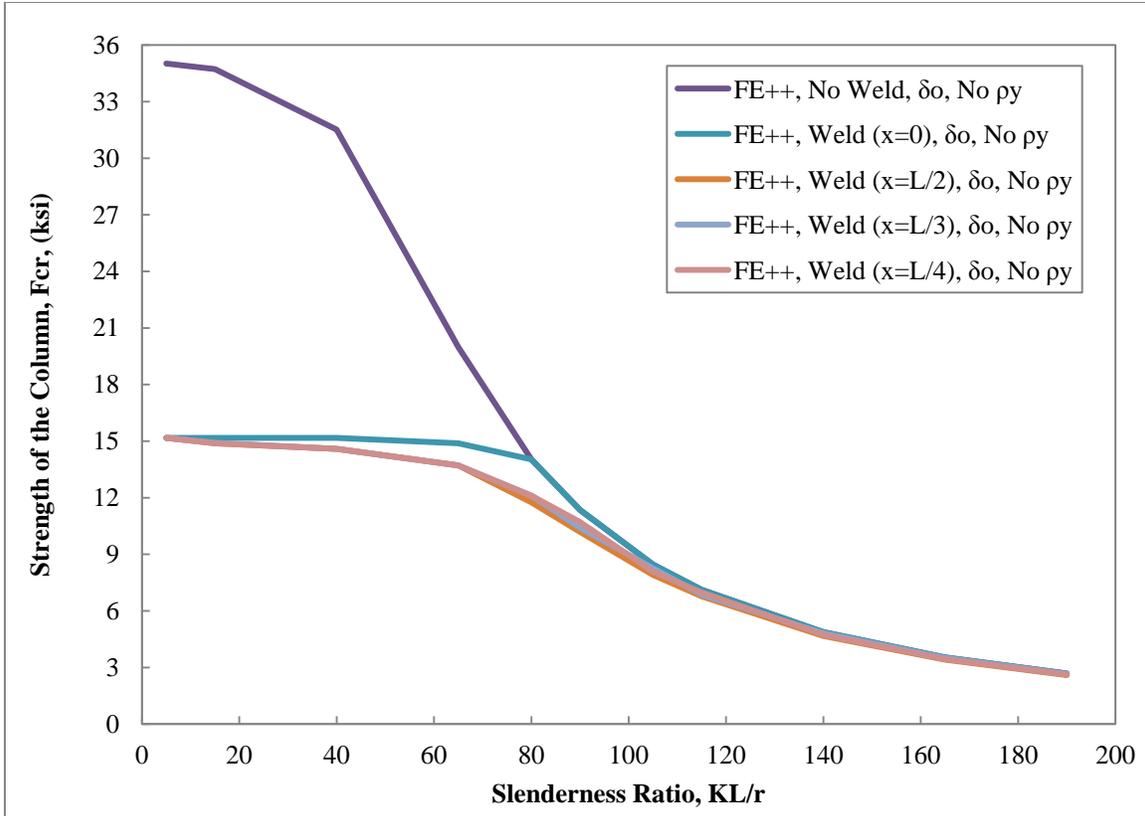


Figure 3.4 Minor-axis column curves with imperfections and weld-affected zone at varying locations (FE++)

The curves in Figure 3.3 to Figure 3.4 show similar shapes and trends. When there is no weld-affected zone in the columns, the compressive strength of the column, F_{cr} , gradually decreases from 35 ksi to 2.6 ksi. When the weld-affected zone is located at the end of the columns, the compressive strength of the column, F_{cr} , stays consistently at 15 ksi until the slender ratio reaches 65. For the columns that have weld-affected zones located at other places, the compressive strength, F_{cr} slowly decreases till the slenderness ratio reaches 65. When the slenderness ratio is larger than 65, all the curves are controlled by the same failure modes, elastic buckling. Additionally, the curve of the weld-affected zone at the mid-span always has the

lowest strength. This is physically reasonable due to the same second-order effect which was described in section 3.1.

The comparison between minor-axis column curves with imperfections and the weld-affected zone at same locations by using MASTAN 2 and FE++ are shown in Figure 3.5 to Figure 3.8.

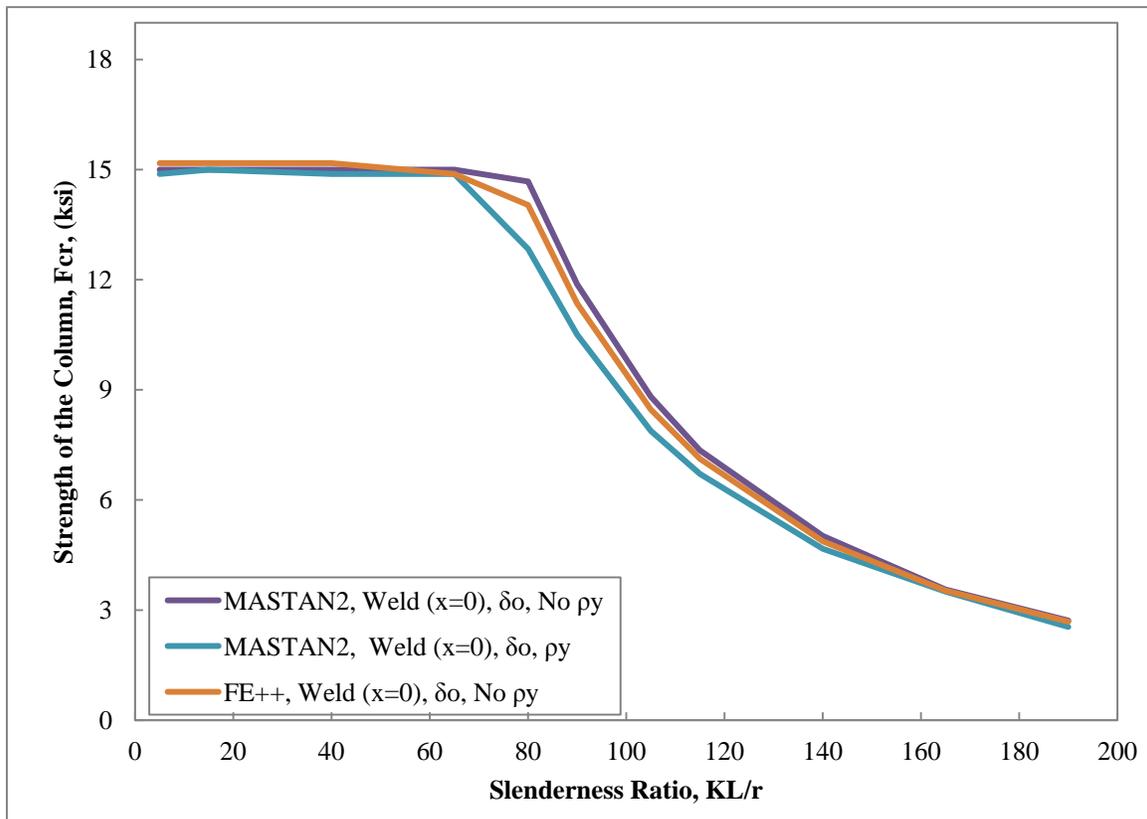


Figure 3.5 Minor-axis column curves with weld-affected zone at the end

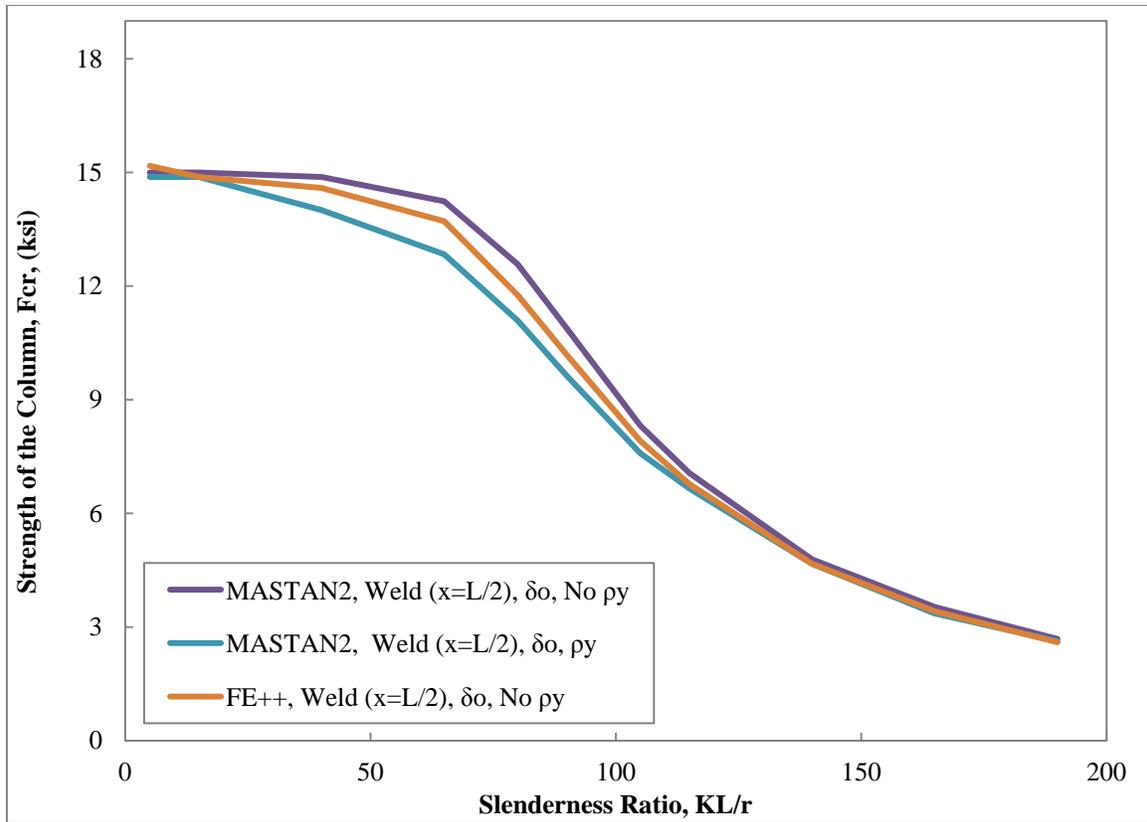


Figure 3.6 Minor-axis column curves with weld-affected zone at the mid-span

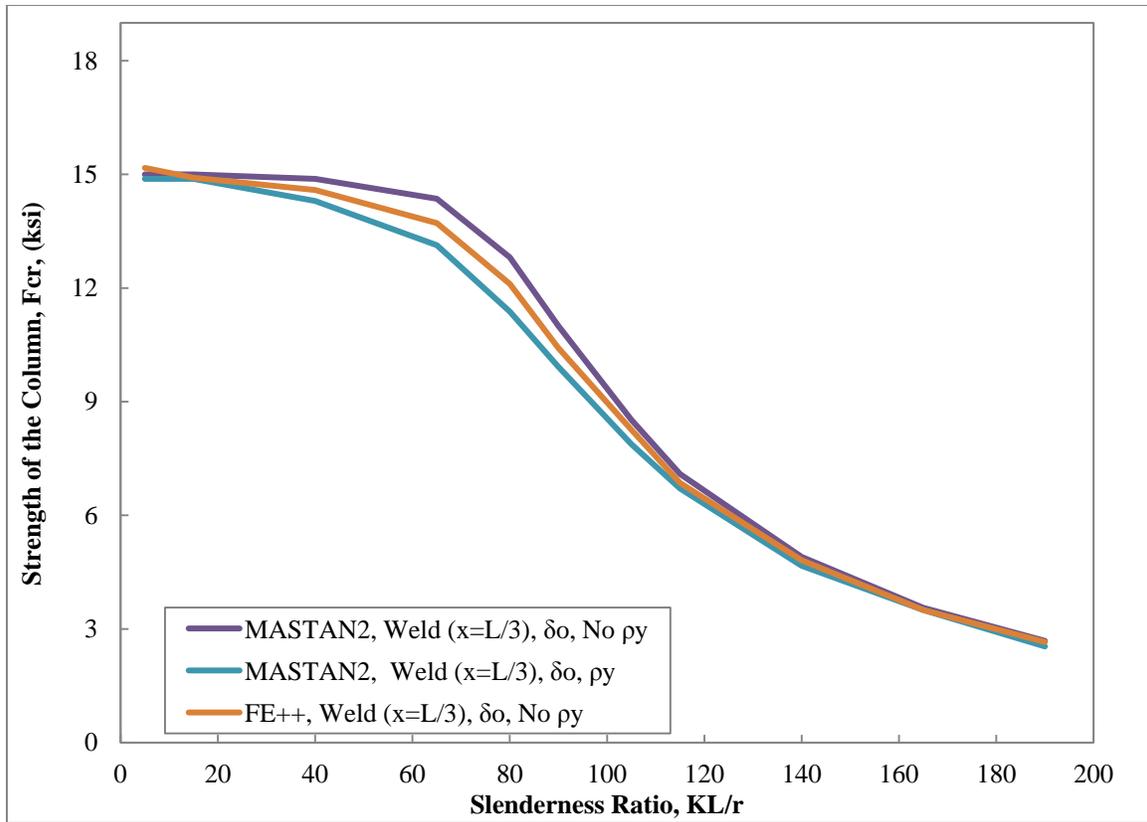


Figure 3.7 Minor-axis column curves with weld-affected zone at one-third of the length

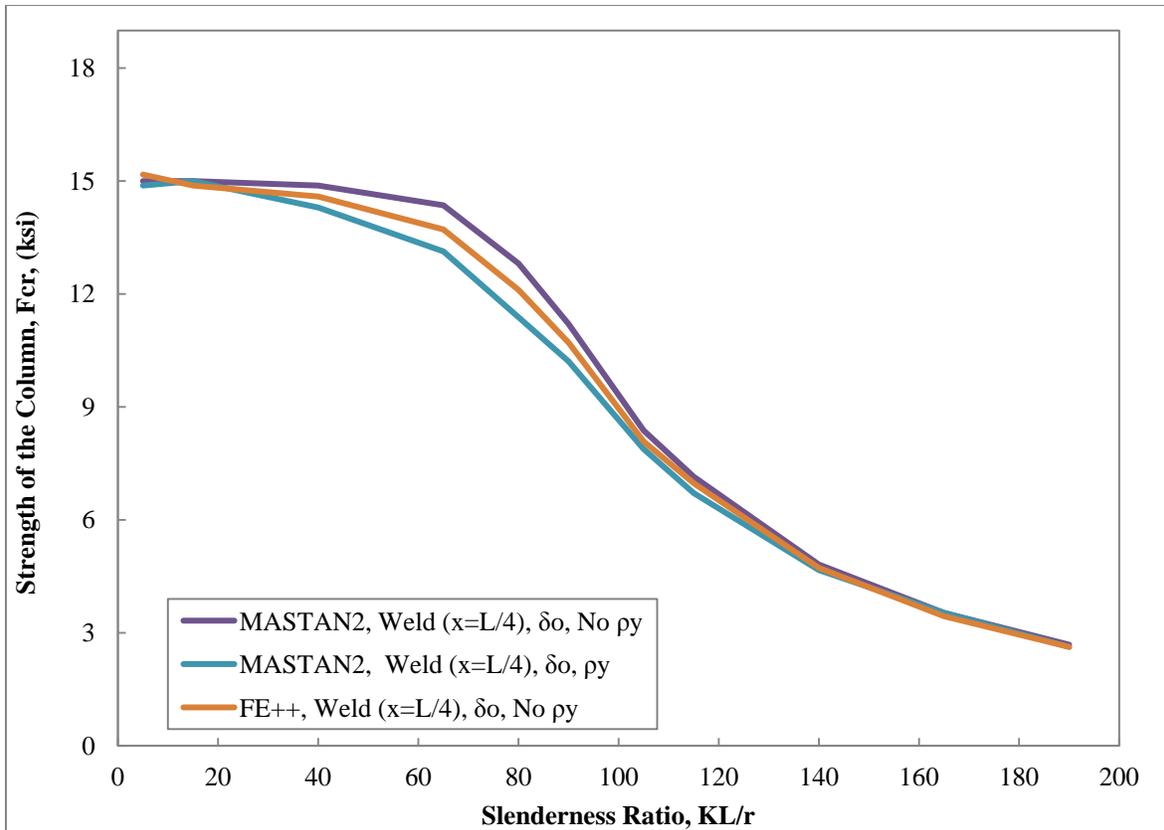


Figure 3.8 Minor-axis column curves with weld-affected zone at the one-fourth of the length

The curves in Figure 3.5 to Figure 3.8 show similar shapes and trends. However, as the weld-affected zones shift from the end (Figure 3.5) to the mid-span (Figure 3.6), the curves tend to split faster. In all three figures, two MASTAN2 curves diverged. FE++ is more sensitive than MASTAN2, so the two results captured by FE++ have smaller differences than those captured by MASTAN2.

3.3 Comparison of MASTAN Compressive Strength Curves with AISC and AA Curves

Figure 3.9 and Figure 4.0 is the summary of the column curves by using AISC and AA computational method and computer modeling of MASTAN2 and FE++.

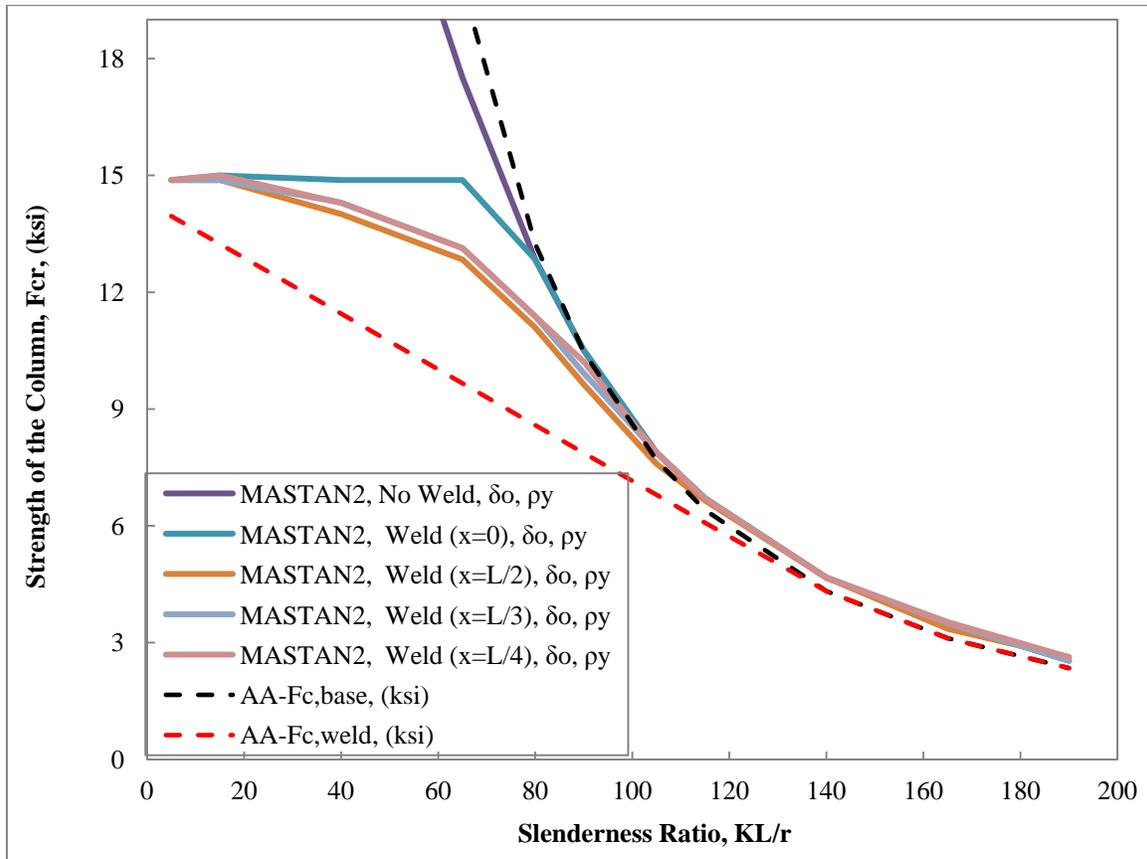


Figure 3.9 Comparison of computational and MASTAN2 computer modeling minor-axis column curves with weld-affected zone at varying locations

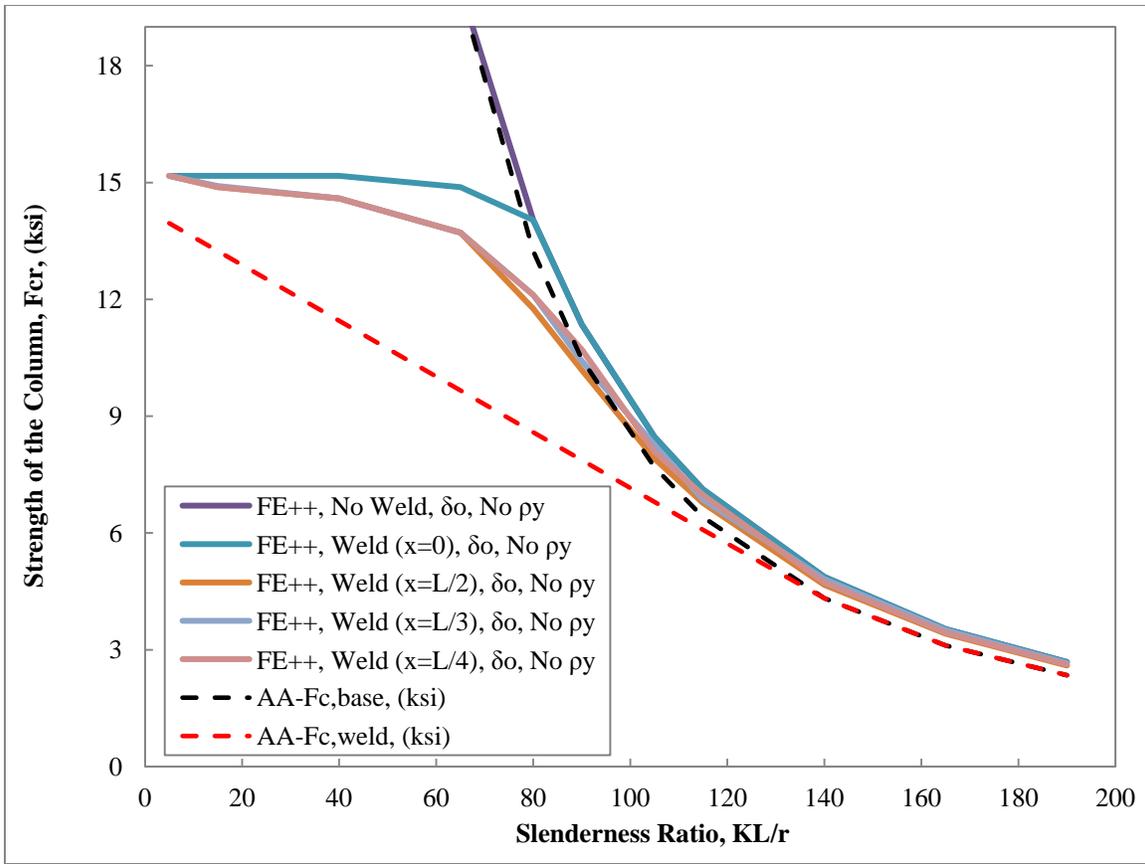


Figure 3.10 Comparison of computational and FE++ computer modeling minor-axis column curves with weld-affected zone at varying locations

Figure 3.9 and Figure 3.10 shows that the AA column curves are very conservative because *AA Specification* assumes that the compressive strength of the weld-affected zone should apply to the entire adjoining members. Additionally, for columns with slenderness ratio less than 130, AA used linear relationships which directly connected the material strength, F_y of the weld-affected zone to the point where the column curve of base material joined the column curve of weld-affected zone. Additionally, the curves of MASTAN2 and FE++ show that the columns with the weld-affected zone at the mid-span have the lowest strength.

Chapter 4

Flexure Strength

Flexural strength is the capacity of a structural member to resist applied loads without excessive deformations and/or instability such as local or lateral-torsional buckling. In this chapter, two relationships will be addressed: the relationship between lateral-torsional buckling strength of beams and the locations of the weld-affected zone, and the relationship between the moment gradients and the lateral-torsional buckling strength. MASTAN 2 computer modeling results of lateral-torsional buckling strength of beams will be compared to the computational results of AISC and AA specifications.

4.1 Lateral-Torsional Buckling of Beams with Uniform Moment

Nominal moment strength is determined by three types of behaviors: full yielding, inelastic lateral-torsional buckling and elastic lateral-torsional buckling. A series of curves that show the major-axis flexural strength of I6x4.03 (T6061) is prepared by employing theoretical elastic lateral-torsional buckling, *AISC Specification*, *AA Specification*, and computational analyses. The beam curves were plotted with length L as the abscissa and the nominal moment, M_n as the ordinate. In all cases, the beam was subjected to uniform major-axis bending. Unbraced lengths ranged from 0.4 to 44.0 ft. These lengths were selected to show the yielding, inelastic lateral-torsional buckling and elastic lateral-torsional buckling behaviors of the beams. To investigate the influence of a weld-affected zone, a two-inch weld is placed at the end, mid-span, one third of the length and one fourth of the length. The Modulus of Elasticity, E , is 10100 ksi, which is the same for both welding and non-welding areas. The yield strengths, F_y , are 15 ksi and 35 ksi for welding and non-welding area, respectively.

To improve the accuracy of the results, beams were subdivided into twenty elements and the welding areas were subdivided into two elements. The impact of imperfection and partial yielding were included in all of the computer models. An imperfection was modeled by bending the beam by $L/1000$ in the out-of-plane direction. A moment of 100 kip-in was applied at both ends of the beam. The load increment size was 0.01 and the maximum number of increments was 10,000. When the beam had undergone significant change in deformation, the analysis was stopped due to the form of a plastic hinge. All the analyses were performed in Space Frame (3D) analysis mode. The major-axis beam curve with imperfections for each weld-affected zone location generated using MASTAN 2, and the computational methods employed are both shown in Figures 4.1 and 4.2. The notation δ represents imperfection and the notation ρ_y represents partial yielding. Figure 4.1 shows a beam curve for each different weld-affected zone location under uniform moment at both ends.

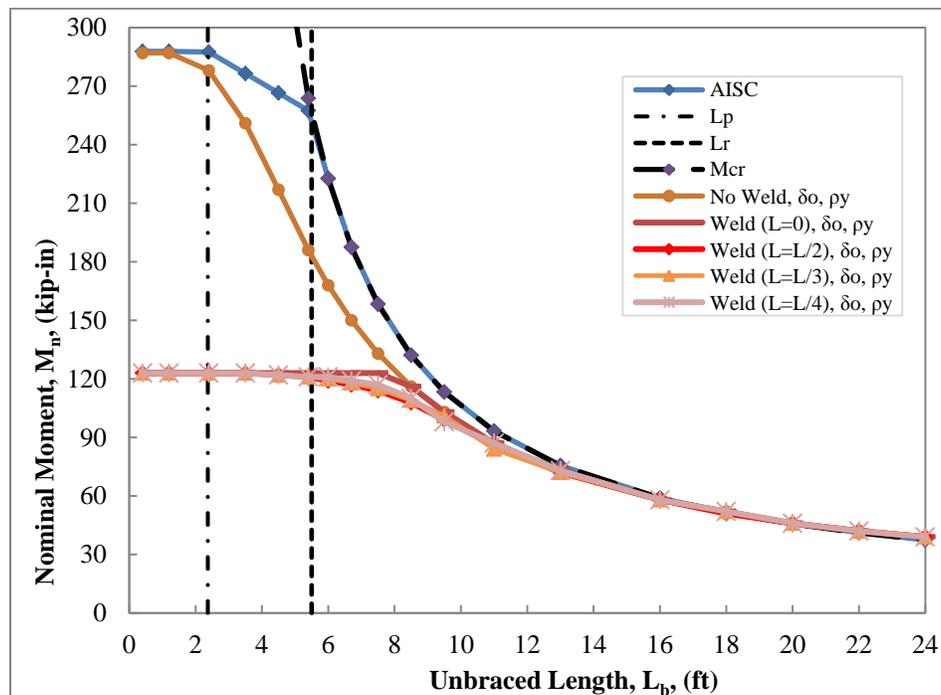


Figure 4.1 Summary of beam curves with uniform moment

As we can see from the figure, the AISC beam curve (blue) agrees with the computer modeling beam curve with no weld-affected zone and no partial yielding (Weld ($x=0$), No δ_o , ρ_y). The magnitude of the nominal moment of the beam with the partial yielding but without the weld-affected zone (Weld ($x=0$), δ_o , ρ_y) is less than that without partial yielding (Weld ($x=0$), δ_o , No ρ_y). However, for all the curves with weld-affected zones, the maximum of the curves all started at 120 kip-in which is controlled by the material yield strength, F_{cy} of the weld-affected zone. As soon as the unbraced length of the beam gets longer than L_r , the nominal moment curve of both AISC and MASTAN2 modeling curves without weld-affected zone started to decrease significantly, and converged with the other curves with partial yielding. This is because when the length is longer than L_r , the failure mode is controlled by elastic lateral-torsional buckling, which is only affected by the Elasticity of Modulus, E , which is the same for both the base material and the weld-affected zone.

Figure 4.2 shows beam curves by using MASTAN 2 only. It is shown by the plot that the curve with the weld-affected zone located at the mid-span (Weld ($x=L/2$), δ_o , ρ_y) has the minimal nominal moment while the curve with the weld-affected zone located at the end (Weld ($x=0$), δ_o , ρ_y) of the beam has the maximum nominal moment.

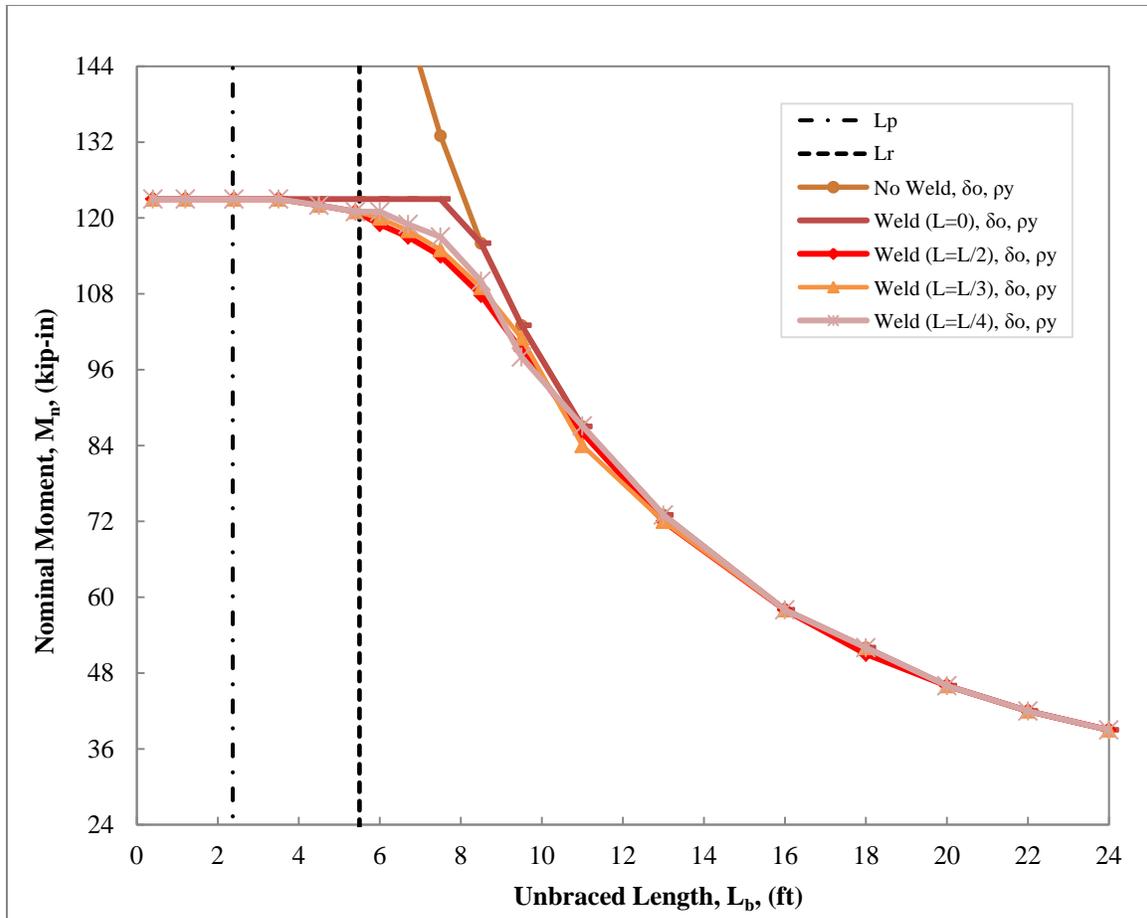


Figure 4.2 Second-order inelastic analysis of flexural members with HAZ at different location and $C_b=1.00$

4.2 Lateral-Torsional Buckling of Beams with Moment Gradient

The same model described in section 4.1 was also adopted in this section with changing moments applied at the ends of the beams. In this section, the beam models were tested using four different values of C_b , the lateral-torsional buckling modification factor for non-uniform moment diagrams were selected to be 1.25, 1.67, 2.17 and 2.27 because these range can show the inelastic behavior, partial yielding behavior and elastic behavior of the beams. These four

moment scenarios were achieved by applying a 100 kip-in moment at the left end of the beam and a 50 kip-in, 0 kip-in, -50 kip-in or a -100 kip-in moment at the right end of the beam. Second-order inelastic analysis with both partial yielding and a material imperfection equal to $L/1000$ were included in all MASTAN2 models.

Figure 4.3 shows the nominal moment beam curves with $C_b=1.25$. The moment capacity of the beam is very small when the beam is very short no matter where the location of the weld-affected zone is located. However, because a short beam has really short unbraced length, the 2-inch weld is the large majority of the entire member hence controls the moment capacity. The same situation applies to all the other cases that will be described in this chapter. Figure 4.3 also shows that as the location of the weld-affected zone switched from the left end to the right end, the beam curves split and the magnitude increased. The curve with the weld-affected zone located at the left end ($x=0$) has the lowest magnitude, and the curve with the weld-affected zone located at the right end ($x=L$) has the highest magnitude. This is reasonable because when $C_b=1.25$, the left end of the beam has a moment of 100 kip-in and the right end of the beam has a moment of 50 kip-in.

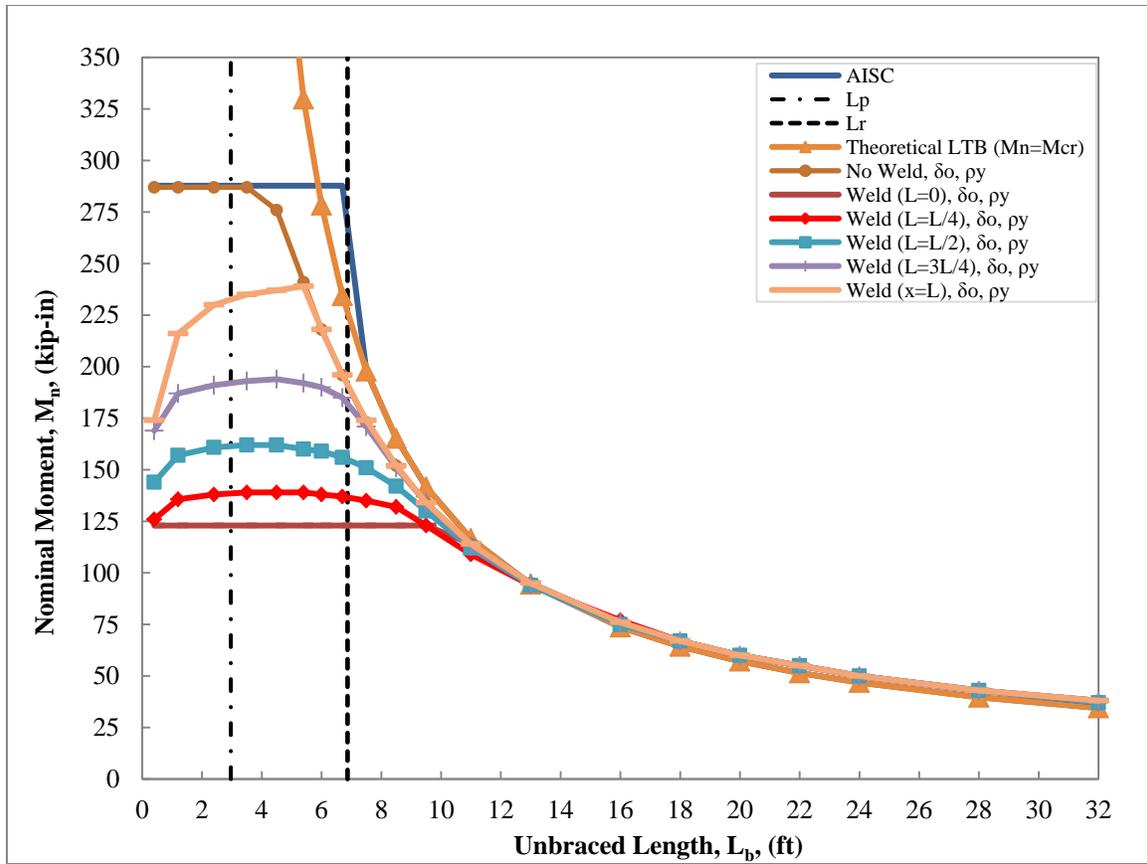


Figure 4.3 Second-order inelastic analysis of flexural members

with HAZ at different location and $C_b=1.25$

Therefore, the moment diagram of the beam is a trapezoidal shape with the top at the right end and the base at the left end of the beam, as shown in Figure 4.4 and 4.5:

**** Moment Z: 2nd-Order Inelastic, Incr # 124, Applied Load Ratio = 1.24 ****

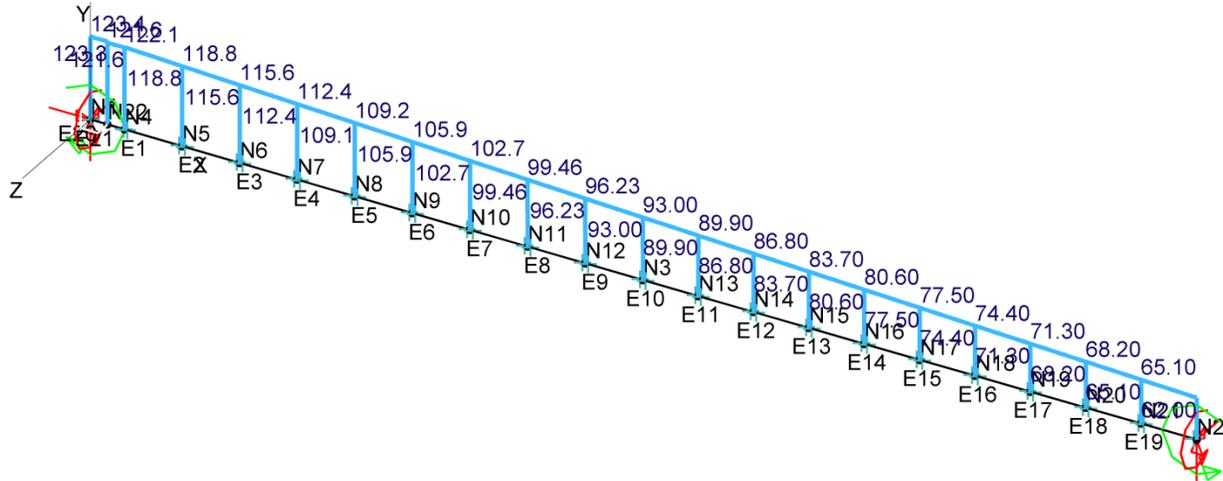


Figure 4.4 Moment diagram ($C_b=1.25$) of welded affected zone at the left end of the beam

**** Moment Z: 2nd-Order Inelastic, Incr # 239, Applied Load Ratio = 2.39 ****

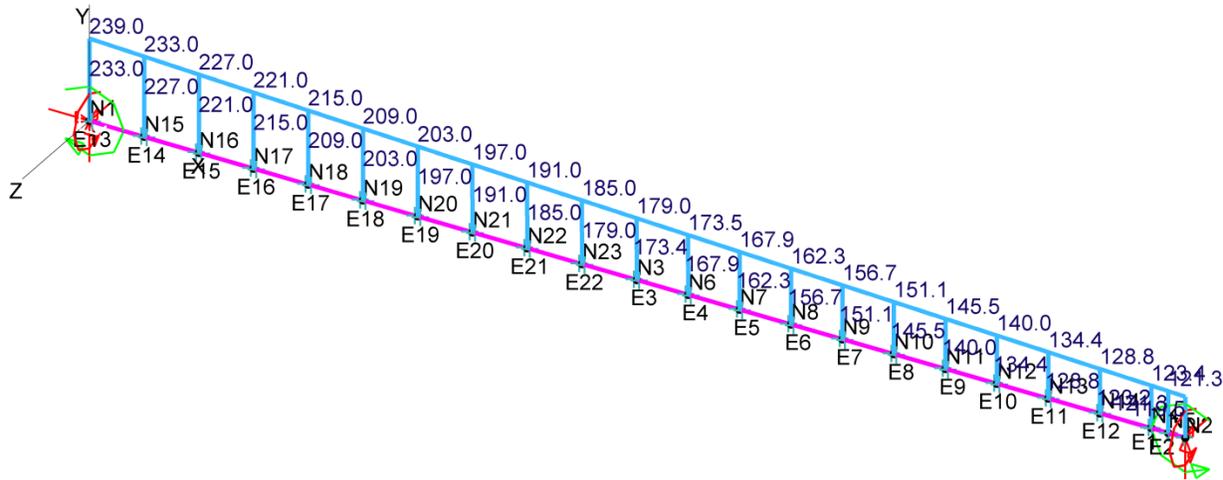


Figure 4.5 Moment diagram ($C_b=1.25$) of welded affected zone at the right end of the beam

The left end and the right end moment in Figure 4.4 and Figure 4.5 are both 123.4 kip-in, which is the nominal flexural strength M_n of the weld-affected zone, as defined by E2-9. Because the moment at the left end of the beam will always be the maximum moment, as shown in Figure

4.4, when the weld-affected zone is located at the left end, the nominal flexural strength M_n of the weld-affected zone will control the nominal strength of the entire beam. In Figure 4.5, the weld-affected zone is at the right end, so the left-end moment is much larger.

Figure 4.6 below shows the split of the curves when C_b is 1.67. In this figure, we can see that the beam curves with the weld-affected zone at the right end ($X=L$) and three quarters of the length ($X=3L/4$) are in the same pattern. This is because the moment at the right end of the beam is 0, so the maximum moment at the left end, which has an applied moment of 100 kip-in, reached the nominal flexural strength M_n of the base material before the yielding nominal flexural strength, M_n of the weld-affected zone was reached.

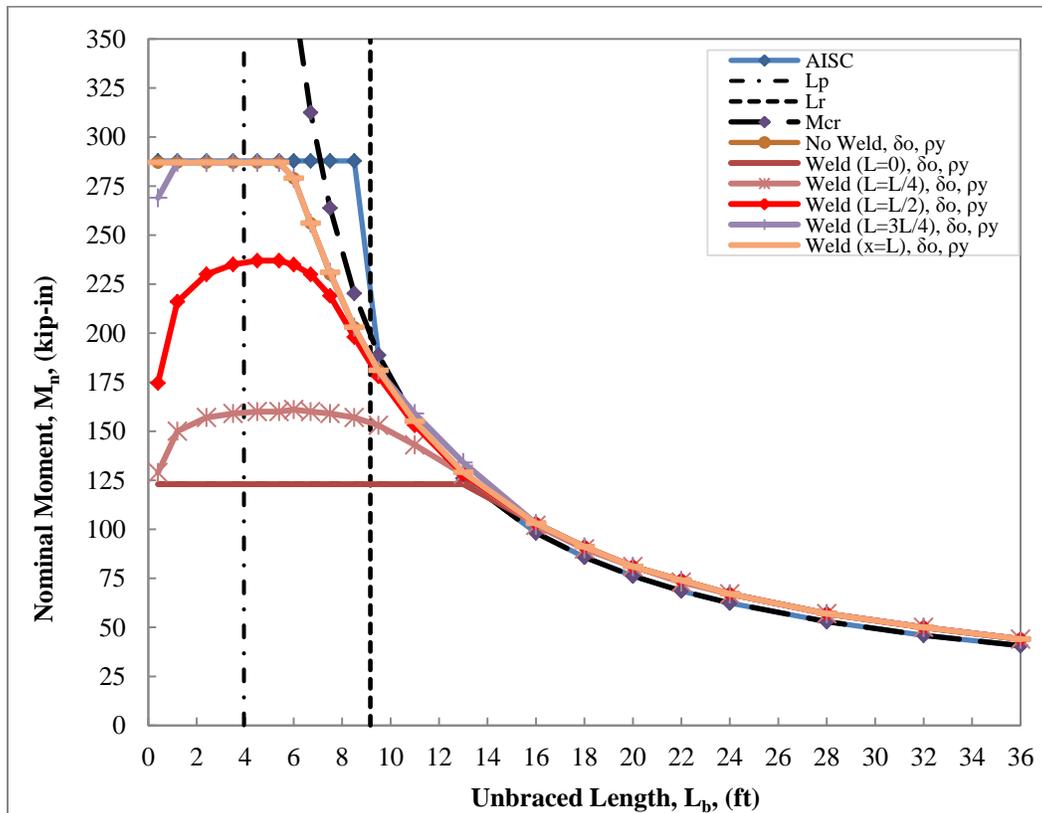


Figure 4.6 Second-order inelastic analysis of flexural members
with HAZ at different location and $C_b=1.67$

Examples of the moment diagram are shown in Figures 4.7 and 4.8:

**** Moment Z: 2nd-Order Inelastic, Incr # 288, Applied Load Ratio = 2.88 ****

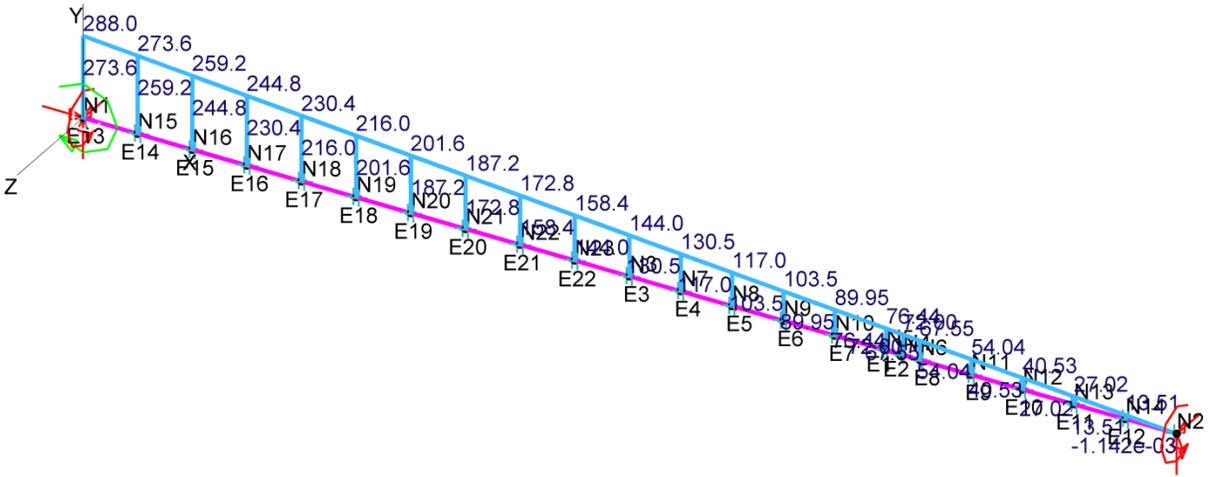


Figure 4.7 Moment diagram ($C_b=1.67$) of welded affected zone at the three quarter of the beam

**** Moment Z: 2nd-Order Inelastic, Incr # 288, Applied Load Ratio = 2.88 ****

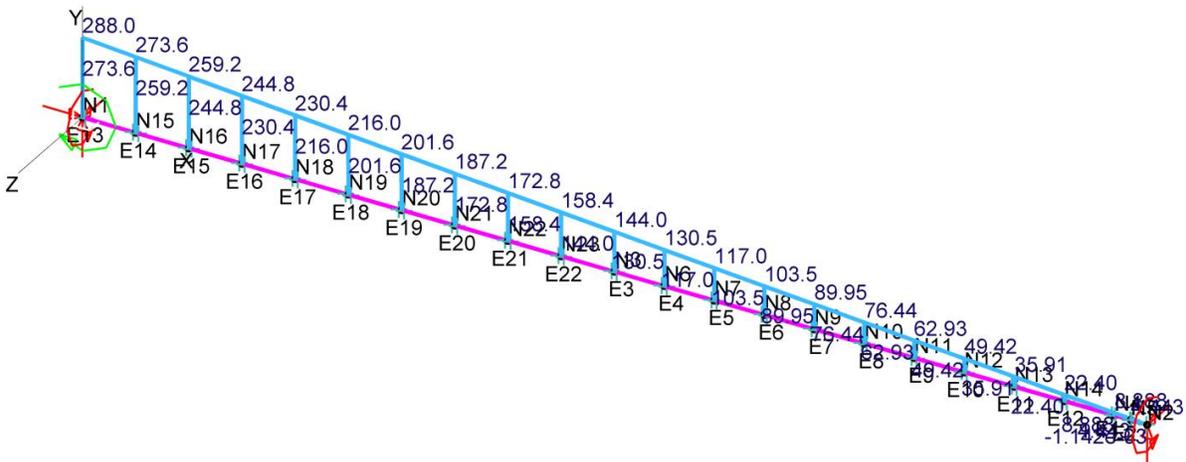


Figure 4.8 Moment diagram ($C_b=1.67$) of welded affected zone at the right end of the beam

The left end moment of the beams in Figure 4.7 and Figure 4.8 are both 288 kip-in, which is the yielding nominal flexural strength, M_n of the base material, as defined by E2-9. Because the moment at the left end of the beam reached the yielding nominal flexural strength M_n of the base material, the nominal flexural strength M_n of the entire structural beam will not be able to get higher than 288 kip-in. Therefore, when the weld-affected zone is at the mid-span ($X=L/2$) or one fourth of the length ($X=L/4$), the weld-affected zone reached its yielding nominal flexural strength before the moment at the right end reached the yielding nominal flexural strength M_n of the base material, thus the nominal flexural strengths of the entire beam of these two cases were lower. However, as the beams get longer, the failure modes are controlled by the elastic lateral-torsional buckling thus all the curves converge into one curve.

Figure 4.6 below shows the nominal moment curves when C_b is 2.17. The beam curves with varying locations of weld-affected zones split into four different patterns. The beams with the weld-affected zone at the mid-span ($X=L/2$) and the three quarters ($X=3L/4$) of the length have the highest nominal strength due to the different moment applied at both ends.

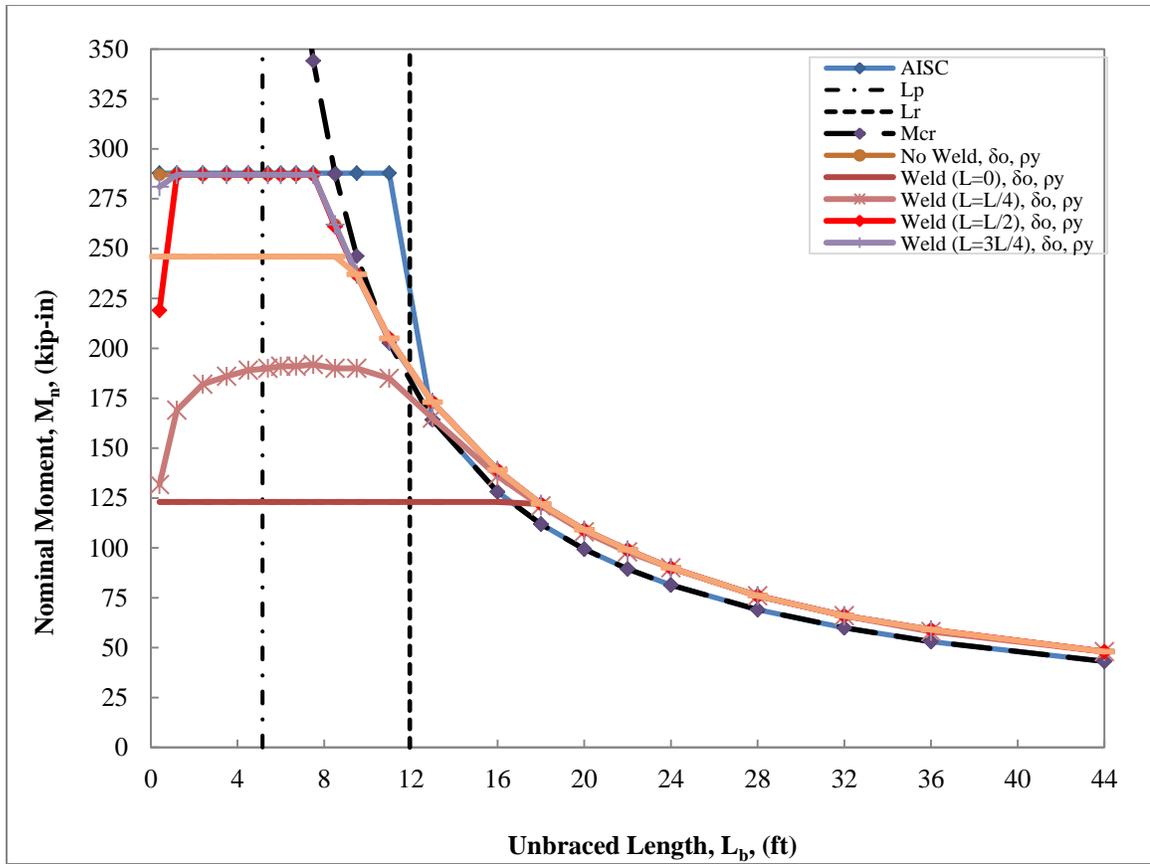


Figure 4.9 Second-order inelastic analysis of flexural members
with HAZ at different location and $C_b=2.17$

Examples of the moment diagram of beams with weld-affected zone at the mid-span ($X=L/2$) and the three quarters ($X=3L/4$) of the length are shown in Figure 4.10 and Figure 4.11:

**** Moment Z: 2nd-Order Inelastic, Incr # 289, Applied Load Ratio = 2.89 ****

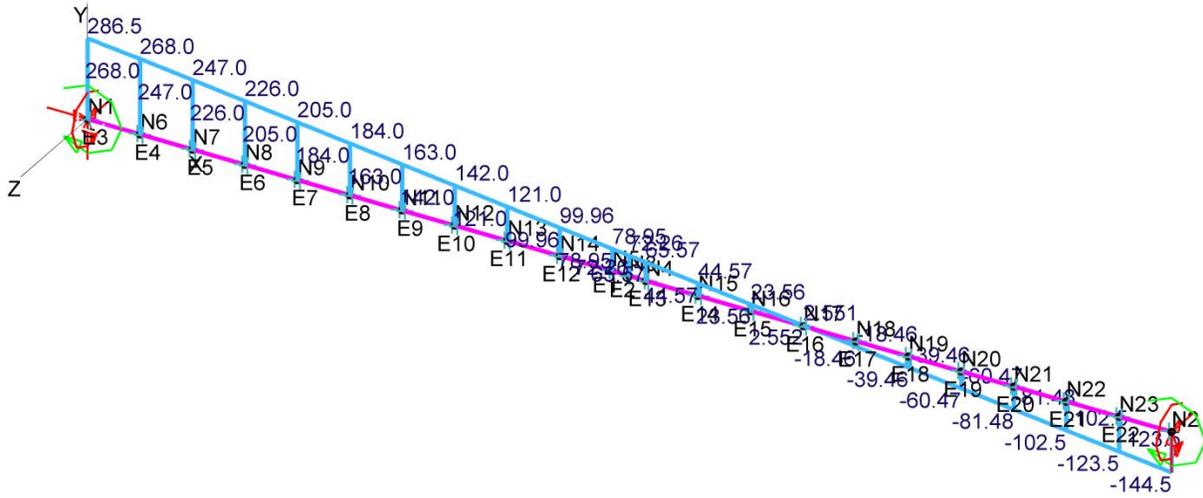


Figure 4.10 Moment diagram ($C_b=2.17$) of welded affected zone at mid-span of the beam

**** Moment Z: 2nd-Order Inelastic, Incr # 289, Applied Load Ratio = 2.89 ****

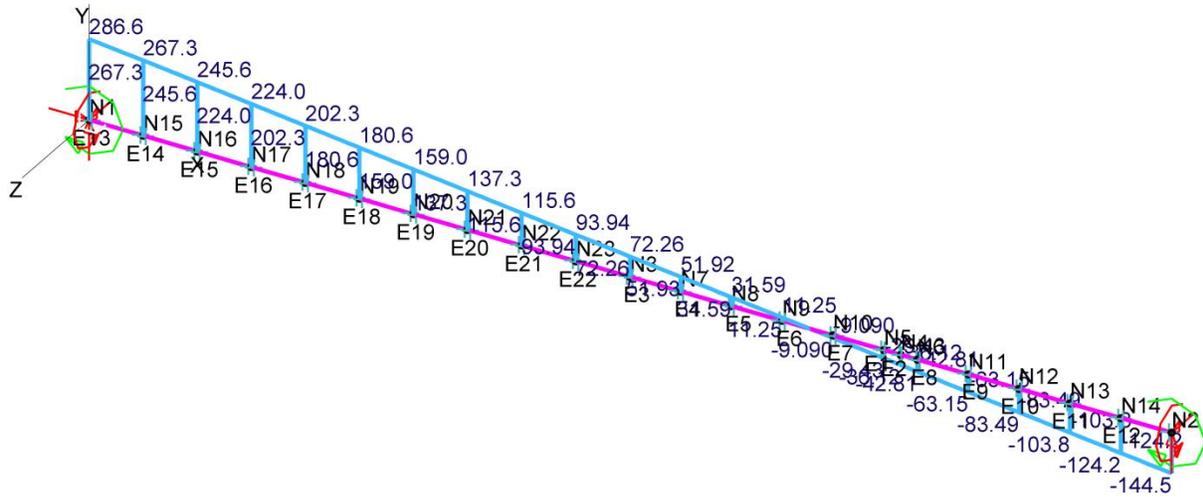


Figure 4.11 Moment diagram ($C_b=2.17$) of welded affected zone at the three quarter of the beam

As shown in the Figures 4.10 and 4.11, the nominal flexural strengths of the entire beam were both controlled by the moment at the left end because the moment reached the yielding nominal flexural strength M_n of the base material.

Figure 4.12 shows the nominal moment curves when C_b is 2.27. In Figure 4.12, the beam curves with varying locations of weld-affected zone split into three different patterns. The beams with the weld-affected zones at the one fourth ($X=L/4$) and three quarters ($X=3L/4$) of the length have the same nominal strength. The beams with the weld-affected zone at the either end ($X=L$ and $X=0$) of the length also have the same lowest nominal strength curve. The beams with the weld-affected zone at the mid-span ($X=L/2$) have the highest nominal strength. Due to the equal opposite moment at both ends, the moment diagrams of the beams are symmetrical.

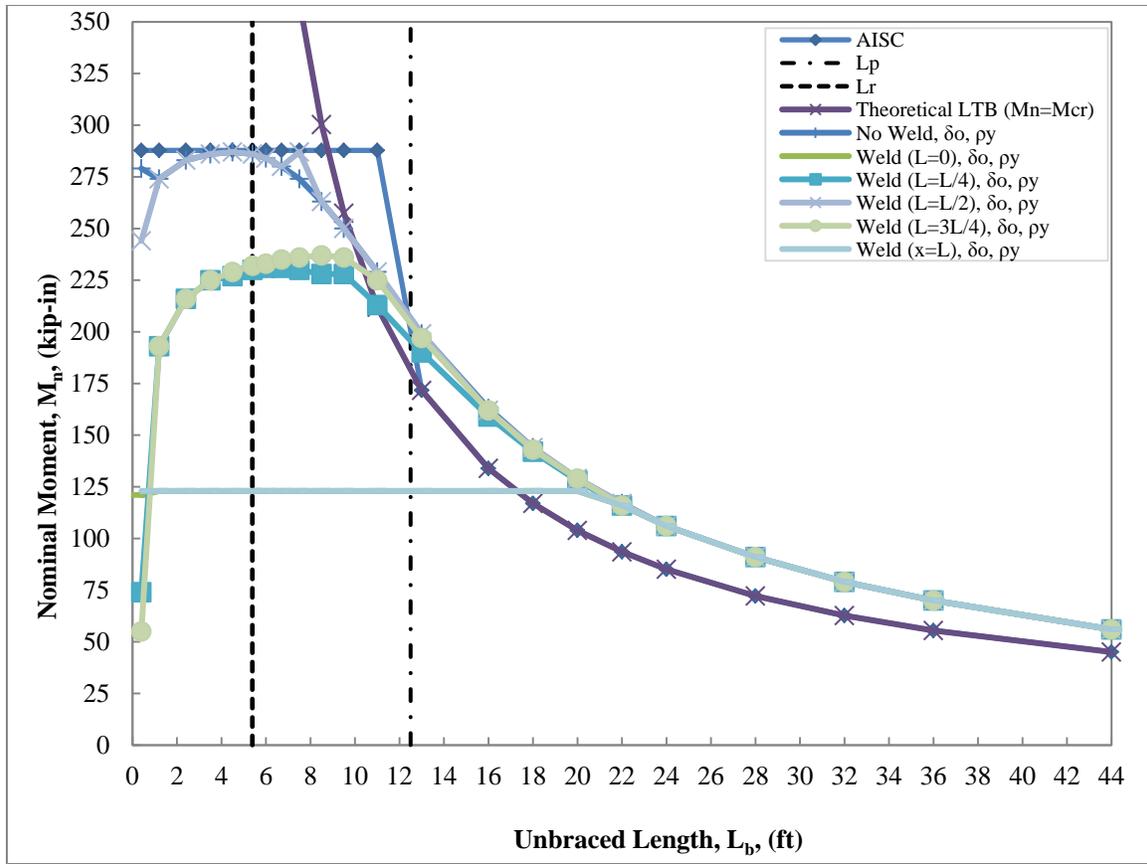


Figure 4.12 Second-order inelastic analysis of flexural members

with HAZ at different location and $C_b=2.27$

Examples of the moment diagram of beams with weld-affected zone at the mid-span ($X=L/4$) and the three-quarter ($X=3L/4$) of the length are shown in the Figure 4.13 and Figure 4.14:

**** Moment Z: 2nd-Order Inelastic, Incr # 230, Applied Load Ratio = 2.3 ****

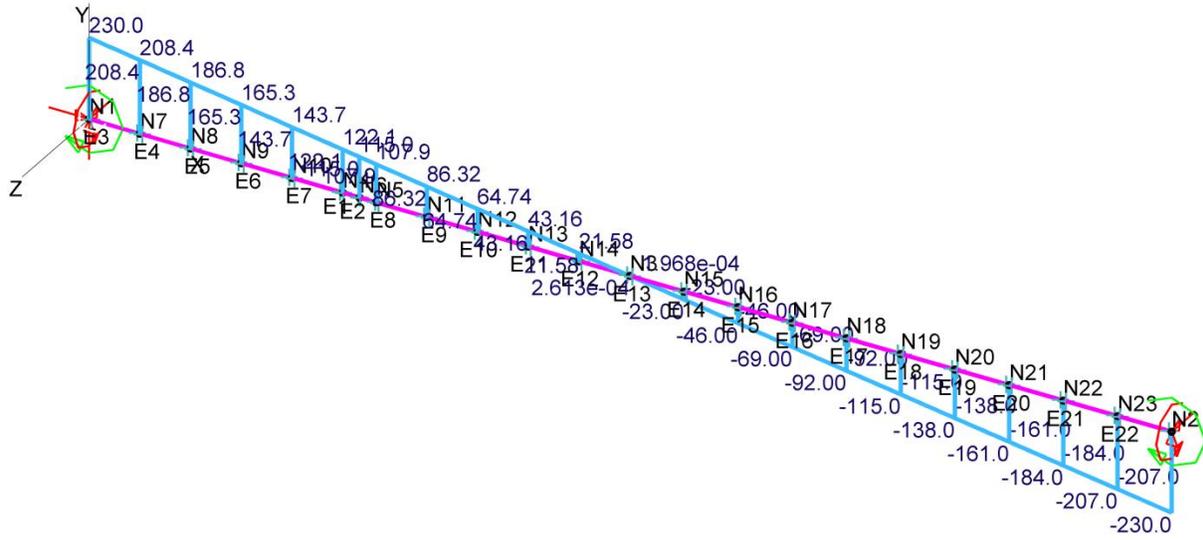


Figure 4.13 Moment diagram ($C_b=2.27$) of welded affected zone at the one-quarter of the beam

**** Moment Z: 2nd-Order Inelastic, Incr # 232, Applied Load Ratio = 2.32 ****

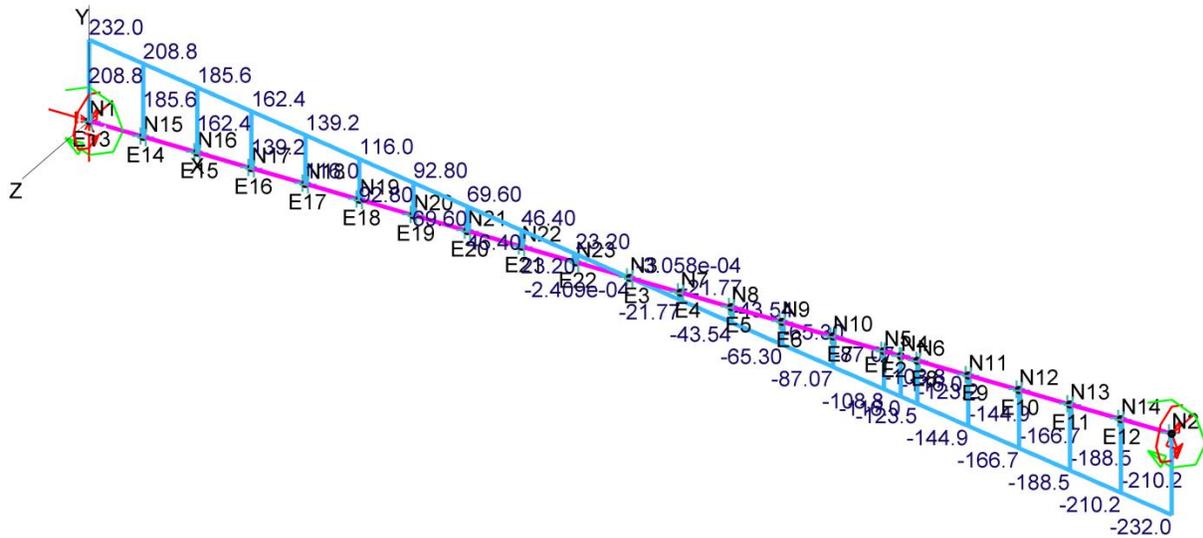


Figure 4.14 Moment diagram ($C_b=2.27$) of welded affected zone at the three-quarter of the beam

As shown in Figure 4.11 and Figure 4.12, the moment diagrams are symmetric and the same for both moment scenarios. Therefore, the nominal flexural strengths of the entire beam were deformed both by the moment at the both ends. When the weld-affected zone is at either the mid-span ($X=L/4$) and the three-quarter ($X=3L/4$) of the length, the weld-affected zone reached its nominal flexural strength, M_n , before the end moment reached the nominal flexural strength of the base material. Consequently, when the weld-affected zone was at either end of the beam, the nominal flexural strength, M_n of the entire structure were controlled by the the nominal flexural strengths, M_n of the weld-affected zone. Additionally, because the moment at the mid-span was always zero, the nominal flexural strength, M_n of the entire structure when the weld-affected zone was at the mid-span was controlled by the nominal flexural strength, M_n of the base material.

4.3 Comparison of MASTAN Flexural Strength Curves with AISC and AA Curves

The comparisons of nominal beam curves generated by MASTAN2 and AISC and AA computational methods with different moment gradients and varying locations of weld-affected zones are shown in Figures 4.13 to 4.17.

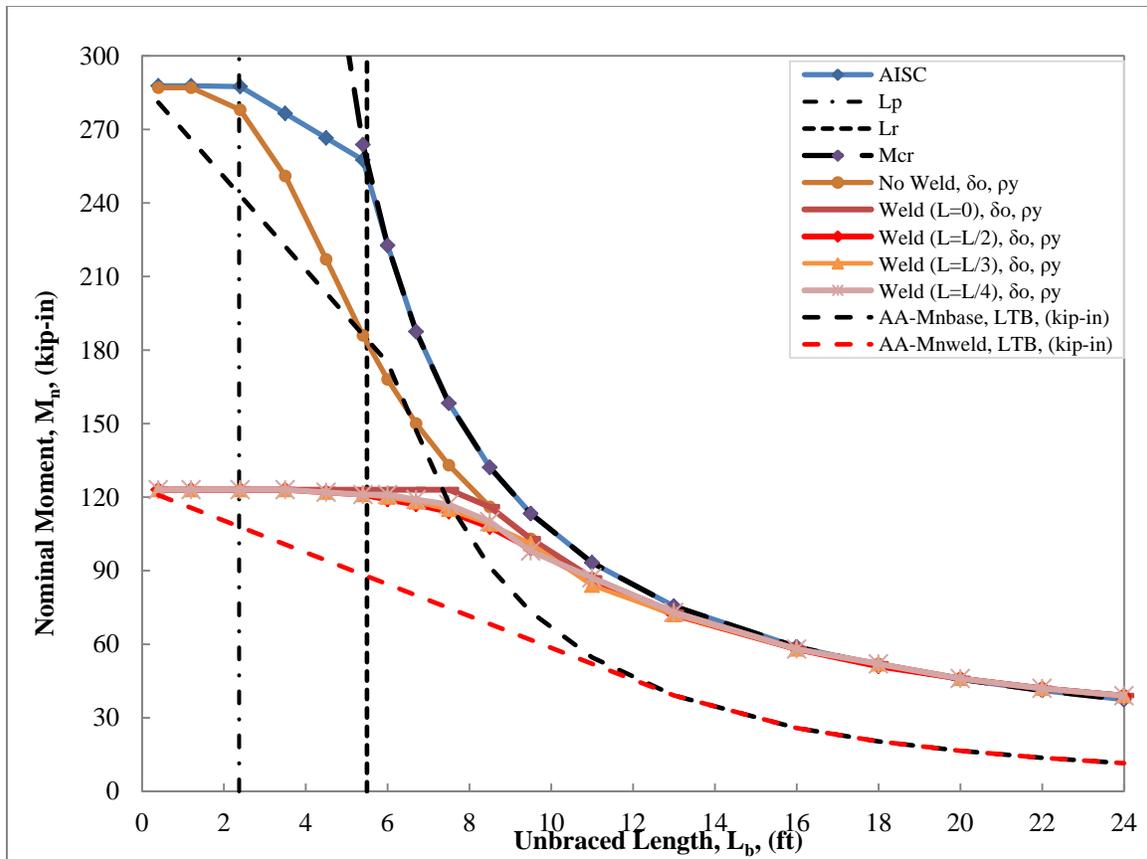


Figure 4.15 Comparison of nominal moment beam curves ($C_b=1.0$)

generated by MASTAN2 and AISC and AA computational methods

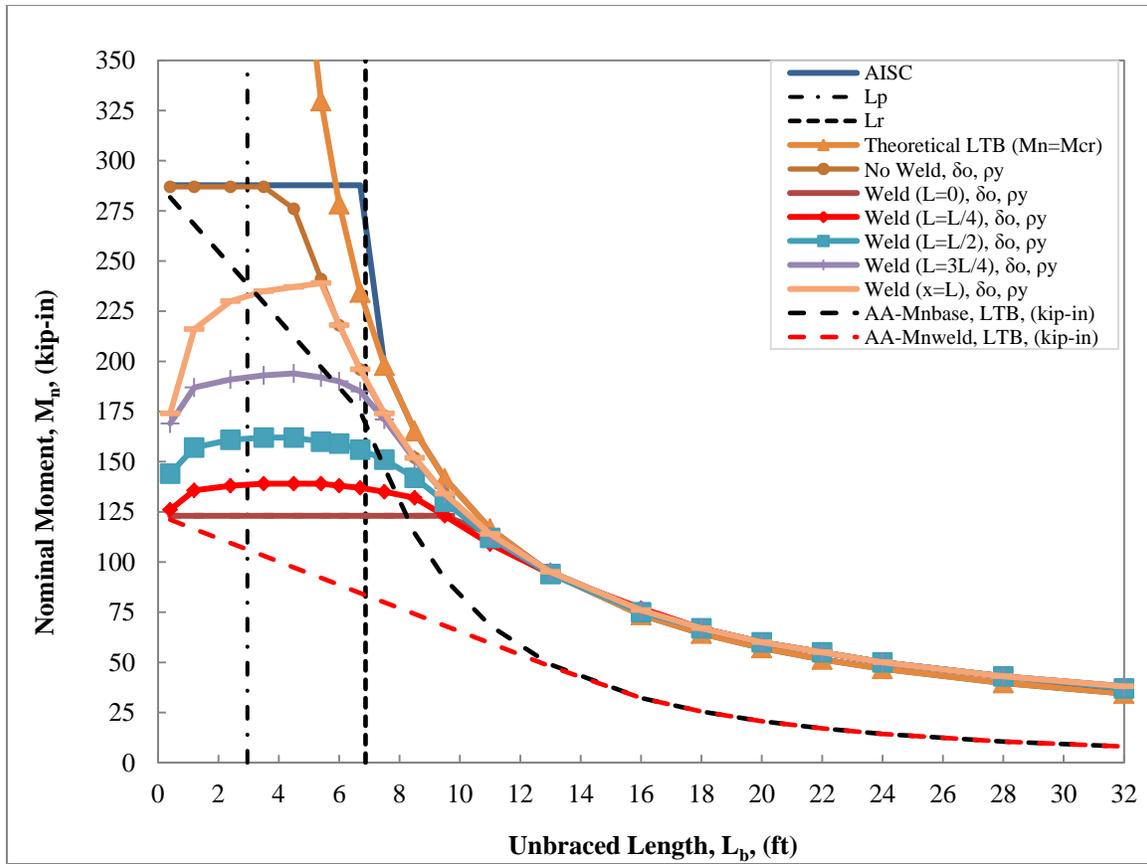


Figure 4.16 Comparison of nominal moment beam curves ($C_b=1.25$) generated by MASTAN2 and AISC and AA computational methods

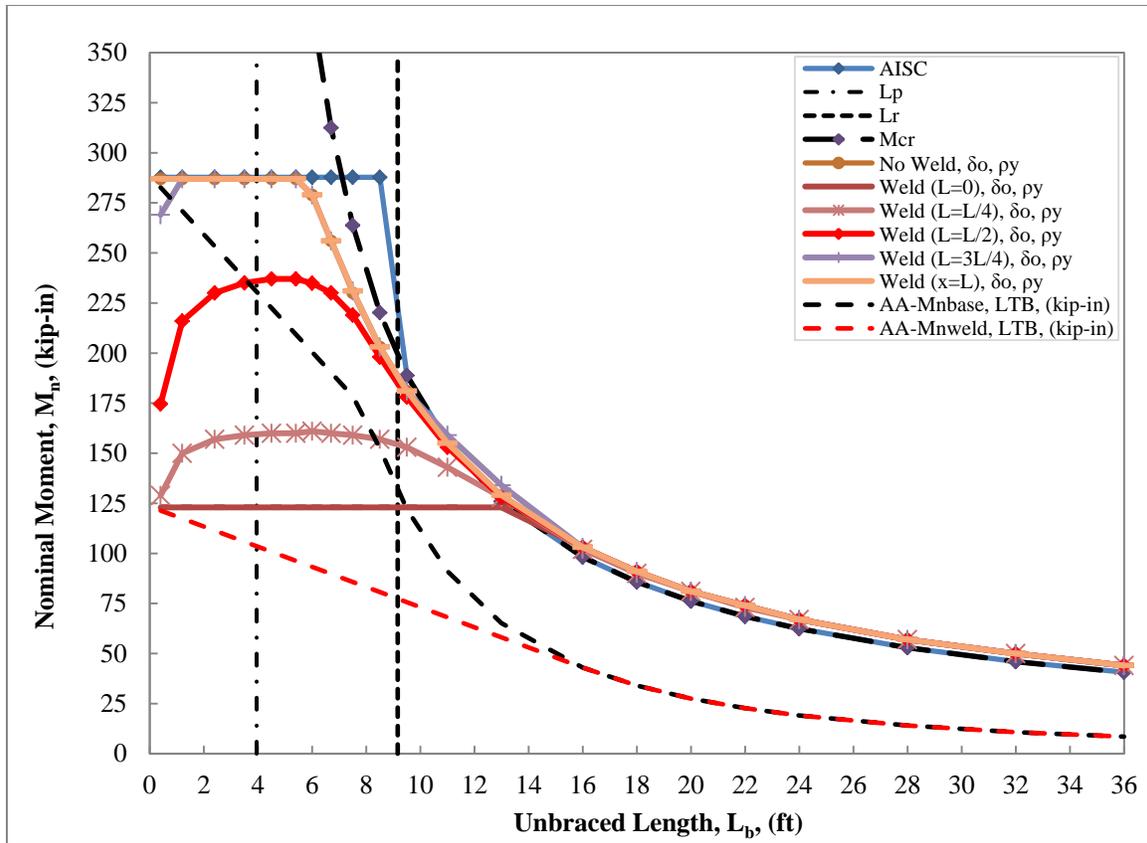


Figure 4.17 Comparison of nominal moment beam curves ($C_b=1.67$)

generated by MASTAN2 and AISC and AA computational methods

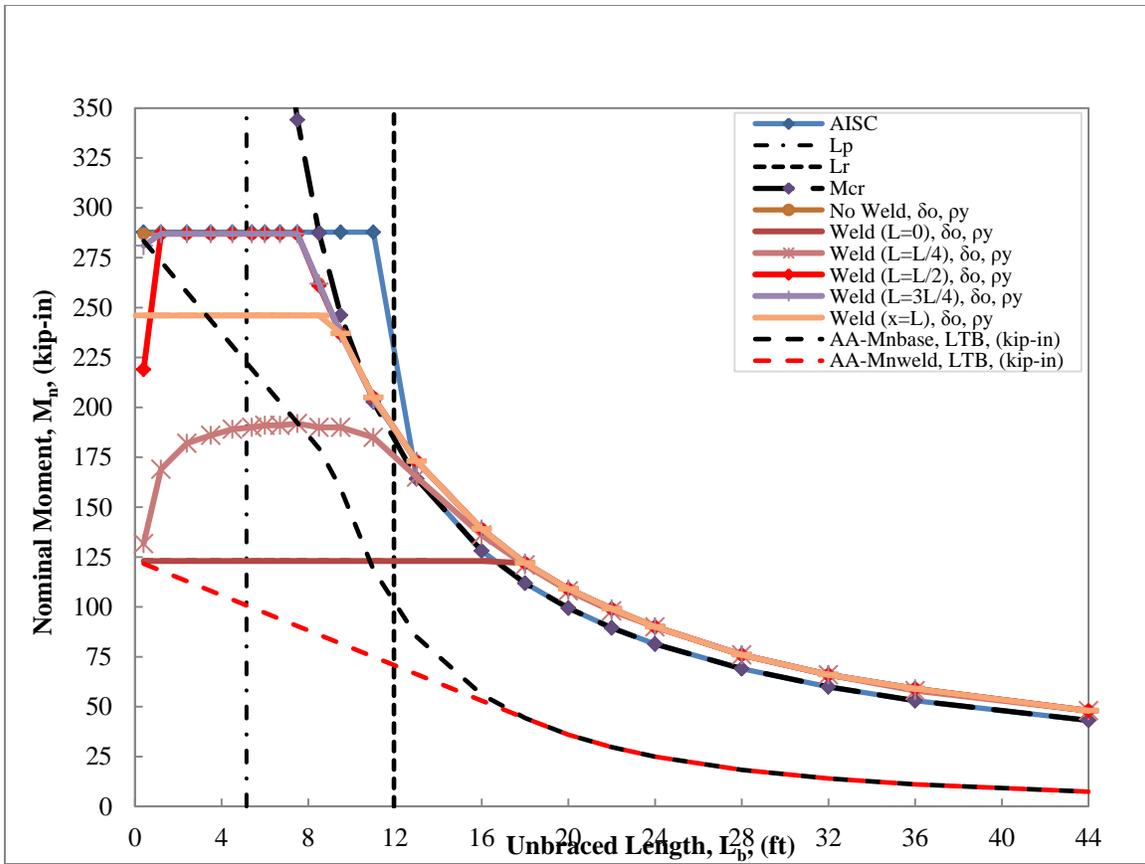


Figure 4.18 Comparison of nominal moment beam curves ($C_b=2.17$)

generated by MASTAN2 and AISC and AA computational methods

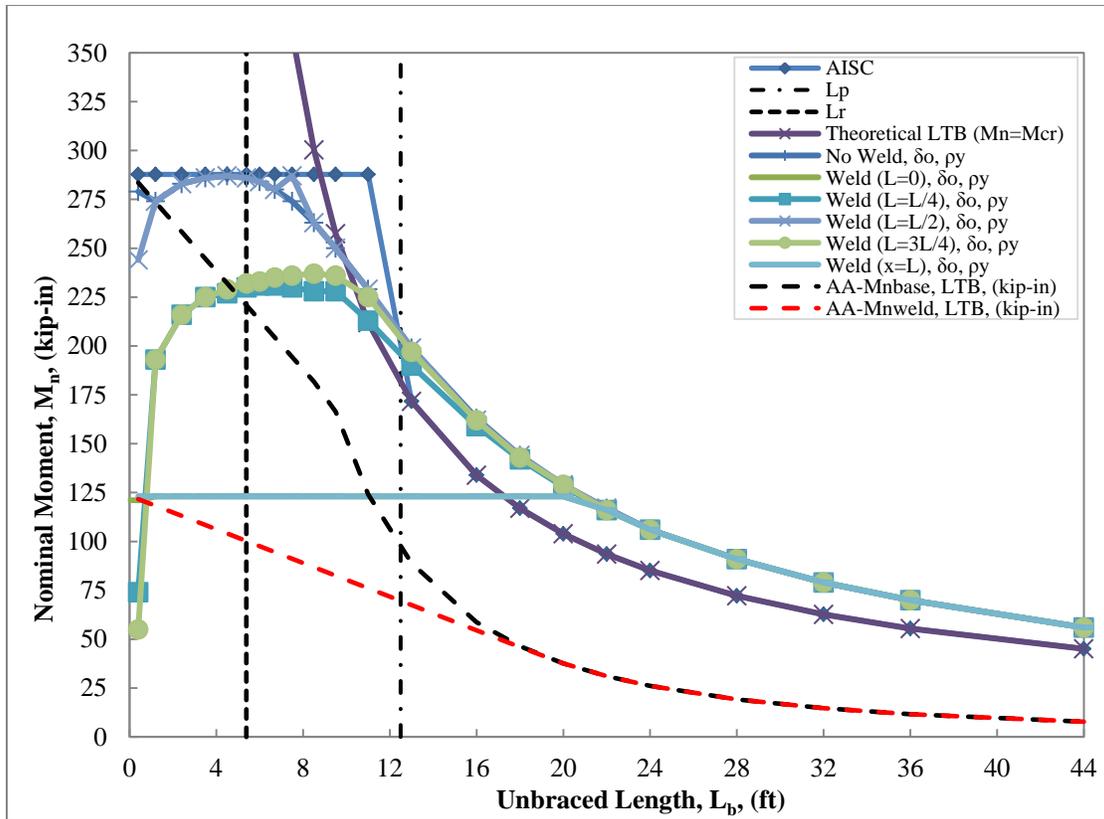


Figure 4.19 Comparison of nominal moment beam curves ($C_b=2.27$) generated by MASTAN2 and AISC and AA computational methods

First, the beam curves generated by AISC computational methods always agree with the maximum nominal moment curves generated by MASTAN2 because the latter cases were controlled by the yield strength, F_y of the base material. Second, the yield strength, F_y of the weld-affected zone is applied to the entire beam, so the beam curves generated by AA computational methods were very conservative comparing with all the other curves. Third, when the length of the beam exceeded L_r , the curves except the curves generated by AA computational method converged into one curve due to elastic lateral-torsional buckling, which is only controlled by the Modulus of Elasticity, E . Last, beam curves generated by both AISC and AA computational methods were not able to show the impact of the varying locations of weld-affected zone.

Chapter 5

Results and Discussions

The compressive strength curves of the columns with varying locations of the weld-affected zone generated by *AA Specification* are conservative. Comparing the AA curves with the compressive strength curves generated by MASTAN2, FE++ and the *AISC Specification*, the compressive strengths of the AA curves have the smallest magnitude due to use of a strength reduction factor of 0.85 in Eq2-14 and Eq2-15. Also, comparing the compressive strength curves generated by MASTAN2 and FE++, the shape and the trends of the curves are very similar, and the only slight differences are shown when varying the location of the weld-affected zone. The strength curves of the columns with a weld-affected zone at the mid-span were always at a lower strength compared to other curves. As a result, to be conservative, the curves representing a column with a weld-affected zone at the mid-span should be used as the compressive strength designed curves.

The flexural strength curves of the beams with varying weld-affected zone locations and moment gradients generated by *AA Specification* are very conservative. Comparing the AA curves with the flexural strength curves generated by MASTAN2 and *AISC Specification*, AA flexural strength curves have the lowest strength. Additionally, curves generated by both *AA Specification* and *AISC Specification* are not capable of showing the impact of the locations of the weld-affected zone on the flexural strength. Therefore, a new computational method is needed for calculating the nominal moment strength. Based on the flexural strength curves generated by MASTAN2, the new nominal moment strength, M_n is defined as follows:

When $L_b < L_r$

$$M_n = 0.95 \left[\frac{F_{cy,weld} Z_x}{G + LF(1-G)} \right] \leq F_{cy,base} Z_y \quad (E6-1)$$

where $F_{cy,weld}$ and $F_{cy,base}$ are the yield stress of the weld-affected zone and the base material, respectively, Z_x is the plastic section modulus about the x-axis, G is the ratio of the moment at the right end divided by the moment at the left end of the beam, and LF is the location factor.

When $L_b \geq L_r$

$$M_n = 0.95 F_{cr} S_x \leq M_p \quad (E6-2)$$

where

L_b = length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_0} \left(\frac{L_b}{r_{ts}}\right)^2} \quad (E2-12)$$

and where

J = torsional constant

S_x = elastic section modulus taken about the x-axis

h_0 = distance between the flange centroids

Figure 6.2 is an example of the nominal moment strength curves with varying locations of the weld-affected zone of a T6061 Aluminum member by using the new computational method described above:

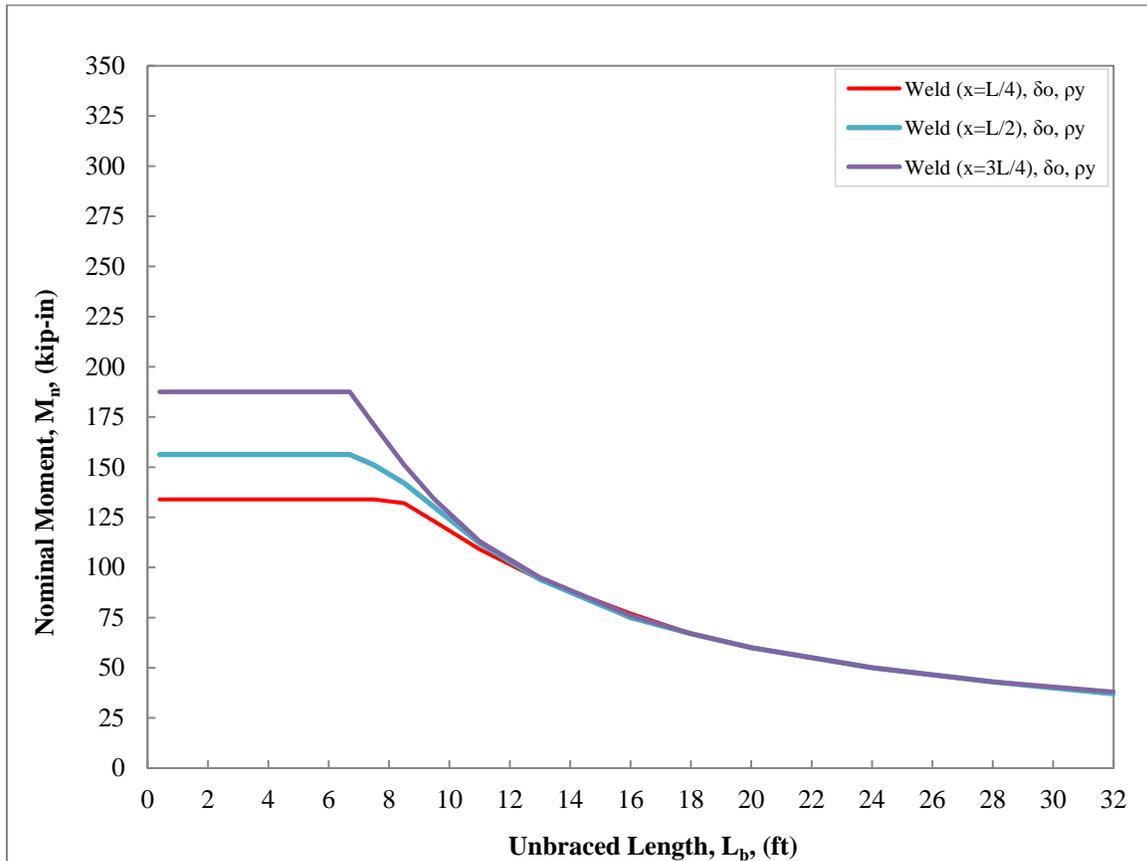


Figure 5.1 Nominal moment strength ($C_b=1.25$) with varying locations of weld-affected zone

Figure 5.2 compares the nominal moment strength curves ($C_b=1.25$) generated by new relationship with all the other curves.

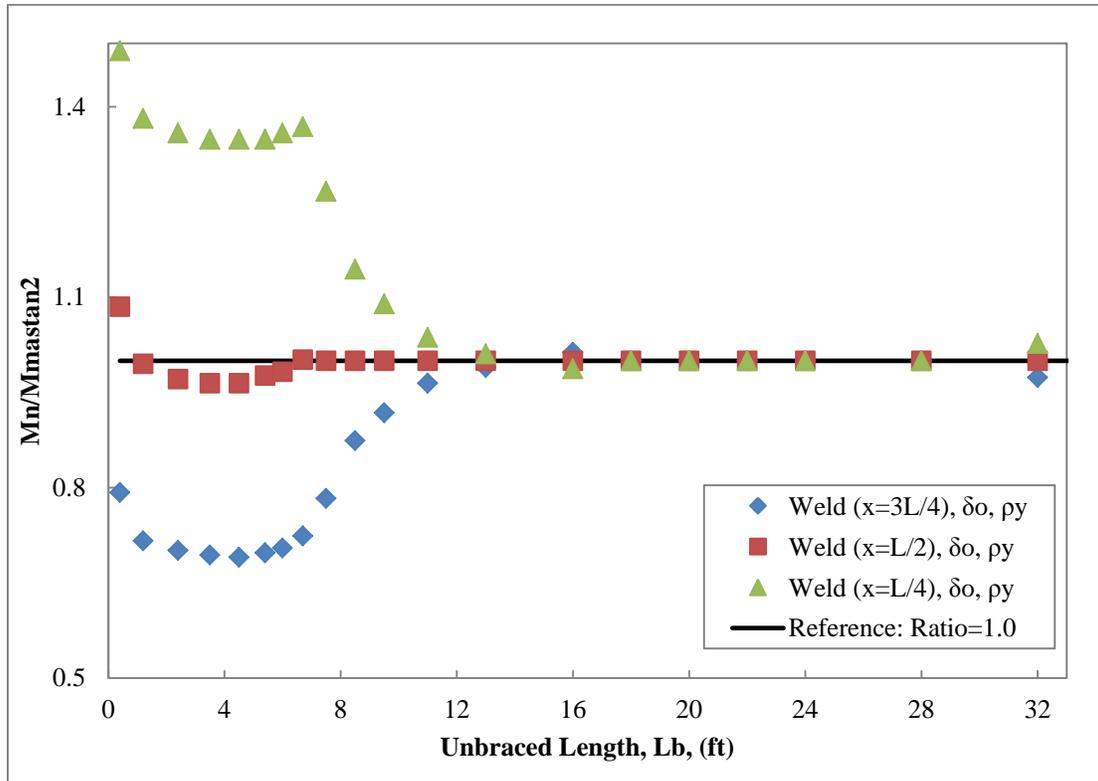


Figure 5.2 Ratio of nominal moment strength curves ($C_b=1.25$)
of new method to moment strength curves of MASTAN2

As shown in the Figure 5.2, the nominal strength curves generated by the new relationship roughly agree with the curves generated by MASTAN2, with some fluctuations when the unbraced length is short. This is due to the reason mentioned earlier. Due to the short unbraced length of a short beam, the 2-inch weld is the large majority of the entire member hence controls the moment capacity. Therefore, the moment capacity for the short member generated by MASTAN is not accurate. Thus, the new method is able to reflect the true behavior of the beams with a weld-affected zone better than both AISC and AA methods.

Chapter 6

Conclusion and Recommendations

The strength curves of the columns with a weld-affected zone at the mid-span were always at a lower strength compared to other curves. As a result, as expected to be conservative, the curves representing a column with a weld-affected zone at the mid-span should be used as the compressive strength designed curves.

A new computational method for the nominal moment strength of beams, in addition to those provided by *AA* and *AISC*, is needed for calculating the nominal moment strength. Based on the flexural strength curves generated by MASTAN2, the new nominal moment strength, M_n has been defined. The new nominal strength curves generated by the new relationship roughly agree with the curves generated by MASTAN2 with a comparatively smaller magnitude of the fluctuation. This model should be further studied and improved to provide better recommendations for the true strength of the beams with a weld-affected zone.

To expand the findings of this study, more sophisticated software could be used, and more tests with different loading scenarios should be run. There were only two types of frame analysis software being used in the research described in this thesis. In the future, a sophisticated finite element analysis software, such as ADINA, could be used, in which three-dimensional modeling is done using shell elements considering yielding and second order effects. This would be used to study and confirm the current results, and update the newly-defined relationship. It is anticipated that the results will generate a series of three-dimensional plots with the x-axis defining the weld location, y-axis the slenderness ratio, and z-axis the member buckling strength.

A second recommendation is that the load scenario can be varied to develop a more precise understanding of the beam flexural behaviors. The newly-defined relationship can only be applied when only one moment is applied to both ends of the beams. If a horizontal or vertical point load is applied to either end of the beam, the relationship will be invalid. Therefore, computer models with more loading scenarios should be studied to form a more comprehensive relationship between the locations of the weld-affected zone and the nominal moment strength.

REFERENCES

- [1] Aluminum Association (2010). *Specification for Aluminum Structures*, Arlington, VA.
- [2] American Institute of Steel Construction (2010). *Specification for Structural Steel Buildings*, Chicago, IL.
- [3] ADINA, Theory manual. ADINA R and D, Inc., Watertown, MA, 2012.
- [4] Alemdar, B. N. (2001) *Distributed plasticity analysis of steel building structural systems* (Doctoral dissertation and basis for FE++2012), 379pp. School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.
- [5] Ziemian, R.D., and McGuire, W. (2012) *MASTAN2*, www.mastan2.com
- [6] Kissell, R., (2012) Aluminum Association
- [7] Alemdar, B.N., (2004), “An Object-Oriented Computational Framework for Nonlinear Analysis of Structures”, *6th International Congress on Advances in Civil Engineering*, 6-8 October 2004, Bogazici University, Istanbul, Turkey
- [8] Lu, John, (1994), “An Object-Oriented Application Framework for Finite Element Analysis in Structural Engineering”, PhD thesis, Purdue University, 280 pp.
- [9] Lu, John, (1994), “An Object-Oriented Application Framework for Finite Element Analysis in Structural Engineering”, PhD thesis, Purdue University, 280 pp.
- [10] Lu J., White D.W., Chen, W.F., Dunsmore, E.H., and Sotelino, E.D., (1994), “FE++: An Object-Oriented Application Framework for Finite Element Programming”, Second Annual Object-Oriented Numerics Conference, Sunriver, OR, April 24-27

- [11] Ziemian, R.D., Seo D.W., and McGuire, W., “On the Inelastic Strength of Beam-Columns under Biaxial Bending”, *P4-5*
- [12] Bulent N.Alemdar., (2010), “FE++ 2010”, *P12-19*

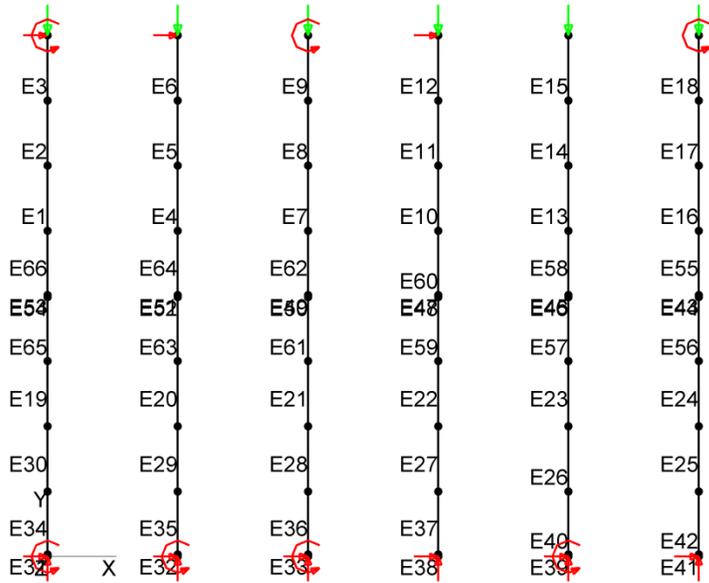
APPENDIX A

COMPUTER MODELS

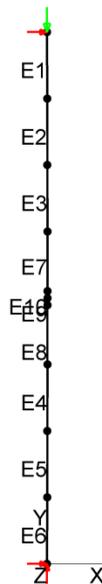
A1. Section Property of T6 x 4.03 (T6061)

κ	1	ksi
$F_{ty,base}$	35	ksi
$F_{tu,base}$	38	ksi
$F_{cy,base}$	35	ksi
$F_{su,base}$	24	ksi
$F_{e,base\&weld}$	204.6714	ksi
$F_{ty,weld}$	15	ksi
$F_{tu,weld}$	24	ksi
$F_{cy,weld}$	15	ksi
$F_{su,weld}$	15	ksi
Z_x	8.223	in ³
E	10100	ksi
I_y	3.1	in ⁴
d	6	in
J	0.089	in ⁴
C_w	25.26818	in ⁶
G	3884.615	ksi
L_p	28.43281	in
L_r	65.97274	in
S_x	7.33	in ³
c	1	
C_b	1	
r_{ts}	1.098834	in
h_o	5.71	in
r_y	0.951	in
$B_{c,base}$	39.36527	
$D_{c,base}$	0.245759	
$C_{c,base}$	65.67316	
$S_{2,base}$	78.8078	
$B_{c,weld}$	16.83712	
$D_{c,weld}$	0.084195	
$C_{c,weld}$	133.3183	
$S_{2,weld}$	159.982	
b	4	in
t	0.29	in

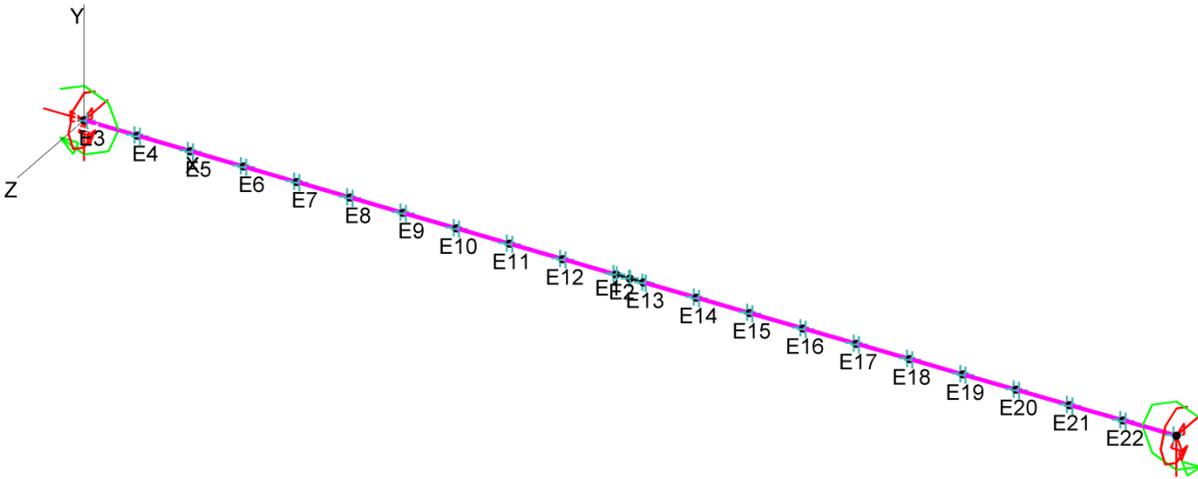
A.2. Elastic Column Buckling Models of different End Restraints



A.3. Column Buckling Models with Weld-Affected Zone



A.4. Lateral-Torsional Buckling of Beams Models with Weld-Affected Zone



APPENDIX B

COMPRESSION STRENGTH

B.1. Flexural Buckling Strength of Column with No Weld-Affected Zone at the End

L/r	None			
	MASTAN2		FE++	
	Fcr W/ imperfection (NO HAZ)	Fcr W/ partial yielding & imperfection (NO HAZ)	NONE (FE++) (NO HAZ)	KETTER (FE++) (NO HAZ)
5.00	34.987	34.724	35.016	35.016
15.00	34.987	33.557	34.724	34.228
40.00	34.345	26.846	31.514	26.437
65.00	22.206	17.508	19.988	17.946
80	14.940	12.839	14.036	13.248
90	11.876	10.505	11.351	10.826
105	8.812	7.879	8.462	8.200
115	7.353	6.711	7.120	6.916
140	5.019	4.669	4.873	4.786
165	3.560	3.502	3.531	3.472
190	2.714	2.539	2.685	2.655

B.2. Flexural Buckling Strength of Column with Weld-Affected Zone at the End

L/r	End			
	MASTAN2		FE++	
	Fcr W/ imperfection (HAZ@END)	Fcr W/ partial yielding & imperfection (HAZ@END)	NONE (FE++) (HAZ@END)	KETTER (FE++) (HAZ@END)
5.00	14.999	14.882	15.174	15.174
15.00	14.999	14.999	15.174	15.174
40.00	14.999	14.882	15.174	15.174
65.00	14.999	14.882	14.882	15.174
80	14.678	12.839	14.036	13.248
90	11.876	10.505	11.351	10.826
105	8.812	7.879	8.462	8.200
115	7.353	6.711	7.120	6.916
140	5.019	4.669	4.873	4.786
165	3.560	3.502	3.531	3.472
190	2.714	2.539	2.685	2.655

B.3. Flexural Buckling Strength of Column with Weld-Affected Zone at the Mid-Span

L/r	Mid-span			
	MASTAN2		FE++	
	Fcr W/ imperfection (HAZ@HALF)	Fcr W/ imperfection & partial yielding (HAZ@HALF)	NONE (FE++) (HAZ@HALF)	KETTER (FE++) (HAZ@HALF)
5.00	14.999	14.882	15.174	15.174
15.00	14.999	14.882	14.882	14.969
40.00	14.882	14.006	14.590	14.590
65.00	14.240	12.839	13.715	13.423
80	12.577	11.088	11.760	11.526
90	10.884	9.629	10.184	10.038
105	8.316	7.587	7.908	7.820
115	7.062	6.653	6.770	6.682
140	4.786	4.669	4.669	4.640
165	3.531	3.356	3.414	3.414
190	2.685	2.626	2.597	2.597

B.4. Flexural Buckling Strength of Column with Weld-Affected Zone at One Third of the Length

	One third			
	MASTAN2		FE++	
L/r	Fcr W/ imperfection (HAZ@1/3)	Fcr W/ partial yielding & imperfection (HAZ@1/3)	NONE (FE++) (HAZ@1/3)	KETTER (FE++) (HAZ@1/3)
5.00	14.999	14.882	15.174	15.174
15.00	14.999	14.882	14.911	15.174
40.00	14.882	14.298	14.590	14.590
65.00	14.502	13.423	14.006	13.773
80	13.043	11.672	12.402	12.256
90	11.205	10.213	10.709	10.505
105	8.521	7.879	8.258	8.083
115	7.091	6.711	6.857	6.770
140	4.902	4.669	4.815	4.756
165	3.560	3.502	3.502	3.472
190	2.685	2.539	2.655	2.626

B.5. Flexural Buckling Strength of Column with Weld-Affected Zone at One Fourth of the Length

L/r	One fourth			
	MASTAN2		FE++	
	Fcr W/ imperfection (HAZ@1/4)	Fcr W/ partial yielding & imperfection (HAZ@1/4)	NONE (FE++) (HAZ@1/4)	KETTER (FE++) (HAZ@1/4)
5.00	14.999	14.882	15.174	15.174
15.00	14.999	14.999	14.882	14.969
40.00	14.882	14.298	14.590	14.590
65.00	14.357	13.131	13.715	13.715
80	12.810	11.380	12.110	11.876
90	11.205	10.213	10.709	10.505
105	8.375	7.879	8.083	7.966
115	7.149	6.711	6.974	6.857
140	4.815	4.669	4.727	4.698
165	3.531	3.531	3.443	3.443
190	2.685	2.626	2.626	2.626

B.6. Flexural Buckling Strength of Column by Using AA and AISC Specifications

Member for Compression				
L/r	AA		AISC	
	Fc,base (AA)	Fc,weld (AA)	Fc,base (AISC)	Fc,weld (AISC)
5	32.416	13.954	34.872	14.976
15	30.327	13.238	33.862	14.789
40	25.105	11.449	27.666	13.562
65	19.882	9.660	18.811	11.495
80	13.239	8.586	13.665	10.024
90	10.461	7.871	10.793	9.006
105	7.685	6.797	7.929	7.491
115	6.407	6.081	6.610	6.522
140	4.323	4.323	4.460	4.460
165	3.112	3.112	3.211	3.211
190	2.347	2.347	2.422	2.422
200	2.118	2.118	2.186	2.186

APPENDIX C
FLEXURAL STRENGTH

C.1. MASTAN2: Flexural Strength of Beams with $C_b=1.0$

C.1.1. Flexural Strength of Beams with No Weld-Affected Zone

Table 1 Cb=1											
Length (ft.)	Length (in.)	Plastic Capacity ($M_n=M_p$)		Theoretical LTB ($M_n=M_{cr}$)		AISC		No Weld, δ_o , No σ_{res}		No Weld, δ_o , σ_{res}	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	287.805	1.00	38352.359	133.258	287.805	1.000	287.00	0.997	287.00	0.997
1.2	14.4	287.805	1.00	4314.774	14.992	287.805	1.000	287.00	0.997	287.00	0.997
2.4	28.8	287.805	1.00	1122.579	3.900	287.499	0.999	287.00	0.997	278.00	0.966
3.5	42	287.805	1.00	557.408	1.937	276.509	0.961	286.00	0.994	251.00	0.872
4.5	54	287.805	1.00	358.017	1.244	266.518	0.926	279.00	0.969	217.00	0.754
5.4	64.8	287.805	1.00	263.795	0.917	257.526	0.895	250.50	0.870	186.00	0.646
6.0	72	287.805	1.00	222.648	0.774	222.684	0.774	220.00	0.764	168.00	0.584
6.7	80.4	287.805	1.00	187.497	0.651	187.531	0.652	185.00	0.643	150.00	0.521
7.5	90	287.805	1.00	158.282	0.550	158.314	0.550	155.00	0.539	133.00	0.462
8.5	102	287.805	1.00	132.135	0.459	132.165	0.459	131.00	0.455	116.00	0.403
9.5	114	287.805	1.00	113.266	0.394	113.294	0.394	112.00	0.389	103.00	0.358
11.0	132	287.805	1.00	93.249	0.324	93.275	0.324	93.00	0.323	87.00	0.302
13.0	156	287.805	1.00	75.527	0.262	75.550	0.263	76.00	0.264	73.00	0.254
16	192	287.805	1.00	58.914	0.205	58.933	0.205	59.00	0.205	58.00	0.202
18	216	287.805	1.00	51.453	0.179	51.471	0.179	52.00	0.181	52.00	0.181
20	240	287.805	1.00	45.710	0.159	45.726	0.159	47.00	0.163	46.00	0.160
22	264	287.805	1.00	41.148	0.143	41.162	0.143	42.00	0.146	42.00	0.146
24	288	287.805	1.00	37.433	0.130	37.446	0.130	39.00	0.136	39.00	0.136

C.1.2. Flexural Strength of Beams with Weld-Affected Zone at the Left End

Table 2 END $C_b=1$

Length (ft.)	Length (in.)	Weld ($x=0$), δ_o , No ρ_y		Weld ($x=0$), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	123.0	0.427	123.0	0.427
1.2	14.4	123.0	0.427	123.0	0.427
2.4	28.8	123.0	0.427	123.0	0.427
3.5	42	123.0	0.427	123.0	0.427
4.5	54	123.0	0.427	123.0	0.427
5.4	64.8	123.0	0.427	123.0	0.427
6.0	72	123.0	0.427	123.0	0.427
6.7	80.4	123.0	0.427	123.0	0.427
7.5	90	123.0	0.427	123.0	0.427
8.5	102	123.0	0.427	116.0	0.403
9.5	114	112.0	0.389	103.0	0.358
11.0	132	92.0	0.320	87.0	0.302
13.0	156	76.0	0.264	73.0	0.254
16.0	192	59.0	0.205	58.0	0.202
18	216	52.0	0.181	52.0	0.181
20	240	47.0	0.163	46.0	0.160
22	264	42.0	0.146	42.0	0.146
24	288	39.0	0.136	39.0	0.136

C.1.3. Flexural Strength of Beams with Weld-Affected Zone at the Mid-Span

Table 3 0.50 Cb=1

Length (ft.)	Length (in.)	Weld ($x=L/2$), δ_o , No ρ_y		Weld ($x=L/2$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	123.0	0.427	123.0	0.427
1.2	14.4	123.0	0.427	123.0	0.427
2.4	28.8	123.0	0.427	123.0	0.427
3.5	42	123.0	0.427	123.0	0.427
4.5	54	123.0	0.427	122.0	0.424
5.4	64.8	122.9	0.427	121.0	0.420
6.0	72	122.0	0.424	119.0	0.413
6.7	80.4	121.0	0.420	117.0	0.407
7.5	90	118.8	0.413	114.0	0.396
8.5	102	113.0	0.393	107.8	0.375
9.5	114	105.3	0.366	99.0	0.344
11.0	132	93.0	0.323	86.0	0.299
13.0	156	76.0	0.264	72.0	0.250
16.0	192	58.0	0.202	58.0	0.202
18	216	52.0	0.181	51.0	0.177
20	240	47.0	0.163	46.0	0.160
22	264	42.0	0.146	42.0	0.146
24	288	39.0	0.136	39.0	0.136

C.1.4. Flexural Strength of Beams with Weld-Affected Zone at one third of the length

Table 4 0.33 Cb=1

Length (ft.)	Length (in.)	Weld ($x=L/3$), δ_o , No ρ_y		Weld ($x=L/3$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	123.0	0.427	123.0	0.427
1.2	14.4	123.0	0.427	123.0	0.427
2.4	28.8	123.0	0.427	123.0	0.427
3.5	42	123.0	0.427	123.0	0.427
4.5	54	123.0	0.427	122.0	0.424
5.4	64.8	123.0	0.427	121.0	0.420
6.0	72	122.6	0.426	120.0	0.417
6.7	80.4	121.7	0.423	118.0	0.410
7.5	90	119.6	0.416	115.0	0.400
8.5	102	114.3	0.397	109.0	0.379
9.5	114	106.5	0.370	101.0	0.351
11.0	132	92.0	0.320	84.0	0.292
13.0	156	76.0	0.264	72.0	0.250
16.0	192	59.0	0.205	58.0	0.202
18	216	52.0	0.181	52.0	0.181
20	240	47.0	0.163	46.0	0.160
22	264	42.0	0.146	42.0	0.146
24	288	39.0	0.136	39.0	0.136

C.1.5. Flexural Strength of Beams with Weld-Affected Zone at one fourth of the length

Table 6 0.25 Cb=1

Length (ft.)	Length (in.)	Weld ($x=L/4$), δ_o , No ρ_y		Weld ($x=L/4$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	123.0	0.427	123.0	0.427
1.2	14.4	123.0	0.427	123.0	0.427
2.4	28.8	123.0	0.427	123.0	0.427
3.5	42	123.0	0.427	123.0	0.427
4.5	54	123.0	0.427	122.0	0.424
5.4	64.8	123.0	0.427	121.0	0.420
6.0	72	122.9	0.427	121.0	0.420
6.7	80.4	122.0	0.424	119.0	0.413
7.5	90	120.6	0.419	117.0	0.407
8.5	102	116.0	0.403	110.0	0.382
9.5	114	104.6	0.363	98.0	0.341
11.0	132	93.0	0.323	87.1	0.303
13	156	76.0	0.264	73.0	0.254
16	192	59.0	0.205	58.0	0.202
18	216	52.0	0.181	52.0	0.181
20	240	47.0	0.163	46.0	0.160
22	264	42.0	0.146	42.0	0.146
24	288	39.0	0.136	39.0	0.136

C.2. MASTAN2: Flexural Strength of Beams with $C_b=1.25$

C.2.1. Flexural Strength of Beams with No Weld-Affected Zone

Table 1 $C_b = 1.25$											
Length (ft.)	Length (in.)	Plastic Capacity ($M_n=M_p$)		Theoretical LTB ($M_n=M_{cr}$)		AISC		No Weld, δ_o , No σ_{res}		No Weld, δ_o , σ_{res}	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	287.805	1.00	47940.449	166.573	287.805	1.000	287.000	0.997	287.000	0.997
1.2	14.4	287.805	1.00	5393.468	18.740	287.805	1.000	287.000	0.997	287.000	0.997
2.4	28.8	287.805	1.00	1403.224	4.876	287.805	1.000	287.000	0.997	287.000	0.997
3.5	42	287.805	1.00	696.760	2.421	287.805	1.000	287.000	0.997	287.000	0.997
4.5	54	287.805	1.00	447.522	1.555	287.805	1.000	287.000	0.997	276.000	0.959
5.4	64.8	287.805	1.00	329.744	1.146	287.805	1.000	287.000	0.997	241.000	0.837
6.0	72	287.805	1.00	278.310	0.967	287.805	1.000	286.000	0.994	218.000	0.757
6.7	80.4	287.805	1.00	234.371	0.814	287.805	1.000	244.000	0.848	196.000	0.681
7.5	90	287.805	1.00	197.852	0.687	197.893	0.688	204.000	0.709	173.000	0.601
8.5	102	287.805	1.00	165.169	0.574	165.206	0.574	172.000	0.598	152.000	0.528
9.5	114	287.805	1.00	141.583	0.492	141.618	0.492	147.000	0.511	134.000	0.466
11.0	132	287.805	1.00	116.561	0.405	116.593	0.405	121.000	0.420	114.000	0.396
13.0	156	287.805	1.00	94.409	0.328	94.437	0.328	99.000	0.344	95.000	0.330
16	192	287.805	1.00	73.643	0.256	73.667	0.256	77.000	0.268	76.000	0.264
18	216	287.805	1.00	64.317	0.223	64.338	0.224	68.000	0.236	67.000	0.233
20	240	287.805	1.00	57.138	0.199	57.157	0.199	61.000	0.212	60.000	0.208
22	264	287.805	1.00	51.435	0.179	51.453	0.179	55.000	0.191	55.000	0.191
24	288	287.805	1.00	46.791	0.163	46.807	0.163	50.000	0.174	50.000	0.174
28	336	287.805	1.00	39.674	0.138	39.689	0.138	43.000	0.149	43.000	0.149
32	384	287.805	1.00	34.468	0.120	34.480	0.120	38.000	0.132	38.000	0.132

C.2.2. Flexural Strength of Beams with Weld-Affected Zone at the Left End

Table 2 0 $C_b = 1.25$

Length (ft.)	Length (in.)	Weld ($x=0$), δ_o , No ρ_y		Weld ($x=0$), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	123.0	0.427	123.0	0.427
1.2	14.4	123.0	0.427	123.0	0.427
2.4	28.8	123.0	0.427	123.0	0.427
3.5	42	123.0	0.427	123.0	0.427
4.5	54	123.0	0.427	123.0	0.427
5.4	64.8	123.0	0.427	123.0	0.427
6.0	72	123.0	0.427	123.0	0.427
6.7	80.4	123.0	0.427	123.0	0.427
7.5	90	123.0	0.427	123.0	0.427
8.5	102	123.0	0.427	123.0	0.427
9.5	114	123.0	0.427	123.0	0.427
11.0	132	121.0	0.420	114.0	0.396
13.0	156	99.0	0.344	95.0	0.330
16.0	192	77.0	0.268	76.0	0.264
18	216	68.0	0.236	67.0	0.233
20	240	61.0	0.212	60.0	0.208
22	264	55.0	0.191	55.0	0.191
24	288	50.0	0.174	50.0	0.174
28	336	43.0	0.149	43.0	0.149
32	384	38.0	0.132	38.0	0.132

C.2.3. Flexural Strength of Beams with Weld-Affected Zone at one fourth of the length

Table 3 0.25 $C_b = 1.25$

Length (ft.)	Length (in.)	Weld ($x=L/4$), δ_o , No ρ_y		Weld ($x=L/4$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	126.0	0.438	126.0	0.438
1.2	14.4	135.7	0.471	135.7	0.471
2.4	28.8	138.0	0.479	138.0	0.479
3.5	42	139.0	0.483	139.0	0.483
4.5	54	139.0	0.483	139.0	0.483
5.4	64.8	139.0	0.483	139.0	0.483
6.0	72	139.0	0.483	138.0	0.479
6.7	80.4	139.0	0.483	137.0	0.476
7.5	90	138.0	0.479	135.0	0.469
8.5	102	136.0	0.473	132.0	0.459
9.5	114	128.0	0.445	123.0	0.427
11.0	132	116.3	0.404	109.0	0.379
13.0	156	99.0	0.344	94.0	0.327
16.0	192	77.0	0.268	77.0	0.268
18	216	68.0	0.236	67.0	0.233
20	240	61.0	0.212	60.0	0.208
22	264	55.0	0.191	55.0	0.191
24	288	50.0	0.174	50.0	0.174
28	336	43.0	0.149	43.0	0.149
32	384	38.0	0.132	37.0	0.129

C.2.4. Flexural Strength of Beams with Weld-Affected Zone at the Mid-Span

Table 4 0.50 $C_b = 1.25$

Length (ft.)	Length (in.)	Weld ($x=L/2$), δ_o , No ρ_y		Weld ($x=L/2$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	144.0	0.500	144.0	0.500
1.2	14.4	157.0	0.546	157.0	0.546
2.4	28.8	160.9	0.559	160.9	0.559
3.5	42	162.0	0.563	162.0	0.563
4.5	54	162.0	0.563	162.0	0.563
5.4	64.8	162.0	0.563	160.0	0.556
6.0	72	161.0	0.559	159.0	0.552
6.7	80.4	160.0	0.556	156.0	0.542
7.5	90	157.0	0.546	151.0	0.525
8.5	102	149.9	0.521	142.0	0.493
9.5	114	139.4	0.484	130.0	0.452
11.0	132	121.0	0.420	112.0	0.389
13.0	156	99.0	0.344	94.0	0.327
16.0	192	76.0	0.264	75.0	0.261
18	216	68.0	0.236	67.0	0.233
20	240	61.0	0.212	60.0	0.208
22	264	55.0	0.191	55.0	0.191
24	288	50.0	0.174	50.0	0.174
28	336	43.0	0.149	43.0	0.149
32	384	38.0	0.132	37.0	0.129

C.2.5. Flexural Strength of Beams with Weld-Affected Zone at three quarters of the length

Table 5 0.75 $C_b = 1.25$

Length (ft.)	Length (in.)	Weld ($x=3L/4$), δ_o , No ρ_y		Weld ($x=3L/4$), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	169.0	0.587	169.0	0.587
1.2	14.4	187.0	0.650	187.0	0.650
2.4	28.8	191.0	0.664	191.0	0.664
3.5	42	193.0	0.671	193.0	0.671
4.5	54	194.0	0.674	194.0	0.674
5.4	64.8	194.0	0.674	192.0	0.667
6.0	72	193.0	0.671	190.0	0.660
6.7	80.4	191.0	0.664	185.0	0.643
7.5	90	184.7	0.642	171.0	0.594
8.5	102	169.5	0.589	151.0	0.525
9.5	114	147.0	0.511	134.0	0.466
11.0	132	120.0	0.417	113.0	0.393
13.0	156	99.0	0.344	95.0	0.330
16.0	192	77.0	0.268	76.0	0.264
18	216	68.0	0.236	67.0	0.233
20	240	61.0	0.212	60.0	0.208
22	264	55.0	0.191	55.0	0.191
24	288	50.0	0.174	50.0	0.174
28	336	43.0	0.149	43.0	0.149
32	384	38.0	0.132	38.0	0.132

C.2.6. Flexural Strength of Beams with Weld-Affected Zone at the Right End

Table 6 1.00 $C_b = 1.25$

Length (ft.)	Length (in.)	Weld ($x=L$), δ_o , No ρ_y		Weld ($x=L$), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	174.0	0.605	174.0	0.605
1.2	14.4	216.0	0.751	216.0	0.751
2.4	28.8	230.0	0.799	230.0	0.799
3.5	42	235.0	0.817	235.0	0.817
4.5	54	237.0	0.823	237.0	0.823
5.4	64.8	239.0	0.830	239.0	0.830
6.0	72	240.0	0.834	218.0	0.757
6.7	80.4	240.0	0.834	196.0	0.681
7.5	90	206.0	0.716	174.0	0.605
8.5	102	172.0	0.598	152.0	0.528
9.5	114	147.0	0.511	134.0	0.466
11.0	132	121.0	0.420	114.0	0.396
13	156	99.0	0.344	95.0	0.330
16	192	77.0	0.268	76.0	0.264
18	216	68.0	0.236	67.0	0.233
20	240	61.0	0.212	60.0	0.208
22	264	55.0	0.191	55.0	0.191
24	288	50.0	0.174	50.0	0.174
28	336	43.0	0.149	43.0	0.149
32	384	38.0	0.132	38.0	0.132

C.3. MASTAN2: Flexural Strength of Beams with $C_b=1.67$

C.3.1. Flexural Strength of Beams with No Weld-Affected Zone

Table 1 $C_b=1.667$											
Length (ft.)	Length (in.)	Plastic Capacity ($M_n=M_p$)		Theoretical LTB ($M_n=M_{cr}$)		AISC		No Weld, δ_o , No σ_{res}		No Weld, δ_o , σ_{res}	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	287.805	1.00	63920.599	222.097	287.805	1.000	287.000	0.997	287.000	0.997
1.2	14.4	287.805	1.00	7191.291	24.987	287.805	1.000	287.000	0.997	287.000	0.997
2.4	28.8	287.805	1.00	1870.966	6.501	287.805	1.000	287.000	0.997	287.000	0.997
3.5	42	287.805	1.00	929.013	3.228	287.805	1.000	287.000	0.997	287.000	0.997
4.5	54	287.805	1.00	596.696	2.073	287.805	1.000	287.000	0.997	287.000	0.997
5.4	64.8	287.805	1.00	439.658	1.528	287.805	1.000	287.000	0.997	287.000	0.997
6.0	72	287.805	1.00	371.080	1.289	287.805	1.000	287.000	0.997	279.000	0.969
6.7	80.4	287.805	1.00	312.495	1.086	287.805	1.000	287.000	0.997	256.000	0.889
7.5	90	287.805	1.00	263.803	0.917	287.805	1.000	283.000	0.983	230.000	0.799
8.5	102	287.805	1.00	220.225	0.765	287.805	1.000	239.000	0.830	203.000	0.705
9.5	114	287.805	1.00	188.777	0.656	188.824	0.656	204.000	0.709	181.000	0.629
11.0	132	287.805	1.00	155.415	0.540	155.458	0.540	168.000	0.584	155.000	0.539
13.0	156	287.805	1.00	125.878	0.437	125.916	0.438	136.000	0.473	129.000	0.448
16.0	192	287.805	1.00	98.190	0.341	98.222	0.341	106.000	0.368	103.000	0.358
18	216	287.805	1.00	85.756	0.298	85.785	0.298	92.000	0.320	91.000	0.316
20	240	287.805	1.00	76.183	0.265	76.210	0.265	82.000	0.285	81.000	0.281
22	264	287.805	1.00	68.580	0.238	68.604	0.238	74.000	0.257	74.000	0.257
24	288	287.805	1.00	62.388	0.217	62.410	0.217	67.000	0.233	67.000	0.233
28	336	287.805	1.00	52.899	0.184	52.919	0.184	57.000	0.198	57.000	0.198
32	384	287.805	1.00	45.957	0.160	45.974	0.160	50.000	0.174	50.000	0.174
36	432	287.805	1.00	40.648	0.141	40.663	0.141	44.000	0.153	44.000	0.153

C.3.2. Flexural Strength of Beams with Weld-Affected Zone at the Left End

Table 2 0 $C_b=1.667$

Length (ft.)	Length (in.)	Weld ($x=0$), δ_o , No ρ_y		Weld ($x=0$), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	123.0	0.427	123.0	0.427
1.2	14.4	123.0	0.427	123.0	0.427
2.4	28.8	123.0	0.427	123.0	0.427
3.5	42	123.0	0.427	123.0	0.427
4.5	54	123.0	0.427	123.0	0.427
5.4	64.8	123.0	0.427	123.0	0.427
6.0	72	123.0	0.427	123.0	0.427
6.7	80.4	123.0	0.427	123.0	0.427
7.5	90	123.0	0.427	123.0	0.427
8.5	102	123.0	0.427	123.0	0.427
9.5	114	123.0	0.427	123.0	0.427
11.0	132	123.0	0.427	123.0	0.427
13.0	156	123.0	0.427	123.0	0.427
16.0	192	106.0	0.368	103.0	0.358
18	216	92.0	0.320	91.0	0.316
20	240	82.0	0.285	81.0	0.281
22	264	74.0	0.257	74.0	0.257
24	288	67.0	0.233	67.0	0.233
28	336	57.0	0.198	57.0	0.198
32	384	50.0	0.174	50.0	0.174
36	432	44.0	0.153	44.0	0.153

C.3.3. Flexural Strength of Beams with Weld-Affected Zone at one fourth of the length

Table 3 0.25 Cb=1.667

Length (ft.)	Length (in.)	Weld ($x=L/4$), δ_o , No ρ_y		Weld ($x=L/4$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	128.9	0.448	128.9	0.448
1.2	14.4	150.0	0.521	150.0	0.521
2.4	28.8	157.0	0.546	157.0	0.546
3.5	42	159.0	0.552	159.0	0.552
4.5	54	160.0	0.556	160.0	0.556
5.4	64.8	161.0	0.559	160.0	0.556
6.0	72	161.0	0.559	161.0	0.559
6.7	80.4	161.0	0.559	160.0	0.556
7.5	90	161.0	0.559	159.0	0.552
8.5	102	160.0	0.556	157.0	0.546
9.5	114	157.8	0.548	153.0	0.532
11.0	132	150.0	0.521	143.0	0.497
13.0	156	135.0	0.469	128.0	0.445
16.0	192	106.0	0.368	102.0	0.354
18	216	92.0	0.320	90.0	0.313
20	240	82.0	0.285	81.0	0.281
22	264	74.0	0.257	73.0	0.254
24	288	67.0	0.233	67.0	0.233
28	336	57.0	0.198	57.0	0.198
32	384	50.0	0.174	50.0	0.174
36	432	44.0	0.153	44.0	0.153

C.3.4. Flexural Strength of Beams with Weld-Affected Zone at the Mid-Span

Table 4 0.50 C_b=1.667

Length (ft.)	Length (in.)	Weld ($x=L/2$), δ_o , No ρ_y		Weld ($x=L/2$), δ_o , ρ_y	
		M _n (kip-in)	M _n /M _p	M _n (kip-in)	M _n /M _p
0.4	4.8	174.6	0.607	174.6	0.607
1.2	14.4	216.0	0.751	216.0	0.751
2.4	28.8	230.0	0.799	230.0	0.799
3.5	42	235.0	0.817	235.0	0.817
4.5	54	237.0	0.823	237.0	0.823
5.4	64.8	238.0	0.827	237.0	0.823
6.0	72	237.0	0.823	234.9	0.816
6.7	80.4	236.0	0.820	230.0	0.799
7.5	90	231.0	0.803	219.0	0.761
8.5	102	218.0	0.757	198.0	0.688
9.5	114	200.3	0.696	178.0	0.618
11.0	132	168.0	0.584	153.0	0.532
13.0	156	136.0	0.473	128.0	0.445
16.0	192	104.0	0.361	103.0	0.358
18	216	92.0	0.320	91.0	0.316
20	240	82.0	0.285	81.0	0.281
22	264	74.0	0.257	74.0	0.257
24	288	67.0	0.233	67.0	0.233
28	336	57.0	0.198	57.0	0.198
32	384	50.0	0.174	50.0	0.174
36	432	44.0	0.153	44.0	0.153

C.3.5. Flexural Strength of Beams with Weld-Affected Zone at three quarters of the length

Table 5 0.75 $C_b=1.667$

Length (ft.)	Length (in.)	Weld ($x=3L/4$), δ_o , No ρ_y		Weld ($x=3L/4$), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	269.0	0.935	269.0	0.935
1.2	14.4	287.0	0.997	287.0	0.997
2.4	28.8	287.0	0.997	287.0	0.997
3.5	42	287.0	0.997	287.0	0.997
4.5	54	287.0	0.997	287.0	0.997
5.4	64.8	287.0	0.997	287.0	0.997
6.0	72	287.0	0.997	279.0	0.969
6.7	80.4	287.0	0.997	256.0	0.889
7.5	90	286.0	0.994	231.0	0.803
8.5	102	239.0	0.830	203.0	0.705
9.5	114	204.0	0.709	181.0	0.629
11.0	132	166.0	0.577	159.0	0.552
13.0	156	136.0	0.473	134.0	0.466
16.0	192	106.0	0.368	103.0	0.358
18	216	92.0	0.320	91.0	0.316
20	240	82.0	0.285	81.0	0.281
22	264	74.0	0.257	74.0	0.257
24	288	67.0	0.233	67.0	0.233
28	336	57.0	0.198	57.0	0.198
32	384	50.0	0.174	50.0	0.174
36	432	44.0	0.153	44.0	0.153

C.3.6. Flexural Strength of Beams with Weld-Affected Zone at the Right End

Table 6 1.00 $C_b=1.667$

Length (ft.)	Length (in.)	Weld ($x=L$), δ_o , No ρ_y		Weld ($x=L$), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	287.0	0.997	287.0	0.997
1.2	14.4	287.0	0.997	287.0	0.997
2.4	28.8	287.0	0.997	287.0	0.997
3.5	42	287.0	0.997	287.0	0.997
4.5	54	287.0	0.997	287.0	0.997
5.4	64.8	287.0	0.997	287.0	0.997
6.0	72	287.0	0.997	279.0	0.969
6.7	80.4	287.0	0.997	256.0	0.889
7.5	90	286.0	0.994	231.0	0.803
8.5	102	239.0	0.830	203.0	0.705
9.5	114	204.0	0.709	181.0	0.629
11.0	132	168.0	0.584	155.0	0.539
13	156	136.0	0.473	129.0	0.448
16	192	106.0	0.368	103.0	0.358
18	216	92.0	0.320	91.0	0.316
20	240	82.0	0.285	81.0	0.281
22	264	74.0	0.257	74.0	0.257
24	288	67.0	0.233	67.0	0.233
28	336	57.0	0.198	57.0	0.198
32	384	50.0	0.174	50.0	0.174
36	432	44.0	0.153	44.0	0.153

C.4. MASTAN2: Flexural Strength of Beams with $C_b=2.17$

C.4.1. Flexural Strength of Beams with No Weld-Affected Zone

Table 1 Cb=2.17											
Length (ft.)	Length (in.)	Plastic Capacity ($M_n=M_p$)		Theoretical LTB ($M_n=M_{cr}$)		AISC		No Weld, δ_o , No σ_{res}		No Weld, δ_o , σ_{res}	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	287.805	1.00	83374.695	289.692	287.805	1.000	287.000	0.997	287.000	0.997
1.2	14.4	287.805	1.00	9379.945	32.591	287.805	1.000	287.000	0.997	287.000	0.997
2.4	28.8	287.805	1.00	2440.390	8.479	287.805	1.000	287.000	0.997	287.000	0.997
3.5	42	287.805	1.00	1211.756	4.210	287.805	1.000	287.000	0.997	287.000	0.997
4.5	54	287.805	1.00	778.299	2.704	287.805	1.000	287.000	0.997	287.000	0.997
5.4	64.8	287.805	1.00	573.467	1.993	287.805	1.000	287.000	0.997	287.000	0.997
6.0	72	287.805	1.00	484.018	1.682	287.805	1.000	287.000	0.997	287.000	0.997
6.7	80.4	287.805	1.00	407.602	1.416	287.805	1.000	287.000	0.997	287.000	0.997
7.5	90	287.805	1.00	344.091	1.196	287.805	1.000	287.000	0.997	287.000	0.997
8.5	102	287.805	1.00	287.250	0.998	287.805	1.000	287.000	0.997	262.000	0.910
9.5	114	287.805	1.00	246.231	0.856	287.805	1.000	284.000	0.987	237.000	0.823
11.0	132	287.805	1.00	202.715	0.704	287.805	1.000	232.000	0.806	205.000	0.712
13.0	156	287.805	1.00	164.189	0.570	164.238	0.571	187.000	0.650	173.000	0.601
16	192	287.805	1.00	128.075	0.445	128.116	0.445	144.000	0.500	139.000	0.483
18	216	287.805	1.00	111.855	0.389	111.893	0.389	125.000	0.434	122.000	0.424
20	240	287.805	1.00	99.370	0.345	99.404	0.345	110.000	0.382	109.000	0.379
22	264	287.805	1.00	89.45	0.311	89.483	0.311	99.000	0.344	99.000	0.344
24	288	287.805	1.00	81.375	0.283	81.404	0.283	90.000	0.313	90.000	0.313
28	336	287.805	1.00	68.999	0.240	69.024	0.240	76.000	0.264	76.000	0.264
32	384	287.805	1.00	59.944	0.208	59.966	0.208	66.000	0.229	66.000	0.229
36	432	287.805	1.00	53.019	0.184	53.039	0.184	59.000	0.205	59.000	0.205
44	528	287.805	1.00	43.109	0.150	43.126	0.150	48.000	0.167	48.000	0.167

C.4.2. Flexural Strength of Beams with Weld-Affected Zone at the Left End

Table 2 0 Cb=2.17

Length (ft.)	Length (in.)	Weld (x= 0), δ_o , No ρ_y		Weld (x=0), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	123.0	0.427	123.0	0.427
1.2	14.4	123.0	0.427	123.0	0.427
2.4	28.8	123.0	0.427	123.0	0.427
3.5	42	123.0	0.427	123.0	0.427
4.5	54	123.0	0.427	123.0	0.427
5.4	64.8	123.0	0.427	123.0	0.427
6.0	72	123.0	0.427	123.0	0.427
6.7	80.4	123.0	0.427	123.0	0.427
7.5	90	123.0	0.427	123.0	0.427
8.5	102	123.0	0.427	123.0	0.427
9.5	114	123.0	0.427	123.0	0.427
11.0	132	123.0	0.427	123.0	0.427
13.0	156	123.0	0.427	123.0	0.427
16	192	123.0	0.427	123.0	0.427
18	216	123.0	0.427	122.0	0.424
20	240	111.0	0.386	109.0	0.379
22	264	99.0	0.344	99.0	0.344
24	288	90.0	0.313	90.0	0.313
28	336	76.0	0.264	76.0	0.264
32	384	66.0	0.229	66.0	0.229
36	432	59.0	0.205	59.0	0.205
44	528	48.0	0.167	48.0	0.167

C.4.3. Flexural Strength of Beams with Weld-Affected Zone at one fourth of the length

Table 3 0.25 Cb=2.17

Length (ft.)	Length (in.)	Weld ($x=L/4$), δ_o , No ρ_y		Weld ($x=L/4$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	131.7	0.458	131.7	0.458
1.2	14.4	169.0	0.587	169.0	0.587
2.4	28.8	182.0	0.632	182.0	0.632
3.5	42	186.0	0.646	186.0	0.646
4.5	54	189.0	0.657	189.0	0.657
5.4	64.8	190.0	0.660	190.0	0.660
6.0	72	191.6	0.666	191.0	0.664
6.7	80.4	191.0	0.664	191.0	0.664
7.5	90	192.0	0.667	191.9	0.667
8.5	102	192.0	0.667	190.0	0.660
9.5	114	193.0	0.671	190.0	0.660
11.0	132	192.0	0.667	185.0	0.643
13.0	156	177.6	0.617	165.0	0.573
16	192	144.0	0.500	136.0	0.473
18	216	125.0	0.434	121.0	0.420
20	240	110.0	0.382	108.0	0.375
22	264	99.0	0.344	98.0	0.341
24	288	90.0	0.313	90.0	0.313
28	336	76.0	0.264	76.0	0.264
32	384	66.0	0.229	66.0	0.229
36	432	59.0	0.205	58.0	0.202
44	528	48.0	0.167	48.0	0.167

C.4.4. Flexural Strength of Beams with Weld-Affected Zone at the Mid-Span

Table 4 0.50 Cb=2.17

Length (ft.)	Length (in.)	Weld ($x=L/2$), δ_o , No ρ_y		Weld ($x=L/2$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	219.0	0.761	219.0	0.761
1.2	14.4	287.0	0.997	287.0	0.997
2.4	28.8	287.0	0.997	287.0	0.997
3.5	42	287.0	0.997	287.0	0.997
4.5	54	287.0	0.997	287.0	0.997
5.4	64.8	287.0	0.997	287.0	0.997
6.0	72	287.0	0.997	287.0	0.997
6.7	80.4	287.0	0.997	287.0	0.997
7.5	90	287.0	0.997	287.0	0.997
8.5	102	287.0	0.997	261.0	0.907
9.5	114	284.0	0.987	237.0	0.823
11.0	132	232.0	0.806	205.0	0.712
13.0	156	187.0	0.650	173.0	0.601
16	192	143.0	0.497	139.0	0.483
18	216	125.0	0.434	122.0	0.424
20	240	110.0	0.382	109.0	0.379
22	264	99.0	0.344	99.0	0.344
24	288	90.0	0.313	90.0	0.313
28	336	76.0	0.264	76.0	0.264
32	384	66.0	0.229	66.0	0.229
36	432	59.0	0.205	59.0	0.205
44	528	48.0	0.167	48.0	0.167

C.4.5. Flexural Strength of Beams with Weld-Affected Zone at three quarters of the length

Table 5 0.75 Cb=2.17

Length (ft.)	Length (in.)	Weld ($x=3L/4$), δ_o , No ρ_y		Weld ($x=3L/4$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	281.0	0.976	281.0	0.976
1.2	14.4	287.0	0.997	287.0	0.997
2.4	28.8	287.0	0.997	287.0	0.997
3.5	42	287.0	0.997	287.0	0.997
4.5	54	287.0	0.997	287.0	0.997
5.4	64.8	287.0	0.997	287.0	0.997
6.0	72	287.0	0.997	287.0	0.997
6.7	80.4	287.0	0.997	287.0	0.997
7.5	90	287.0	0.997	287.0	0.997
8.5	102	287.0	0.997	262.0	0.910
9.5	114	284.0	0.987	237.0	0.823
11.0	132	230.0	0.799	204.0	0.709
13.0	156	187.0	0.650	173.0	0.601
16	192	144.0	0.500	139.0	0.483
18	216	125.0	0.434	122.0	0.424
20	240	111.0	0.386	109.0	0.379
22	264	99.0	0.344	99.0	0.344
24	288	90.0	0.313	90.0	0.313
28	336	76.0	0.264	76.0	0.264
32	384	66.0	0.229	66.0	0.229
36	432	59.0	0.205	59.0	0.205
44	528		0.000		0.000

C.4.6. Flexural Strength of Beams with Weld-Affected Zone at the Right End

Table 6 1.00 Cb=2.17

Length (ft.)	Length (in.)	Weld (x=L), δ_o , No ρ_y		Weld (x=L), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	246.0	0.855	246.0	0.855
1.2	14.4	246.0	0.855	246.0	0.855
2.4	28.8	246.0	0.855	246.0	0.855
3.5	42	246.0	0.855	246.0	0.855
4.5	54	246.0	0.855	246.0	0.855
5.4	64.8	246.0	0.855	246.0	0.855
6.0	72	246.0	0.855	246.0	0.855
6.7	80.4	246.0	0.855	246.0	0.855
7.5	90	246.0	0.855	246.0	0.855
8.5	102	246.0	0.855	246.0	0.855
9.5	114	246.0	0.855	237.0	0.823
11.0	132	232.0	0.806	205.0	0.712
13.0	156	187.0	0.650	173.0	0.601
16	192	144.0	0.500	139.0	0.483
18	216	125.0	0.434	122.0	0.424
20	240	111.0	0.386	109.0	0.379
22	264	99.0	0.344	99.0	0.344
24	288	90.0	0.313	90.0	0.313
28	336	76.0	0.264	76.0	0.264
32	384	66.0	0.229	66.0	0.229
36	432	59.0	0.205	59.0	0.205
44	528	48.0	0.167	48.0	0.167

C.5. MASTAN2: Flexural Strength of Beams with $C_b=2.27$

C.5.1. Flexural Strength of Beams with No Weld-Affected Zone

Table 1 Cb=2.27											
Length (ft.)	Length (in.)	Plastic Capacity ($M_n=M_p$)		Theoretical LTB ($M_n=M_{cr}$)		AISC		No Weld, δ_o , No σ_{res}		No Weld, δ_o , σ_{res}	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	287.805	1.00	87164.453	302.859	287.805	1.000	287.000	0.997	279.000	0.969
1.2	14.4	287.805	1.00	9806.306	34.073	287.805	1.000	287.000	0.997	274.000	0.952
2.4	28.8	287.805	1.00	2551.317	8.865	287.805	1.000	287.000	0.997	283.000	0.983
3.5	42	287.805	1.00	1266.836	4.402	287.805	1.000	287.000	0.997	286.000	0.994
4.5	54	287.805	1.00	813.676	2.827	287.805	1.000	287.000	0.997	287.000	0.997
5.4	64.8	287.805	1.00	599.534	2.083	287.805	1.000	287.000	0.997	286.000	0.994
6.0	72	287.805	1.00	506.019	1.758	287.805	1.000	287.000	0.997	284.000	0.987
6.7	80.4	287.805	1.00	426.129	1.481	287.805	1.000	287.000	0.997	280.000	0.973
7.5	90	287.805	1.00	359.731	1.250	287.805	1.000	287.000	0.997	274.000	0.952
8.5	102	287.805	1.00	300.306	1.043	287.805	1.000	287.000	0.997	263.000	0.914
9.5	114	287.805	1.00	257.423	0.894	287.805	1.000	287.000	0.997	250.000	0.869
11.0	132	287.805	1.00	211.930	0.736	287.805	1.000	272.000	0.945	229.000	0.796
13.0	156	287.805	1.00	171.652	0.596	171.704	0.597	220.000	0.764	199.000	0.691
16	192	287.805	1.00	133.896	0.465	133.940	0.465	171.000	0.594	163.000	0.566
18	216	287.805	1.00	116.940	0.406	116.979	0.406	149.000	0.518	144.000	0.500
20	240	287.805	1.00	103.886	0.361	103.922	0.361	132.000	0.459	129.000	0.448
22	264	287.805	1.00	93.518	0.325	93.550	0.325	118.000	0.410	117.000	0.407
24	288	287.805	1.00	85.074	0.296	85.104	0.296	107.000	0.372	106.000	0.368
28	336	287.805	1.00	72.135	0.251	72.162	0.251	91.000	0.316	91.000	0.316
32	384	287.805	1.00	62.668	0.218	62.692	0.218	79.000	0.274	79.000	0.274
36	432	287.805	1.00	55.429	0.193	55.450	0.193	70.000	0.243	70.000	0.243
44	528	287.805	1.00	45.069	0.157	45.086	0.157	56.000	0.195	56.000	0.195

C.5.2. Flexural Strength of Beams with Weld-Affected Zone at the Left End

Table 2 0 Cb=2.27

Length (ft.)	Length (in.)	Weld (x= 0), δ_o , No ρ_y		Weld (x=0), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	123.0	0.427	121.0	0.420
1.2	14.4	123.0	0.427	123.0	0.427
2.4	28.8	123.0	0.427	123.0	0.427
3.5	42	123.0	0.427	123.0	0.427
4.5	54	123.0	0.427	123.0	0.427
5.4	64.8	123.0	0.427	123.0	0.427
6.0	72	123.0	0.427	123.0	0.427
6.7	80.4	123.0	0.427	123.0	0.427
7.5	90	123.0	0.427	123.0	0.427
8.5	102	123.0	0.427	123.0	0.427
9.5	114	123.0	0.427	123.0	0.427
11.0	132	123.0	0.427	123.0	0.427
13.0	156	123.0	0.427	123.0	0.427
16	192	123.0	0.427	123.0	0.427
18	216	123.0	0.427	123.0	0.427
20	240	123.0	0.427	123.0	0.427
22	264	118.0	0.410	116.0	0.403
24	288	107.0	0.372	106.0	0.368
28	336	91.0	0.316	91.0	0.316
32	384	79.0	0.274	79.0	0.274
36	432	70.0	0.243	70.0	0.243
44	528	56.0	0.195	56.0	0.195

C.5.3. Flexural Strength of Beams with Weld-Affected Zone at one fourth of the length

Table 3 0.25 Cb=2.27

Length (ft.)	Length (in.)	Weld ($x=L/4$), δ_o , No ρ_y		Weld ($x=L/4$), δ_o , ρ_y	
		M_n (kip- in)	M_n/M_p	M_n (kip- in)	M_n/M_p
0.4	4.8	134.7	0.468	74.0	0.257
1.2	14.4	193.0	0.671	193.0	0.671
2.4	28.8	216.0	0.751	216.0	0.751
3.5	42	225.6	0.784	225.0	0.782
4.5	54	229.0	0.796	227.0	0.789
5.4	64.8	232.0	0.806	230.0	0.799
6.0	72	233.0	0.810	231.0	0.803
6.7	80.4	235.0	0.817	231.0	0.803
7.5	90	236.0	0.820	230.0	0.799
8.5	102	237.0	0.823	228.0	0.792
9.5	114	234.0	0.813	228.0	0.792
11.0	132	226.0	0.785	213.0	0.740
13.0	156	218.3	0.758	190.0	0.660
16	192	171.0	0.594	159.0	0.552
18	216	149.0	0.518	142.0	0.493
20	240	132.0	0.459	128.0	0.445
22	264	118.0	0.410	116.0	0.403
24	288	107.0	0.372	106.0	0.368
28	336	91.0	0.316	91.0	0.316
32	384	79.0	0.274	79.0	0.274
36	432	70.0	0.243	70.0	0.243
44	528	56.0	0.195	56.0	0.195

C.5.4. Flexural Strength of Beams with Weld-Affected Zone at the Mid-Span

Table 4 0.50 Cb=2.27

Length (ft.)	Length (in.)	Weld ($x=L/2$), δ_o , No ρ_y		Weld ($x=L/2$), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	287.0	0.997	244.0	0.848
1.2	14.4	287.0	0.997	274.0	0.952
2.4	28.8	287.0	0.997	283.0	0.983
3.5	42	287.0	0.997	286.0	0.994
4.5	54	287.0	0.997	287.0	0.997
5.4	64.8	287.0	0.997	286.0	0.994
6.0	72	287.0	0.997	284.0	0.987
6.7	80.4	287.0	0.997	280.0	0.973
7.5	90	287.0	0.997	287.0	0.997
8.5	102	287.0	0.997	263.0	0.914
9.5	114	287.0	0.997	250.0	0.869
11.0	132	272.0	0.945	229.0	0.796
13.0	156	220.0	0.764	199.0	0.691
16	192	167.0	0.580	162.0	0.563
18	216	149.0	0.518	144.0	0.500
20	240	132.0	0.459	129.0	0.448
22	264	118.0	0.410	117.0	0.407
24	288	107.0	0.372	106.0	0.368
28	336	91.0	0.316	91.0	0.316
32	384	79.0	0.274	79.0	0.274
36	432	70.0	0.243	70.0	0.243
44	528	56.0	0.195	56.0	0.195

C.5.5. Flexural Strength of Beams with Weld-Affected Zone at three quarters of the length

Table 5 0.75 Cb=2.27

Length (ft.)	Length (in.)	Weld ($x=3L/4$), δ_o , No ρ_y		Weld ($x=3L/4$), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	94.0	0.327	55.0	0.191
1.2	14.4	193.0	0.671	193.0	0.671
2.4	28.8	216.0	0.751	216.0	0.751
3.5	42	226.5	0.787	225.0	0.782
4.5	54	229.0	0.796	229.0	0.796
5.4	64.8	232.0	0.806	232.0	0.806
6.0	72	233.0	0.810	233.0	0.810
6.7	80.4	235.0	0.817	235.0	0.817
7.5	90	236.0	0.820	236.0	0.820
8.5	102	237.0	0.823	237.0	0.823
9.5	114	238.0	0.827	236.0	0.820
11.0	132	239.0	0.830	225.0	0.782
13.0	156	219.0	0.761	197.0	0.684
16	192	171.0	0.594	162.0	0.563
18	216	149.0	0.518	143.0	0.497
20	240	132.0	0.459	129.0	0.448
22	264	118.0	0.410	116.0	0.403
24	288	107.0	0.372	106.0	0.368
28	336	91.0	0.316	91.0	0.316
32	384	79.0	0.274	79.0	0.274
36	432	70.0	0.243	70.0	0.243
44	528	56.0	0.195	56.0	0.195

C.5.6. Flexural Strength of Beams with Weld-Affected Zone at the Right End

Table 5 1.00 Cb=2.27

Length (ft.)	Length (in.)	Weld (x=L), δ_o , No ρ_y		Weld (x=L), δ_o , ρ_y	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
0.4	4.8	123.0	0.427	123.0	0.427
1.2	14.4	123.0	0.427	123.0	0.427
2.4	28.8	123.0	0.427	123.0	0.427
3.5	42	123.0	0.427	123.0	0.427
4.5	54	123.0	0.427	123.0	0.427
5.4	64.8	123.0	0.427	123.0	0.427
6.0	72	123.0	0.427	123.0	0.427
6.7	80.4	123.0	0.427	123.0	0.427
7.5	90	123.0	0.427	123.0	0.427
8.5	102	123.0	0.427	123.0	0.427
9.5	114	123.0	0.427	123.0	0.427
11.0	132	123.0	0.427	123.0	0.427
13.0	156	123.0	0.427	123.0	0.427
16	192	123.0	0.427	123.0	0.427
18	216	123.0	0.427	123.0	0.427
20	240	123.0	0.427	123.0	0.427
22	264	118.0	0.410	116.0	0.403
24	288	107.0	0.372	106.0	0.368
28	336	91	0.316	91	0.316
32	384	79	0.274	79	0.274
36	432	70	0.243	70	0.243
44	528	56	0.195	56	0.195

C.6. AA Specification: Flexural Strength of Beams

C.6.1. AA Specification: Flexural Strength of Beams with $C_b = 1.0$

Strength of Flexural Members ($C_b=1.0$)						
Length (ft.)	Length (in.)	Slenderness	$F_{b, \text{base}}$, (ksi)	$F_{b, \text{weld}}$, (ksi)	AA- $M_{\text{LTB, nbase}}$, (kip-in)	AA- $M_{\text{LTB, nweld}}$, (kip-in)
0.4	4.8	5.0473	38.332	16.483	280.970	120.820
1.2	14.4	15.142	36.264	15.775	265.817	115.629
2.4	28.8	30.284	33.163	14.712	243.086	107.841
3.5	42	44.164	30.321	13.738	222.249	100.703
4.5	54	56.782	27.736	12.853	203.307	94.213
5.4	64.8	68.139	25.411	12.056	186.259	88.373
6.0	72	75.710	23.860	11.525	174.894	84.479
6.7	80.4	84.543	20.083	10.905	147.210	79.937
7.5	90	94.637	16.027	10.197	117.480	74.745
8.5	102	107.256	12.478	9.312	91.464	68.255
9.5	114	119.874	9.989	8.426	73.222	61.766
11.0	132	138.801	7.451	7.098	54.614	52.032
13.0	156	164.038	5.335	5.335	39.102	39.102
16	192	201.893	3.522	3.522	25.813	25.813
18	216	227.129	2.783	2.783	20.396	20.396
20	240	252.366	2.254	2.254	16.521	16.521
22	264	277.603	1.863	1.863	13.653	13.653
24	288	302.839	1.565	1.565	11.473	11.473
28	336	353.312	1.150	1.150	8.429	8.429
32	384	403.785	0.880	0.880	6.453	6.453
36	432	454.259	0.696	0.696	5.099	5.099
44	528	555.205	0.466	0.466	3.413	3.413

C.6.2. AA Specification: Flexural Strength of Beams with $C_b = 1.25$

Strength of Flexural Members ($C_b=1.25$)						
Length (ft.)	Length (in.)	Slenderness	$F_{b, \text{base}}$, (ksi)	$F_{b, \text{weld}}$, (ksi)	AA- M_{nbase} , LTB, (kip- in)	AA- M_{nweld} , LTB, (kip- in)
0.4	4.8	4.514	38.441	16.520	281.770	121.094
1.2	14.4	13.543	36.592	15.887	268.216	116.451
2.4	28.8	27.087	33.818	14.937	247.885	109.486
3.5	42	39.502	31.275	14.066	229.249	103.101
4.5	54	50.788	28.964	13.274	212.306	97.296
5.4	64.8	60.945	26.884	12.561	197.058	92.072
6.0	72	67.717	25.497	12.086	186.892	88.590
6.7	80.4	75.617	23.879	11.532	175.033	84.527
7.5	90	84.646	20.034	10.898	146.850	79.883
8.5	102	95.932	15.597	10.106	114.329	74.079
9.5	114	107.218	12.487	9.314	91.527	68.275
11.0	132	124.148	9.313	8.127	68.267	59.568
13.0	156	146.720	6.668	6.543	48.878	47.959
16	192	180.578	4.402	4.402	32.267	32.267
18	216	203.151	3.478	3.478	25.495	25.495
20	240	225.723	2.817	2.817	20.651	20.651
22	264	248.295	2.328	2.328	17.067	17.067
24	288	270.868	1.956	1.956	14.341	14.341
28	336	316.012	1.437	1.437	10.536	10.536
32	384	361.157	1.101	1.101	8.067	8.067
36	432	406.301	0.870	0.870	6.374	6.374
44	528	496.590	0.582	0.582	4.267	4.267

C.6.3. AA Specification: Flexural Strength of Beams with $C_b = 1.67$

Strength of Flexural Members ($C_b=1.67$)						
Length (ft.)	Length (in.)	Slenderness	$F_{b, \text{base}}$, (ksi)	$F_{b, \text{weld}}$, (ksi)	AA- $M_{n\text{base}}$, LTB, (kip- in)	AA- $M_{n\text{weld}}$, LTB, (kip- in)
0.4	4.8	3.910	38.565	16.563	282.678	121.405
1.2	14.4	11.729	36.963	16.014	270.940	117.384
2.4	28.8	23.458	34.561	15.191	253.333	111.352
3.5	42	34.209	32.359	14.437	237.193	105.823
4.5	54	43.983	30.358	13.751	222.521	100.796
5.4	64.8	52.780	28.556	13.134	209.315	96.272
6.0	72	58.645	27.355	12.722	200.512	93.256
6.7	80.4	65.486	25.954	12.242	190.241	89.737
7.5	90	73.306	24.352	11.694	178.503	85.716
8.5	102	83.080	20.797	11.008	152.439	80.689
9.5	114	92.854	16.649	10.322	122.036	75.662
11.0	132	107.515	12.418	9.294	91.023	68.122
13.0	156	127.063	8.891	7.922	65.170	58.069
16	192	156.385	5.869	5.865	43.022	42.988
18	216	175.934	4.638	4.638	33.993	33.993
20	240	195.482	3.756	3.756	27.534	27.534
22	264	215.030	3.104	3.104	22.756	22.756
24	288	234.578	2.609	2.609	19.121	19.121
28	336	273.675	1.917	1.917	14.048	14.048
32	384	312.771	1.467	1.467	10.756	10.756
36	432	351.867	1.159	1.159	8.498	8.498
44	528	430.060	0.776	0.776	5.689	5.689

C.6.4. AA Specification: Flexural Strength of Beams with $C_b = 2.17$

Strength of Flexural Members ($C_b=2.17$)						
Length (ft.)	Length (in.)	Slenderness	$F_{b, \text{base}}$ (ksi)	$F_{b, \text{weld}}$ (ksi)	AA- $M_{\text{nbase, LTB}}$ (kip-in)	AA- $M_{\text{nweld, LTB}}$ (kip-in)
0.4	4.8	3.423	38.664	16.597	283.408	121.656
1.2	14.4	10.270	37.262	16.117	273.131	118.134
2.4	28.8	20.540	35.159	15.396	257.714	112.853
3.5	42	29.954	33.231	14.736	243.582	108.011
4.5	54	38.512	31.478	14.135	230.735	103.610
5.4	64.8	46.214	29.901	13.595	219.172	99.649
6.0	72	51.349	28.849	13.234	211.464	97.008
6.7	80.4	57.340	27.622	12.814	202.471	93.927
7.5	90	64.186	26.220	12.334	192.193	90.406
8.5	102	72.744	24.467	11.733	179.345	86.004
9.5	114	81.302	21.716	11.133	159.177	81.603
11.0	132	94.140	16.197	10.232	118.725	75.001
13.0	156	111.256	11.597	9.031	85.004	66.198
16	192	136.930	7.656	7.230	56.116	52.994
18	216	154.047	6.049	6.029	44.339	44.191
20	240	171.163	4.900	4.900	35.914	35.914
22	264	188.279	4.049	4.049	29.681	29.681
24	288	205.395	3.403	3.403	24.941	24.941
28	336	239.628	2.500	2.500	18.324	18.324
32	384	273.861	1.914	1.914	14.029	14.029
36	432	308.093	1.512	1.512	11.085	11.085
44	528	376.558	1.012	1.012	7.420	7.420

C.6.5. AA Specification: Flexural Strength of Beams with $C_b = 2.27$

Strength of Flexural Members ($C_b=2.27$)						
Length (ft.)	Length (in.)	Slenderness	$F_{b, \text{base}}$, (ksi)	$F_{b, \text{weld}}$, (ksi)	AA- $M_{n\text{base}}$, LTB, (kip- in)	AA- $M_{n\text{weld}}$, LTB, (kip- in)
0.4	4.8	3.348	38.680	16.602	283.521	121.694
1.2	14.4	10.044	37.308	16.132	273.470	118.251
2.4	28.8	20.088	35.251	15.428	258.392	113.085
3.5	42	29.295	33.366	14.782	244.570	108.350
4.5	54	37.665	31.651	14.194	232.005	104.045
5.4	64.8	45.198	30.109	13.666	220.697	100.171
6.0	72	50.220	29.080	13.314	213.158	97.588
6.7	80.4	56.079	27.880	12.902	204.363	94.575
7.5	90	62.775	26.509	12.433	194.311	91.131
8.5	102	71.145	24.795	11.845	181.746	86.827
9.5	114	79.515	22.703	11.258	166.413	82.522
11.0	132	92.070	16.933	10.377	124.122	76.065
13.0	156	108.810	12.124	9.203	88.868	67.456
16	192	133.920	8.004	7.441	58.667	54.542
18	216	150.661	6.324	6.266	46.354	45.933
20	240	167.401	5.122	5.122	37.547	37.547
22	264	184.141	4.233	4.233	31.030	31.030
24	288	200.881	3.557	3.557	26.074	26.074
28	336	234.361	2.613	2.613	19.157	19.157
32	384	267.841	2.001	2.001	14.667	14.667
36	432	301.321	1.581	1.581	11.589	11.589
44	528	368.281	1.058	1.058	7.758	7.758

C.7. AA Specification: Flexural Strength of Beams

AISC Flexure Strength						
Length (ft.)	Length (in.)	$C_b=1.0$	$C_b=1.25$	$C_b=1.67$	$C_b=2.17$	$C_b=2.27$
		Mn (kip-in)				
0.4	4.8	287.805	287.805	287.805	287.805	287.805
1.2	14.4	287.805	287.805	287.805	287.805	287.805
2.4	28.8	287.499	287.805	287.805	287.805	287.805
3.5	42	276.509	287.805	287.805	287.805	287.805
4.5	54	266.518	287.805	287.805	287.805	287.805
5.4	64.8	257.526	287.805	287.805	287.805	287.805
6.0	72	222.684	287.805	287.805	287.805	287.805
6.7	80.4	187.531	287.805	287.805	287.805	287.805
7.5	90	158.314	197.893	287.805	287.805	287.805
8.5	102	132.165	165.206	287.805	287.805	287.805
9.5	114	113.294	141.618	188.824	287.805	287.805
11.0	132	93.275	116.593	155.458	287.805	287.805
13.0	156	75.550	94.437	125.916	164.238	171.704
16	192	58.933	73.667	98.222	128.116	133.940
18	216	51.471	64.338	85.785	111.893	116.979
20	240	45.726	57.157	76.210	99.404	103.922
22	264	41.162	51.453	68.604	89.483	93.550
24	288	37.446	46.807	62.410	81.404	85.104
28	336	31.751	39.689	52.919	69.024	72.162
32	384	27.584	34.480	45.974	59.966	62.692
36	432	24.398	30.498	40.663	53.039	55.450
44	528	19.838	24.797	33.063	43.126	45.086

C.7. Newly-Defined Method: Flexural Strength of Beams $C_b=1.25$

Strength of Flexural Members Using Newly-Defined Method ($C_b=1.25$)			
Length (ft.)	Weld ($x=3L/4$), δ_o, ρ_y	Weld ($x=L/2$), δ_o, ρ_y	Weld ($x=L/4$), δ_o, ρ_y
0.4	133.92	156.24	187.48
1.2	133.92	156.24	187.48
2.4	133.92	156.24	187.48
3.5	133.92	156.24	187.48
4.5	133.92	156.24	187.48
5.4	133.92	156.24	187.48
6.0	133.92	156.24	187.48
6.7	133.92	156.24	187.48
7.5	133.92	151.00	171.00
8.5	132.00	142.00	151.00
9.5	123.00	130.00	134.00
11.0	109.00	112.00	113.00
13.0	94.00	94.00	95.00
16	77.00	75.00	76.00
18	67.00	67.00	67.00
20	60.00	60.00	60.00
22	55.00	55.00	55.00
24	50.00	50.00	50.00
28	43.00	43.00	43.00
32	37.00	37.00	38.00