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Analytical Investigation to Determine Occupant Parameters for Dynamic Modeling of Occupants on a Cantilever Structure

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**Analytical Investigation to Determine Occupant Parameters for
Dynamic Modeling of Occupants on a Cantilever Structure**

by

Aradhana Agarwal

Presented to the Honors Council for Honors
in Civil Engineering at Bucknell University
April 2015

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ABSTRACT

The use of lightweight and slender elements in civil engineering design has been increasing, which in turn has increased the vulnerability of these flexible structures to vibrations. For proper functioning of such structures, it is important to accurately assess the dynamic properties of the occupied structure and to incorporate the effects of human-structure interaction (HSI) in the design process. The dynamic properties refer to natural frequencies, damping ratios, and mode shapes of the occupied structure.

Human occupants are modeled as an additional degree-of-freedom on a single degree-of-freedom model of the structure. Modal analysis of the combined 2DOF system is used to determine the dynamic properties of the occupied structure. This small-scale study utilized previously collected experimental data, from a laboratory test structure built at Bucknell University, to assess the quality of a current method proposed for incorporating HSI into design. In addition, several current models aiming to represent the occupants are assessed through a comparison of modal properties obtained through a series of analytical models with the available experimentally determined properties. Finally, a parametric study is completed to ascertain the most appropriate occupant properties to simulate the experimental dynamic parameters.

The results of this study show that the methodology proposed to incorporate the effects of HSI in design is accurate for the standing-straight occupant posture but not for the standing-with-bent-knees posture. Further, the methodology was not appropriate for seated occupant postures. Thus, the focus was to develop an appropriate seated occupant model that would accurately predict the experimental dynamic properties ascertained from the laboratory structure. Additional seated occupant models from the field of

biomechanics were investigated which informed the range of occupant properties to be used in the parametric modeling. To this end, a Dynamic Graphical User Interface (GUI) that implements modal analysis in the frequency domain was developed in Matlab. A natural frequency range of 5.2-5.9 Hz and damping ratio range of 33-39% was determined. An incremental parametric modeling was completed through the use of a modified version of the GUI code. The results seem to indicate that the frequency of the occupant model should be in the 5.2-5.4 Hz range. However, the results do not definitively suggest a damping ratio for this model. Additional research is necessary.

CHAPTER 1: INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

The successful design of a civil engineering structure is based on the two criteria of strength and serviceability. Strength ties directly with the overall safety of the structure and is of primary importance in any design methodology. Serviceability, on the other hand, focuses on structural functionality and occupant comfort level. Engineers design structures to fulfill safety requirements; however, excessive crowd movements can render the structure vulnerable to the effects of vibrations, thus compromising occupant comfort level. Vibration serviceability is of particular concern in stadiums with cantilevered grandstands in which synchronized crowd motions can cause excessive vibrations and induce panic and fear of structural failure.

Previous research indicates that passive crowds can significantly alter the dynamic or vibrational properties of the occupied structure. To improve the current industry standard for serviceability design involving occupied structures, it is crucial to more accurately identify the dynamic properties of the occupied structure that are currently rendered uncertain as a result of human-structure interaction (HSI). These properties include the natural frequencies, damping ratios, and mode shapes of the occupied structure. A conceptual representation of the occupied structure as a two degree-of-freedom mass-spring-damper system is shown in Figure 1. The human occupants are modeled as a lumped mass, M_H , with stiffness and damping, K_H and C_H respectively, inherent in the flexibility of the human body. This single degree-of-freedom model of the occupant is attached to a similar single degree-of-freedom model of the structure.

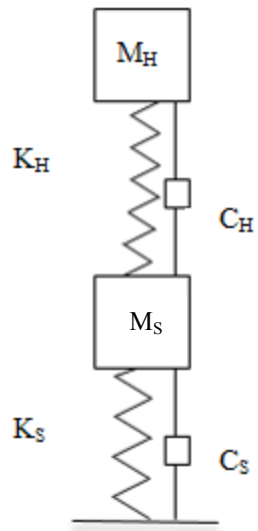


Figure 1. Conceptual representation of 2DOF mass-spring-damper system.

1.2 Purpose of Research

With an accelerating trend toward lighter and more slender civil engineering structures, the vulnerability of these structures to crowd-induced vibrations has increased. Lighter and more slender structures are more flexible and tend to have lower natural frequencies than their traditionally stiffer counterparts. The decrease in natural frequency is potentially problematic as it may drop into the range of human-induced loading, increasing the importance of designing for vibration serviceability (Reynolds 2004, Sim et al. 2006). Consequently, proper design guidelines that incorporate human-structure interaction are needed to achieve the serviceability requirements and to minimize occupant discomfort. However, there is limited knowledge about the effect of HSI on the modal properties of the occupied structure (Sachse 2002, Sachse 2003). This small-scale study utilized previously collected experimental data to assess the quality of a current

method proposed for incorporation HSI into design. In addition, several current models aiming to represent the occupants are assessed through a comparison of modal properties obtained through a series of analytical models with the available experimentally determined properties. The outcomes of this research make significant contribution to the knowledge of dynamic interaction between humans and the structures, and the effect on the modal properties of the structures as a result of this dynamic interaction. In particular, it provides a detailed assessment of an existing design methodology and its appropriateness in representing the results of a small-scale experimental study. Additional occupant models are also investigated and an appropriate range of occupant properties is presented. The graphical user interface (GUI) developed as an analytical tool to complete the project may be used for design purposes and as an educational tool for the study of HSI and dynamics.

1.3 Literature Review

1.3.1 Vibration Serviceability

For civil engineering structures, vibration serviceability is based on the occupant's level of comfort with the dynamic motion of a structure. The level of perception of this motion depends on several factors including the type of loading, the type of activity that the occupant is engaged in, as well as the individual occupant. For instance, the level of perception is more distinct for occupants at rest, such as in office buildings with peak vibration levels of 0.5 percent of gravitational acceleration. On the contrary, for individuals engaged in activities that cause vibrations such as jumping or dancing, the

perception levels are lower and the individual is able to tolerate more (Murray et al. 1997). In order to determine the acceptable level of vibration for occupants, it is necessary to estimate the acceleration response of the structure (Noss 2012). An accurate assessment of this dynamic response requires the estimation of the dynamic properties of the occupied structure.

1.3.2 Human-Structure Interaction

Human-structure interaction is an important phenomenon in the design of civil engineering structures; however, its effects on the occupied structure and the occupants are not fully understood. Human-structure interaction was first acknowledged in 1966 in a study conducted by Lenzen at the University of Kansas. The study involved a group of people occupying a steel-joist-supported floor and it was observed that the dynamic properties of the occupied structure were different from that of the empty structure- there was a decrease in the natural frequency and an increase in the damping ratio of the occupied structure. This phenomenon was termed human-structure interaction (Lenzen 1966) since its effects could not be explained by treating the occupants as an additional lumped mass on the structural system.

Previous research on this phenomenon has shown varying results and that the dynamic properties and response of the occupied structure depends on several factors such as the dynamic properties of the empty structure, the posture and the type of occupant activity, as well as the relative size of the crowd compared with that of the structure (Sachse et al. 2003). The research presented here develops occupant models, based on an in-depth

comparison of experimental and analytical results, that can be used to better understand human-structure interaction.

1.3.3 Laboratory Structure and Experimental Data

The cantilevered structure, Figure 2, was specifically designed and constructed at Bucknell University for the purpose of vibration serviceability research (Noss 2012). Its unique flexibility allows the natural frequency to be varied in the range of 4 to 8 Hz by adjusting the locations and number of supports. The structure was also designed to allow it to be occupied by various group sizes, from one to nine occupants. The occupant mass associated with this range provides the opportunity to investigate a range of mass ratios typical in a stadium construction (Noss 2012). Mass ratio is defined as the ratio of the mass of the occupants to the mass of the structure.

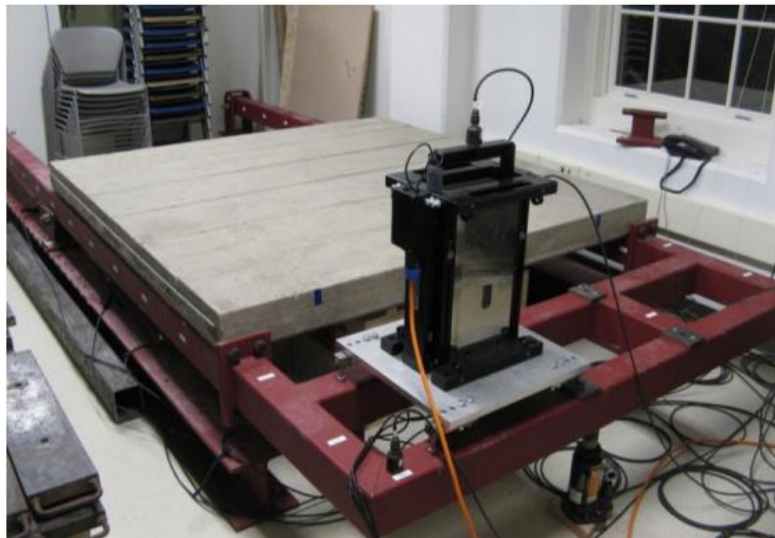
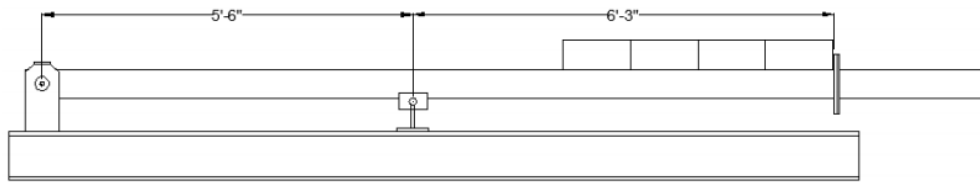


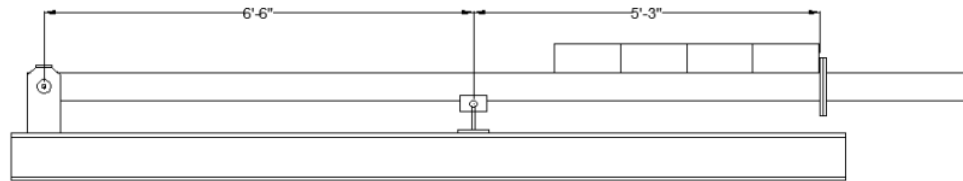
Figure 2. Cantilevered laboratory structure at Bucknell University.

To achieve the range of natural frequencies, the support conditions are adjusted. A previous experimental study (Brennan 2013) examining the modal properties of the

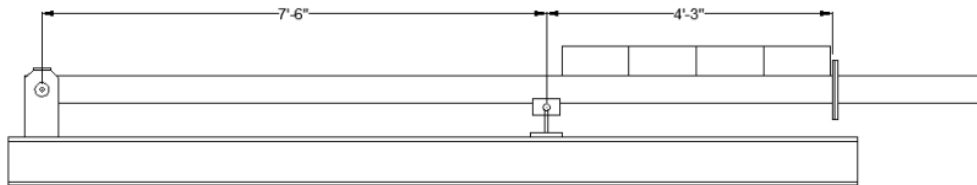
structure occupied by individuals seated on a series of attached benches utilized the five configurations shown in Figure 3. The natural frequency of the empty structure as determined from this study is also noted. In addition, the natural frequency associated with the empty structure with the bench (for seated occupant study) is included in parenthesis alongside.



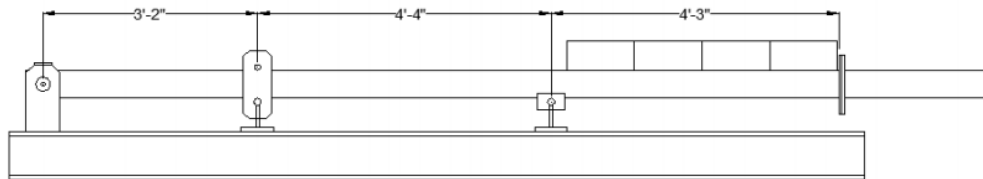
4.21 Hz (4.30 Hz) structural configuration



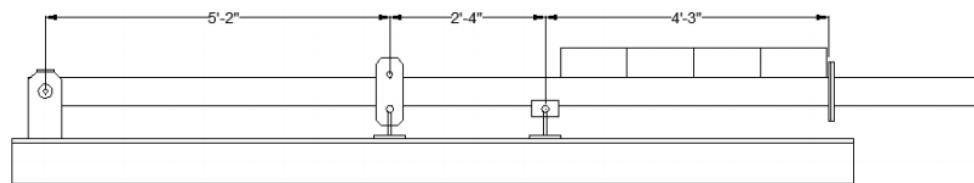
5.41 Hz (5.7 Hz) structural configuration



6.27 Hz (6.8 Hz) structural configuration



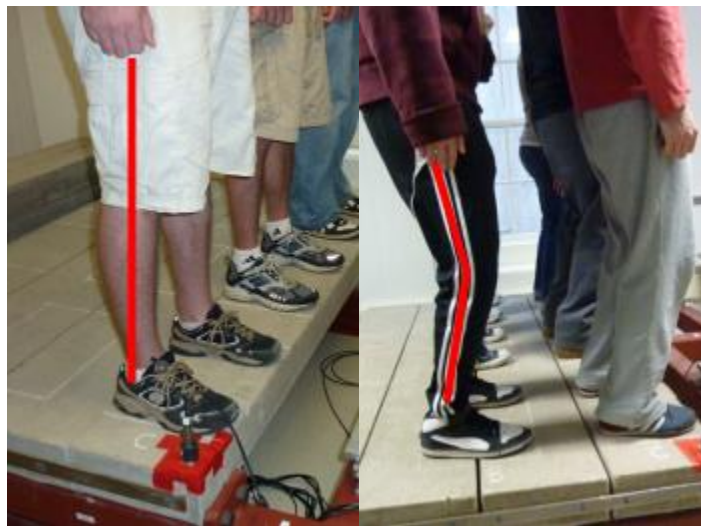
7.30 Hz (7.50 Hz) structural configuration



8.05 Hz (8.1 Hz) structural configuration

Figure 3. Elevation views of the five structural configurations utilized in this study labeled with frequency of empty structure (frequency of empty structure with benches).

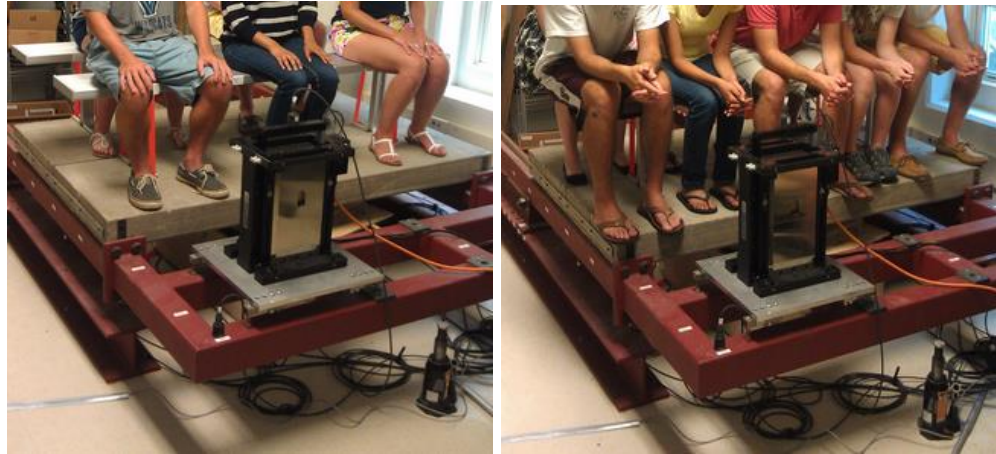
Experimental modal analysis was conducted to ascertain the dynamic properties of the empty structure as well as collection of data for occupants standing with straight as well as with bent knees (Noss 2012), and for occupants seated with straight back as well as seated leaning forward (Brennan 2013) as shown in Figures 4 and 5, respectively. The experimentally obtained modal parameters are the actual dynamic properties of the occupied structure.



(a) Straight

(b) Bent knees

Figure 4. Standing occupant postures.



(a) Straight

(b) Leaning forward

Figure 5. Seated occupant postures.

1.3.4 Sim et al. Methodology for Incorporating HSI into Design

Sim, Blakeborough, and Williams (2006) present a design methodology to consider the effect of HSI based on dynamic analyses on a crowd-structure model developed to investigate the interaction between a passive crowd and the SDOF structural system. The analysis was implemented for structures with natural frequencies in the range of 1 to 10 Hz and crowd mass ratios in the range of 5 to 40%. The effect of the crowd on the properties of the structure is presented in terms of numerical frequency reduction factors that purportedly account for changes in the natural frequency and the response of the occupied structure. The results are graphically summarized for a 100% standing crowd and 100% seated crowd in Figures 6 and 7, respectively. These are based on a reinterpretation of the models of single seated and standing individuals developed by Griffin et al (Sim et al. 2006)

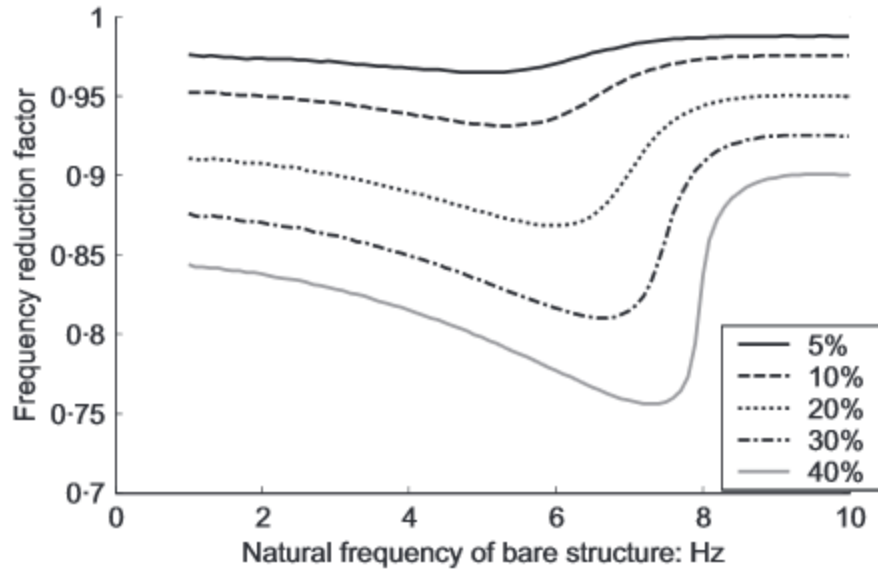


Figure 6. Frequency reduction factor for 100% standing crowd (from Sim et al. 2006).

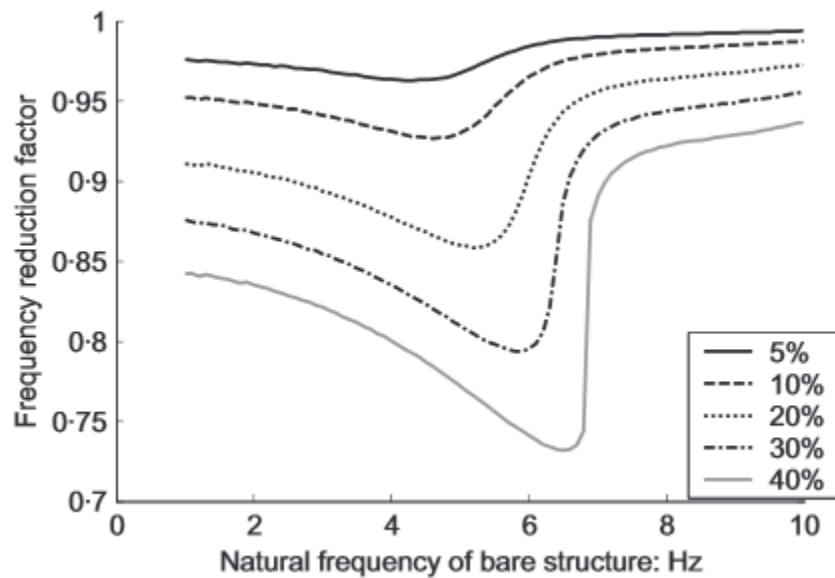


Figure 7. Frequency reduction factor for 100% seated crowd (from Sim et al. 2006).

This methodology requires the structural designer to predict the lowest natural frequency of the empty structure through analytical modeling and estimate the mass ratio expected for a variety of design conditions. Utilizing these two figures for the scenarios of 100% seated crowd or 100% standing crowd, the designer would determine the expected

reduction in natural frequency due to the additional occupants and their posture. However, this methodology does not attempt to address the expected increase in damping due to the occupants. Both the decrease in natural frequency and the increase in damping will impact the dynamic response of the structure when it is subjected to a crowd-induced excitation.

1.3.5 Occupant model proposed by the Joint Working Group

The first recommendation for incorporating the effects of human-structure interaction was provided by the Joint Working Group in 2008 in a vibration serviceability design guidance entitled “Dynamic Performance Requirements for Permanent Grandstands Subject to Crowd Action”. According to the Joint Working Group, one approach to quantifying the impact of crowd motion is to add crowd models on the empty structure. JWG recommends two models, active and passive crowds; active crowd refers to occupants who cause motion in the structure by applying forces through activities such as jumping and bobbing, and the passive crowd refers to people who do not apply a force on the structure, such as a seated crowd. Both models consist of mass-spring-damper systems and the properties of each model are given as natural and damping ratios, developed through analytical modeling. The recommendation for passive crowd model is 5 Hz and 40%, while that for the active crowd is 2.3 Hz and 25%. The focus of this study is on the passive crowd.

CHAPTER 2: METHODOLOGY

This study was organized into three distinct phases. The first phase focused on assessing the design methodology presented by Sim et al. (2006) through a comparison of the predicted frequency reductions with those experimentally obtained through previous studies. The second phase focused on assessing several occupant models proposed in the field of biomechanics utilizing a comparison of analytically predicted modal parameters with those determined through the previous experimental studies. The third and final phase of the study developed from the results of the second phase. A parametric study of possible occupant model parameters was performed to examine the range of occupant models between the two occupant models identified in the second phase that most accurately represented the experimental data available.

2.1 Phase I: Assessment of Sim et al. Methodology

The primary objective was to compare existing experimental data collected from an occupied laboratory structure with the graphical design guideline proposed by Sim et al (2006) for a passive standing as well as seated crowd. Several combinations of natural frequencies in the range of 4-8 Hz and mass ratios in the range of 5-40% were considered. These frequencies are also in the range of human-sensitivity to vibration (Noss 2012). The numerical comparison was made in terms of Frequency Reduction Factor and the following equation was employed:

$$\text{Frequency Reduction Factor} = \frac{f_o}{f_e} \quad (\text{Eq. 1})$$

Where

f_o = natural frequency of the occupied structure (Hz)

f_e = natural frequency of the empty structure (Hz)

The frequency reduction factor for each structural configuration was determined from the applicable graph provided for the Sim et al. methodology. For example, for the 4.21 Hz configuration, mass ratio of 20%, and seated occupant posture, the frequency reduction factor was read as 0.88 as shown in Figure 8.

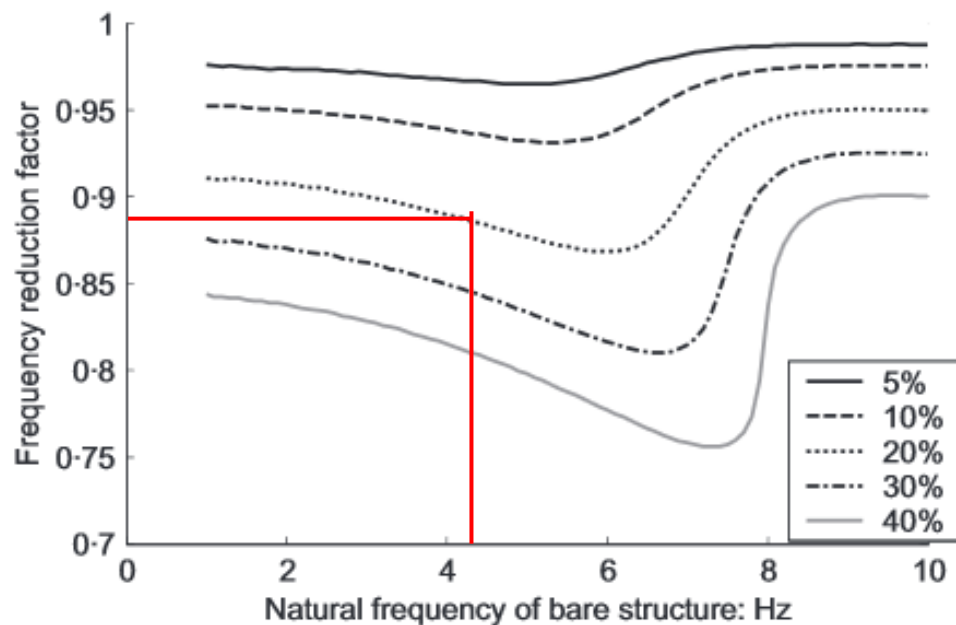


Figure 8. Determining the frequency reduction for 4.21 Hz and 20% mass ratio (from Sim et al. 2006).

This same process was repeated for all structural configurations and for standing as well as seated occupants. The frequency of the occupied structure as predicted by this methodology was computed using Eq. 1 and then quantitatively, in terms of percentage difference, compared with the corresponding experimental parameters.

2.2 Phase II: Assessment of Several Biomechanics Occupant Models

The main purpose of the second phase was to investigate the effects of occupants on the modal properties of the laboratory structure based on analytical models proposed for the field of biomechanics. From the results of Phase I, the conclusion was to focus on seated passive crowds for which the Sim et al. methodology was shown to be less accurate. The analytical models considered are detailed in section 2.2.1.

In order to explore the range of occupant properties and the effects of these on the cantilevered structure, a Dynamic Graphical User Interface (GUI) was developed in MATLAB. The GUI implements modal analyses on the combined human-structure system in the frequency domain to determine the dynamic properties of the occupied structure. This GUI was specifically developed to allow the implementation of modal analysis for several occupant models and to capture the effects of higher order modal responses in determining the dynamic properties of the occupied structure. To this end, the following transfer function, $H(s)$, was developed to mathematically model the combined 2DOF system in Figure 1. A transfer function is a real- or complex- values rational expression that is used to create a mathematical model for the purposes of modal analysis. The function was developed from the following equations of motion that capture the effects of the interaction between the human occupant and the structure.

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f\} \quad (\text{Eq.2})$$

Where $[M]$ is the mass matrix, $[C]$ is the damping matrix, and $[K]$ is the stiffness matrix for the system. $\{ f \}$ refers to the forcing function and $\{ x \}$ refers to the displacement of the system.

The transfer function for 2DOF HSI System is:

$$H(s) = \frac{X(s)}{F(s)} = \frac{1}{[M]s^2 + [C]s + [K]} \quad (\text{Eq.3})$$

Where

$$[M] = \begin{bmatrix} m_s & 0 \\ 0 & m_h \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_h + k_s & -k_h \\ -k_h & k_h \end{bmatrix}$$

$$[C] = \begin{bmatrix} c_h + c_s & -c_h \\ -c_h & c_h \end{bmatrix}$$

m_s = mass of structure (lb)

m_h = mass of human occupants (lb)

k_s = stiffness of structure (lb/ft)

k_h = stiffness of human occupants (lb/ft)

c_s = damping of structure (lbf*sec/ft)

c_h = damping of human occupants (lbf*sec/ft)

s = Laplace variable associated with transfer function

2.2.1 Analytical Occupant Models

For Phase II, a number of existing seated occupant models, summarized in Table 1 (Sachse, Pavic, and Reynolds 2003), were investigated. While there is evidence to show that biomechanical models of human occupants may not apply to civil engineering structures (Noss 2012) because of the difference in expected amplitude magnitude, it is important to mention that the effects of occupants on structures is nonlinear and the current range of occupant properties render the analysis unclear. Further, occupant models vary from multiple-degree-of-freedom (MDOF) to SDOF models. It is highly unlikely that a MDOF model would be utilized in a civil engineering application because the added complexity is not necessary for the aspects of interest in the design process. For this reason only SDOF occupant models are examined in this study.

Table 1. Summary of damped seated SDOF occupant models.

| Model | Frequency (Hz) | Damping Ratio (%) |
|----------------------------|----------------|-------------------|
| Coermann (1962) | 5 | 32 |
| Wei and Griffin (1998) | 4.9 | 53 |
| Zheng and Brownjohn (2001) | 5.24 | 39 |
| Sachse et.al (2002) | 5.9 | 33 |
| Joint Working Group (2008) | 5 | 40 |

The five models were ranked, on the basis of the accuracy of both natural frequency and damping ratio estimates, for a total of eighteen different configurations (combinations of natural frequency of the structure with varying mass ratios) of the test structure detailed in Appendix A. The rankings from each configuration were averaged for each occupant model to determine an overall ranking for natural frequency and damping ratio. The

range so determined served as the basis for the investigation completed in Phase III of this study.

2.2.2 Dynamic Graphical User Interface

Modal analysis for the combined system of each of the SDOF occupant models selected for Phase II with each structural configuration was accomplished using a Dynamic Graphical User Interface or Dynamic GUI. The GUI was developed in Matlab specifically for analytical investigation of the combined 2DOF system. The Matlab code for the GUI is available in Appendix B. It accepts user input in the form of dynamic properties of each of the two DOF, the structure as well as the occupant. A screenshot of the GUI input window is shown in Figure 9. It then computes a frequency response function of the combined system using the transfer function described in Equation 3.

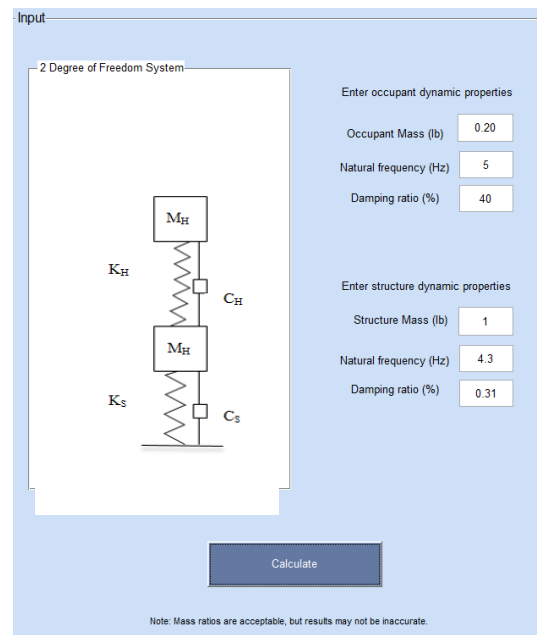


Figure 9. Dynamic GUI input window.

A Matlab function, *invfreqs*, that identifies the numerator and denominator of a transfer function that corresponds to a complex frequency response function was applied to obtain the best-fit model for the original data is applied to the frequency response function generated from the user input to estimate the natural frequency and damping ratio of the 2DOF HSI System. The natural frequencies and damping ratios from the transfer function were identified by the *invfreqs* function. These are associated with two modes, labeled Mode 1 (lower frequency) and Mode 2 (higher frequency) of the dynamic response. It is crucial to select the appropriate pair of values for comparison purposes. This selection, which is dependent on the structural configuration, is discussed in detail in Chapter 4, section 4.2. Both, the analytical FRF (dashed blue) and the curve-fitted results (solid red) are graphically displayed in the output window shown in Figure 10.

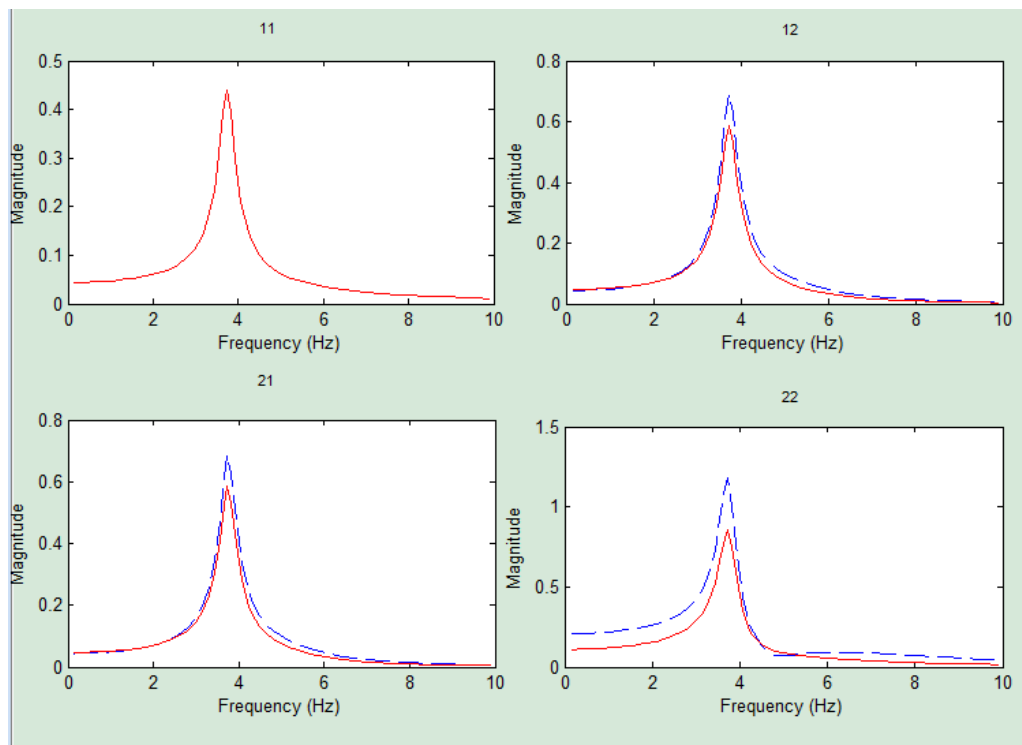


Figure 10. Dynamic GUI output.

As discussed previously, the natural frequencies and damping ratios of the occupied structure were determined by fitting a polynomial to the complex FRFs generated within the GUI for all four combinations of the two degrees of freedoms (11, 12, 21, and 22). As expected, the results from 12 and 21 were identical. The results from 11 and 12 were comparable with differences on the order of 1000th which are negligible for the purposes of this project and future design. Thus, the MATLAB GUI code was modified and only the results from 11 were used in future comparisons.

2.3 Phase III: Parametric modeling to develop a seated occupant model

The primary objective of the third phase was to develop a seated occupant model that is applicable across all configurations of the laboratory test structure. To this end, a range of occupant properties, determined in Phase II, was investigated. All combinations of natural frequencies in 0.1 Hz increments, and damping ratios in 1% increments were explored. This resulted in a total of 56 different occupant models that were combined with each of the 18 structural/mass ratio configurations detailed in Appendix A. This was implemented through the use of a modified version of the GUI code created in MATLAB, and the analytical results were numerically compared with the experimental data to converge to a narrower range of occupant properties. The modified GUI code is available in Appendix C.

CHAPTER 3: RESULTS

This chapter summarizes the results obtained from each of the three phases of the project described in Chapter 2. Section 3.1 presents the results of the numerical comparison of experimental dynamic properties with those predicted by the Sim et al. (2006) design methodology. Results for standing and seated occupants are detailed separately. In section 3.2, results obtained from the comparison of five seated occupant models in section 2.2.1 with the experimental modal properties along with a range of applicable occupant properties for design purposes are presented. Section 3.3 presents the results of a comparison of experimental dynamic properties with several combinations of natural frequency and damping ratio for the occupant model within the range detailed in the previous section 3.2. In addition, a narrower range of seated occupant properties is presented for all configurations of the laboratory test structure.

3.1 Summary of Phase I Results

The experimental natural frequency of the laboratory test structure (Noss 2012) was numerically compared with the frequency after the frequency reduction recommended by Sim et al. (2006) was applied. This was done for two different occupant postures, standing and seated. For the standing occupants, the Sim et al. model was compared with experimental data for occupants standing with straight knees as well as with bent knees. For the seated occupant posture, the Sim et al. model was compared with experimental data for occupants seated straight as well as leaning forward with elbows on the thighs.

Various combinations of six natural frequency configurations of the test structure and several different mass ratios were used in the investigation.

3.1.1 Comparison with Standing Occupant Model

Table 2 summarizes the results for the 4.21 Hz structural configuration. The first column describes the occupant posture, the second details the experimentally determined natural frequency of the occupied structure and the third column lists the frequency reduction factor determined from Figures 7 and 8 for the natural frequency/mass ratio combination. The fourth column shows the frequency computed from the Sim methodology and the last column summarizes the percentage difference between the experimental parameters and those determined by applying the Sim methodology. The percentage differences indicate best fit with the standing with straight knees posture across all mass ratios. A deviation is seen for the mass ratio of 56.4% with error being introduced due to graphical interpolation.

Table 2. Comparison of experimental and Sim et al. frequency for 4.21 Hz configuration.

| Occupant Posture | | Experimentally Determined Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | Percentage Difference |
|------------------|----------------|--|----------------------------|---------------------------|-----------------------|
| Mass Ratio | | 0.167 | | | |
| Standing | Straight Knees | 3.79 | 0.91 | 3.83 | -1.08 |
| | Bent Knees | 3.60 | 0.91 | 3.83 | -6.42 |
| Mass Ratio | | 0.281 | | | |
| Standing | Straight Knees | 3.57 | 0.86 | 3.62 | -1.42 |
| | Bent Knees | 3.78 | 0.86 | 3.62 | 4.22 |
| Mass Ratio | | 0.362 | | | |
| Standing | Straight Knees | 3.50 | 0.83 | 3.47 | 0.76 |
| | Bent Knees | 3.53 | 0.83 | 3.47 | 1.61 |
| Mass Ratio | | 0.437 | | | |
| Standing | Straight Knees | 3.50 | 0.8 | 3.37 | 3.77 |
| | Bent Knees | 3.53 | 0.8 | 3.37 | 4.59 |
| Mass Ratio | | 0.564 | | | |
| Standing | Straight Knees | 3.44 | 0.78 | 3.26 | 5.15 |
| | Bent Knees | 3.33 | 0.78 | 3.26 | 2.02 |

Table 3 summarizes the results for the 4.80 Hz structural configuration, organized in the same manner as Table 2. The analytical results best fit the experimental data obtained from the straight knees posture.

Table 3. Comparison of experimental and Sim et al. frequency for 4.80 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | Percentage Difference |
|------------------|----------------|----------------|----------------------------|---------------------------|-----------------------|
| Mass Ratio | | 0.362 | | | |
| Standing | Straight Knees | 3.97 | 0.8125 | 3.9 | 1.76 |
| | Bent Knees | 5.38 | 0.8125 | 3.9 | 27.51 |

Table 4 summarizes the results for the 5.41 Hz structural configuration, organized in the same manner as Table 2. The percentage differences show a similar trend as that for the 4.21 Hz configuration, thus indicating best fit with the straight knees posture.

Table 4. Comparison of experimental and Sim et al. frequency for 5.41 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | Percentage Difference |
|------------------|----------------|----------------|----------------------------|---------------------------|-----------------------|
| Mass Ratio | | 0.167 | | | |
| Standing | Straight Knees | 4.74 | 0.8875 | 4.8 | -1.29 |
| | Bent Knees | 5.69 | 0.8875 | 4.8 | 15.62 |
| Mass Ratio | | 0.281 | | | |
| Standing | Straight Knees | 4.48 | 0.83 | 4.49 | -0.23 |
| | Bent Knees | 5.86 | 0.83 | 4.49 | 23.37 |
| Mass Ratio | | 0.362 | | | |
| Standing | Straight Knees | 4.46 | 0.8125 | 4.4 | 1.44 |
| | Bent Knees | 5.85 | 0.8125 | 4.4 | 24.86 |
| Mass Ratio | | 0.437 | | | |
| Standing | Straight Knees | 4.51 | 0.78 | 4.22 | 6.43 |
| | Bent Knees | 5.8 | 0.78 | 4.22 | 27.24 |

Table 5 summarizes the results for the 6.27 Hz structural configuration, organized in the same manner as Table 2. The percentage differences show a similar trend as that for the 4.21 Hz and 5.41 Hz configurations, thus indicating best fit with the straight knees posture.

Table 5. Comparison of experimental and Sim et al. frequency for 6.27 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | Percentage Difference |
|------------------|----------------|----------------|----------------------------|---------------------------|-----------------------|
| Mass Ratio | | 0.167 | | | |
| Standing | Straight Knees | 5.55 | 0.89 | 5.58 | -0.55 |
| | Bent Knees | 6.45 | 0.89 | 5.58 | 13.48 |
| Mass Ratio | | 0.281 | | | |
| Standing | Straight Knees | 5.07 | 0.82 | 5.14 | -1.41 |
| | Bent Knees | 6.62 | 0.82 | 5.14 | 22.34 |
| Mass Ratio | | 0.362 | | | |
| Standing | Straight Knees | 4.96 | 0.7875 | 4.94 | 0.45 |
| | Bent Knees | 6.45 | 0.7875 | 4.94 | 23.45 |
| Mass Ratio | | 0.437 | | | |
| Standing | Straight Knees | 5.01 | 0.76 | 4.76 | 4.89 |
| | Bent Knees | 6.55 | 0.76 | 4.76 | 27.25 |
| Mass Ratio | | 0.564 | | | |
| Standing | Straight Knees | 4.92 | 0.7375 | 4.62 | 6.01 |
| | Bent Knees | 6.64 | 0.7375 | 4.62 | 30.36 |

Table 6 summarizes the results for the 7.30 Hz structural configuration, organized in the same manner as Table 2. The percentage differences show a deviation in trend so that the maximum difference in assessing occupant parameters is seen for the straight knees posture rather than the bent knees posture.

Table 6. Comparison of experimental and Sim et al. frequency for 7.30 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | Percentage Difference |
|------------------|----------------|----------------|----------------------------|---------------------------|-----------------------|
| Mass Ratio | | 0.362 | | | |
| Standing | Straight Knees | 7.98 | 0.78 | 5.69 | 28.65 |
| | Bent Knees | 7.42 | 0.78 | 5.69 | 23.26 |
| Mass Ratio | | 0.437 | | | |
| Standing | Straight Knees | 7.76 | 0.74 | 5.4 | 30.39 |
| | Bent Knees | 7.54 | 0.74 | 5.4 | 28.36 |

Table 7 summarizes the results for the 8.05 Hz structural configuration, organized in the same manner as Table 2. The percentage differences show a deviation in trend so that the maximum difference in assessing occupant parameters is seen for the straight knees posture rather than the bent knees posture.

Table 7. Comparison of experimental and Sim et al. frequency for 8.05 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | Percentage Difference |
|------------------|----------------|----------------|----------------------------|---------------------------|-----------------------|
| Mass Ratio | | 0.564 | | | |
| Standing | Straight Knees | 8.28 | 0.725 | 5.84 | 29.51 |
| | Bent Knees | 8.04 | 0.725 | 5.84 | 27.41 |

3.1.2 Comparison with Seated Occupant Model

Table 8 summarizes the results for the 4.21 Hz structural configuration. The first column describes the occupant posture, the second details the experimentally determined natural frequency of the occupied structure and the third column lists the frequency reduction factor determined from Figure 8 for the natural frequency/mass ratio combination. The fourth column shows the frequency computed from the Sim methodology and the last column summarizes the percentage difference between the experimental parameters and those determined by applying the Sim methodology. The percentage differences indicate best fit with experimental data collected from the seated-straight posture.

Table 8. Comparison of experimental and Sim et al. frequency for 4.21 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | % Difference |
|------------------|----------|----------------|----------------------------|---------------------------|--------------|
| Mass Ratio | | 0.2 | | | |
| Seated | Forward | 3.99 | 0.87 | 3.66 | 8.20 |
| | Straight | 3.92 | 0.87 | 3.66 | 6.56 |
| Mass Ratio | | 0.42 | | | |
| Seated | Forward | 3.96 | 0.78 | 3.28 | 17.08 |
| | Straight | 3.61 | 0.78 | 3.28 | 9.04 |
| Mass Ratio | | 0.53 | | | |
| Seated | Forward | 3.83 | 0.75 | 3.16 | 17.56 |
| | Straight | 3.68 | 0.75 | 3.16 | 14.20 |

Table 9 summarizes the results for the 5.41 Hz structural configuration, organized in the same manner as Table 8. The percentage differences indicate best fit with experimental data collected from the seated-straight posture.

Table 9. Comparison of experimental and Sim et al. frequency for 5.41 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | Percentage Difference |
|------------------|----------|----------------|----------------------------|---------------------------|-----------------------|
| Mass Ratio | | 0.21 | | | |
| Seated | Forward | 5.06 | 0.86 | 4.65 | 8.05 |
| | Straight | 5.45 | 0.86 | 4.65 | 14.63 |
| Mass Ratio | | 0.31 | | | |
| Seated | Forward | 5.16 | 0.78 | 4.22 | 18.22 |
| | Straight | 5.29 | 0.78 | 4.22 | 20.23 |
| Mass Ratio | | 0.46 | | | |
| Seated | Forward | 5.08 | 0.73 | 3.95 | 22.26 |
| | Straight | 5.24 | 0.73 | 3.95 | 24.63 |
| Mass Ratio | | 0.60 | | | |
| Seated | Forward | 4.88 | 0.70 | 3.79 | 22.4 |
| | Straight | 5.06 | 0.70 | 3.79 | 25.16 |

Table 10 summarizes the results for the 6.27 Hz structural configuration, organized in the same manner as Table 8. The percentage differences indicate best fit with experimental data collected from the seated-straight posture.

Table 10. Comparison of experimental and Sim et al. frequency for 6.27 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | % Difference |
|------------------|----------|----------------|----------------------------|---------------------------|--------------|
| Mass Ratio | | 0.21 | | | |
| Seated | Forward | 6.07 | 0.94 | 5.89 | 2.90 |
| | Straight | 6.76 | 0.94 | 5.89 | 12.81 |
| Mass Ratio | | 0.34 | | | |
| Seated | Forward | 6.15 | 0.87 | 5.45 | 11.30 |
| | Straight | 6.77 | 0.87 | 5.45 | 19.43 |
| Mass Ratio | | 0.48 | | | |
| Seated | Forward | 6.01 | 0.72 | 4.51 | 24.89 |
| | Straight | 6.88 | 0.72 | 4.51 | 34.38 |
| Mass Ratio | | 0.63 | | | |
| Seated | Forward | 5.61 | 0.68 | 4.26 | 24 |
| | Straight | 6.87 | 0.68 | 4.26 | 37.94 |

Table 11 summarizes the results for the 7.30 Hz structural configuration, organized in the same manner as Table 2. The percentage differences indicate a deviation with the previously described trend. The best fit is with the seated-forward posture.

Table 11. Comparison of experimental and Sim et al. frequency for 7.30 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | % Difference |
|------------------|----------|----------------|----------------------------|---------------------------|--------------|
| Mass Ratio | | 0.19 | | | |
| Seated | Forward | 7.31 | 0.97 | 7.08 | 3.13 |
| | Straight | 7.53 | 0.97 | 7.08 | 5.96 |
| Mass Ratio | | 0.34 | | | |
| Seated | Forward | 7.29 | 0.93 | 6.78 | 6.87 |
| | Straight | 7.55 | 0.93 | 6.78 | 10.08 |
| Mass Ratio | | 0.52 | | | |
| Seated | Forward | 7.41 | 0.74 | 5.40 | 27.10 |
| | Straight | 7.97 | 0.74 | 5.40 | 32.22 |

Table 12 summarizes the results for the 8.05 Hz structural configuration, organized in the same manner as Table 8. The percentage differences indicate best fit with experimental data collected from the seated-straight posture.

Table 12. Comparison of experimental and Sim et al. frequency for 8.05 Hz configuration.

| Occupant Posture | | Frequency (Hz) | Frequency Reduction Factor | Sim et al. Frequency (Hz) | % Difference |
|------------------|----------|----------------|----------------------------|---------------------------|--------------|
| Mass Ratio | | 0.24 | | | |
| Seated | Forward | 8.27 | 0.95 | 7.65 | 7.53 |
| | Straight | 8.05 | 0.95 | 7.65 | 5.00 |
| Mass Ratio | | 0.36 | | | |
| Seated | Forward | 8.38 | 0.925 | 7.45 | 11.14 |
| | Straight | 8.06 | 0.925 | 7.45 | 7.62 |
| Mass Ratio | | 0.41 | | | |
| Seated | Forward | 8.27 | 0.92 | 7.41 | 10.45 |
| | Straight | 8.05 | 0.92 | 7.41 | 8.00 |
| Mass Ratio | | 0.56 | | | |
| Seated | Forward | 8.78 | 0.86 | 6.92 | 21.15 |
| | Straight | 8.11 | 0.86 | 6.92 | 14.64 |

3.2 Summary of Phase II Results

A numerical comparison of the results from the comparison of the five seated occupant models from the field of biomechanics with the experimental data, in terms of percentage difference, across all structural configurations was completed. The results were compared only with the experimental data for the seated-straight posture since the models best approximated dynamic properties determined from this occupant posture. The models were then ranked based on average percentage difference- for both frequency and damping estimates. The overall rank of each occupant model, for both natural frequency

and damping ratio, is summarized in Table 13. The lowest number corresponds to the highest rank (best fit with experimental data) and the highest corresponds to the lowest rank (worst fit with experimental data). It is seen that the Zheng and Brownjohn (2001) model (5.24 Hz; 39%) was the best fit across all configurations for the natural frequency, while the Coermann (1962) model (5 Hz; 32%) was the best fit for damping ratio. However, since the natural frequency ranking is more reliable than the damping ratio rank, 32% was not included in the range. Thus, natural frequencies in the range of 5.24 Hz to 5.9 Hz based on the second best model for frequency from Sachse et al (2002), and damping ratios in the range of 33% to 39% were selected for the purposes of parametric modeling in Phase III.

Table 13. Rankings of seated occupant models based on average percentage difference.

| Model | Frequency (Hz) | Damping (%) | Frequency Rank (% difference) | Damping Ratio Rank (% difference) |
|----------------------------|----------------|-------------|-------------------------------|-----------------------------------|
| Coermann (1962) | 5 | 32 | 13.2 | 47.5 |
| Wei and Griffin (1998) | 4.9 | 53 | 14.2 | 91.9 |
| Zheng and Brownjohn (2001) | 5.24 | 39 | 11.6 | 74.3 |
| Sachse et al. (2002) | 5.9 | 33 | 11.7 | 86.8 |
| Joint Working Group (2008) | 5 | 40 | 12.4 | 74.9 |

The quantitative comparison of each configuration is available in Appendix D.

3.3 Summary of Phase III Results

A parametric study in a narrow range, natural frequency in the range of 5.2 Hz to 5.9 Hz and damping ratio in the range of 33% to 39%, of occupant properties was completed.

The results, for both natural frequency and damping ratio, are graphically represented in Figures 11 and 12, respectively.

Figure 11 shows the natural frequency and damping ratio of the occupant models along the horizontal axes, while the percentage difference in approximating the experimental data from the seated straight occupant posture, for all eighteen structural configurations, is represented along the vertical axis. A similar layout is used for Figure 12 which shows the percentage difference in estimating the experimental damping ratio.

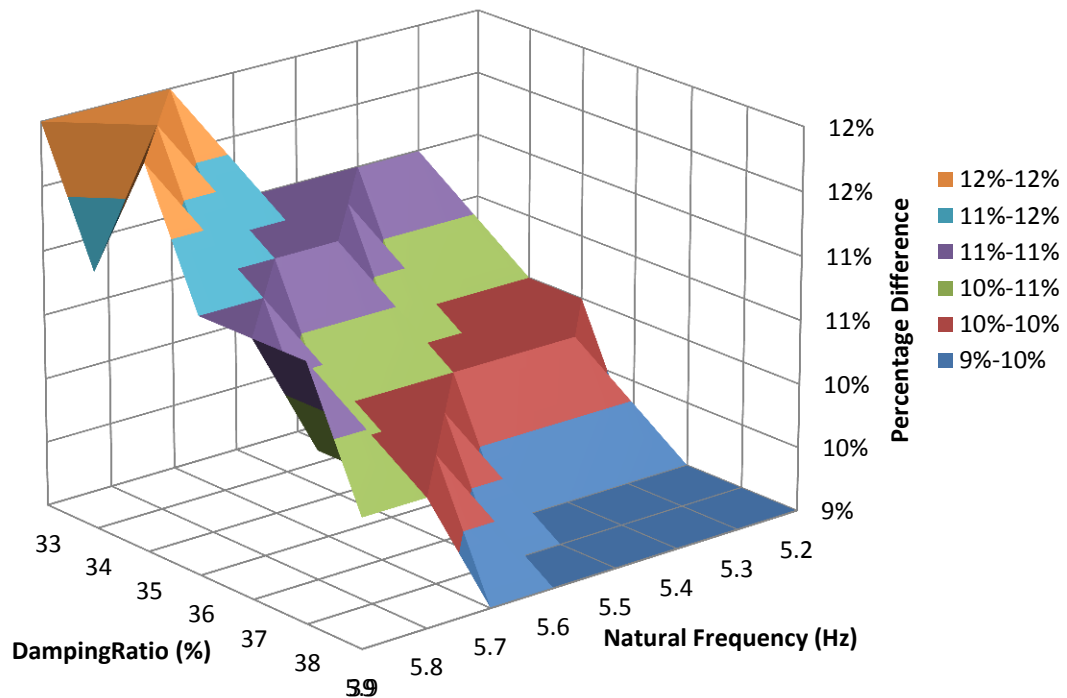


Figure 11. Percentage difference for natural frequency for each parametric model.

The numerical values used to develop Figure 11 are summarized in Table 14. The color scheme in the table shows that the red tones refer to lower percentage difference while

the green tones refer to higher percentage difference. Thus, occupant parameters of 5.2-5.4 Hz and 39% damping ratio are the best fit for natural frequency.

Table 14. Summary of percentage difference for natural frequency.

| Model | 5.2 Hz | 5.3 Hz | 5.4 Hz | 5.5 Hz | 5.6 Hz | 5.7 Hz | 5.8 Hz | 5.9 Hz |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| 33% | 10% | 11% | 11% | 11% | 11% | 12% | 12% | 12% |
| 34% | 10% | 10% | 10% | 11% | 11% | 11% | 12% | 11% |
| 35% | 10% | 10% | 10% | 10% | 11% | 11% | 11% | 12% |
| 36% | 9% | 10% | 10% | 10% | 10% | 10% | 11% | 11% |
| 37% | 9% | 9% | 9% | 9% | 10% | 10% | 10% | 11% |
| 38% | 9% | 9% | 9% | 9% | 9% | 10% | 10% | 11% |
| 39% | 8% | 8% | 8% | 9% | 9% | 9% | 10% | 10% |

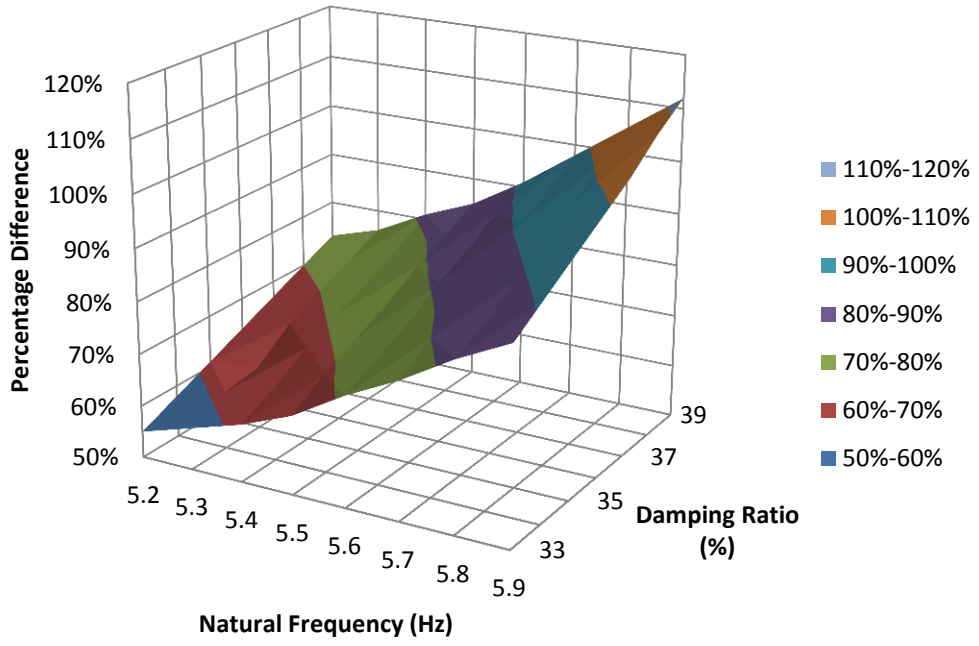


Figure 12. Percentage difference for damping ratio for each parametric model.

The numerical values used to develop Figure 12 are summarized in Table 15. The color scheme in the table shows that the red tones refer to lower percentage difference while the green tones refer to higher percentage difference. Thus, occupant parameters of 5.2-5.3 Hz and 33-34% damping ratio are the best fit for damping ratio.

Table 15. Summary of percentage difference for damping ratio.

| Model | 5.2 Hz | 5.3 Hz | 5.4 Hz | 5.5 Hz | 5.6 Hz | 5.7 Hz | 5.8 Hz | 5.9 Hz |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| 33% | 55% | 58% | 61% | 65% | 71% | 76% | 82% | 87% |
| 34% | 58% | 61% | 64% | 69% | 74% | 80% | 85% | 91% |
| 35% | 61% | 62% | 68% | 72% | 77% | 83% | 89% | 95% |
| 36% | 64% | 67% | 71% | 75% | 81% | 87% | 93% | 99% |
| 37% | 67% | 70% | 74% | 79% | 84% | 91% | 97% | 103% |
| 38% | 70% | 73% | 77% | 82% | 88% | 94% | 101% | 108% |
| 39% | 73% | 76% | 81% | 85% | 91% | 98% | 105% | 112% |

CHAPTER 4: DISCUSSION

This chapter provides a discussion of the results presented in Chapter 3. Section 4.1 presents a discussion of the results from quantitative comparison of the Sim et al. model with experimental data and its appropriateness for estimating the effects of human-structure interaction. Section 4.2 presents a discussion of the additional biomechanics models that were investigated based on conclusions from the first phase of the study. A brief discussion of the acceptable range of variability in occupant properties for design purposes is also presented. Section 4.3 presents a discussion of parametric modeling of HSI within a narrow range of occupant properties previously determined and recommends separate occupant parameters that best fit natural frequency and damping ratio of the occupied structure.

4.1 Assessment of Sim et al. Design Methodology

The design methodology recommended by Sim et al. (2006) was applied and relevant parameters computed for each of the natural frequency and mass ratio configurations. The results were quantitatively compared with the experimental data previously collected from the laboratory test structure. An interpretation of the quantitative results and the conclusions drawn from the same are discussed.

4.1.1 Assessment of Methodology Application for Standing Occupants

The standing occupant model most closely matches the experimental results for occupants standing with straight knees. For lower frequency structural configurations of 4.3 Hz, 5.7 Hz, and 6.8 Hz (for empty structure with bench), the Sim et al. model is appropriate for mass ratios under 45%, with differences in the range of 0-6%. For higher mass ratios, the values were extrapolated since the design guide was limited to 40% mass ratio. This resulted in much greater error. For natural frequency configurations of 7.5 Hz and 8.1 Hz, the frequencies estimated from Sim's design methodology differ from the corresponding experimental values by more than 25% which makes the Sim et al. methodology inappropriate for higher frequency configurations of the test structure. This limit was based on recommendations provided by the Joint Working Group (2008).

4.1.2 Assessment of Methodology Application for Seated Occupants

A comparison of the results from Sim's methodology using the seated occupant model with the corresponding experimental data showed large percentage differences when compared with experimental data for both seated straight and seated leaning forward occupants. For occupants seated straight, the average percentage difference varied in the range of 5-37%, while for occupants seated leaning forward, the percentage difference varied in the range of 3-27% with larger differences for higher mass ratios. Due to the large variation in error across the structural configurations, the Sim et al. methodology is concluded to be less accurate and therefore less appropriate for estimating the effects of human-structure interaction of seated occupants. Thus, the focus of Phase II and III of

this study was selected to evaluate additional seated crowd models and determine more accurate occupant parameters for the same structure. The results could potentially be used in the future to refine Sim's methodology for more accurate estimation of HSI for seated occupants.

4.2 Assessment of Additional Seated Crowd Models from Biomechanics

The results from analytical modal analysis simulating the interaction between the structure and each of five seated occupant models were compared with results from experimental modal analysis of the occupied structure. A numerical comparison was completed for occupants seated straight as well as seated leaning forward. For each configuration, a natural frequency and damping ratio pair was selected for quantitative comparison from the GUI output. As mentioned previously, the GUI outputs dynamic properties for two modes of the combined 2DOF system. For 4.3 Hz and 5.7 Hz, the lower frequency mode (Mode 1) was selected and for 6.8 Hz, 7.5 Hz, and 8.1 Hz, the higher frequency mode (Mode 2) was selected. This selection was based on a graphical comparison of FRFs in which the Mode 1 peak was "visible" only for significantly lower occupant damping in the range of 5-10%. This is also because the response mode associated with a lower damping ratio is more visually apparent than the mode with a higher damping ratio. For these three configurations, the natural frequency of the empty structure was higher than that of the occupant model due to which Mode 1 was not clearly observed for higher levels of occupant damping. The same was not seen for the first three natural frequency configurations because the natural frequency of the test

structure was either lower than or fairly close to that of the occupant model. Thus, in the experimental data only the higher mode parameters were seen which is also confirmed by the analytical FRFs generated.

The results show that the models match the seated-straight posture most accurately for natural frequencies of 4.3 Hz, 5.7 Hz, 6.8 Hz, and 7.5 Hz, while the seated leaning-forward posture was more accurate for the 8.1 Hz configuration. For consistency in Phase III of the project, the parametric modeling results were only compared with data from seated-straight occupant posture. Table 16 displays this information for the 4.3 Hz and 20% mass ratio, where f_n refers to natural frequency and ξ refers to damping ratio. Columns highlighted in blue show the numbers that were compared. Data for all other configurations is provided in Appendix B.

Table 16. Summary of comparison for 4.3 Hz and 20% configuration.

| | | | | | | | Leaning forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 3.99 | 1.99 | 3.92 | 2.42 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 3.906 | 4.55 | 5.801 | 0.304 | 2.1 | 128.6 | 0.3 | 88.0 |
| Wei and Griffin | 4.9 | 53 | 3.800 | 5.06 | 5.541 | 0.53 | 4.7 | 154.3 | 3.0 | 109.1 |
| Zheng and Brownjohn | 5.24 | 39 | 3.754 | 4.13 | 6.002 | 0.385 | 5.9 | 107.5 | 4.2 | 70.7 |
| Sachse et al | 5.9 | 33 | 3.771 | 2.6 | 6.728 | 0.335 | 5.5 | 30.7 | 3.8 | 7.4 |
| JWG | 5 | 40 | 3.746 | 4.86 | 5.739 | 0.389 | 6.1 | 144.2 | 4.4 | 100.8 |

4.3 Parametric Modeling

The parametric study was completed in the seated occupant property range of 5.2-5.9 Hz and 33-39%. The average percentage difference for each natural frequency with the range of damping ratios showed that with increasing damping ratio, the error in approximating the experimental damping also increased while that in approximating the experimental natural frequency decreased. Thus, natural frequency and damping ratio estimates were inversely related in terms of accuracy. The parametric modeling proves that the occupant parameters for accurate natural frequency are determined more accurately and with smaller variation than for damping ratio. The results show that occupant parameters of 5.2-5.4 Hz and 39% damping ratio are the best fit for natural frequency, while occupant parameters of 5.2-5.3 Hz and 33-34% damping ratio are the best fit for damping ratio. Thus, a frequency range of 5.2-5.4 Hz can be recommended, but a definite damping ratio range cannot be. Further investigation is needed to converge the two ranges to a single seated occupant model.

For the purposes of design, it is more critical to accurately predict the natural frequency of the occupied structure than predicting the damping ratio. One reason for this difference in the emphasis on the two parameters is that the latter is more difficult to measure and is more variable. Another reason is that the behavior of the structure can be very closely estimated with an accurate natural frequency.

CHAPTER 5: CONCLUSIONS

5.1 Summary and Limitations

The effects of human-structure interaction on the dynamic properties of the occupied structure are evident in the results of this study. The dependency of the effect on the posture of the human occupant is also validated. The results indicate that the Sim et al. methodology, while appropriate for the standing-straight occupant posture, is not applicable to the seated occupant experimental data. It should be noted that this conclusion is limited to the experimental data collected from the small-scale laboratory structure built for the study. Further, the Sim et al. methodology is limited to mass ratios in the range of 5-40% which limit the scope of the study and inform the greater errors associated with higher mass ratio configurations. In addition, the results of this study are based on five natural frequency configurations of test structure and thus limit the applicability of the results to structures with natural frequencies in the same range and with similar configuration.

This study determined a range of occupant parameters through investigation of five seated occupant models which must be kept in mind when interpreting the results from the parametric modeling. It is further concluded that determining an occupant model to accurately predict the damping ratio of the occupied structure is relatively difficult. The results seem to indicate that the frequency of the occupant model should be in the 5.2-5.4 Hz range. However, the results do not definitively suggest a damping ratio for this model. Accurate experimental measurements analytical modeling is necessary to bridge the large percentage differences in predicting the damping ratio.

5.2 Future Research Work

The results of this study are limited as discussed in Section 4.1. Thus, future work related to this study is necessary. Two major areas of work may be considered: experimental data collection and analytical modal analysis to develop occupant models. The accurate measurement of damping ratio in the experimental procedure is important in order to reduce the difference from analytical modeling. The scope of future research work also includes refinement of the occupant parameters suggested in this study, for both natural frequency and damping ratio. This may be based on the investigation of more than five seated occupant models that were used in this study, as well as extending the parametric modeling to include occupant frequencies less than 5.2 Hz.

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APPENDICES

Appendix A: Summary of Structural Configurations

| Empty Frequency (Hz) | Empty Damping (%) | Mass Ratio |
|----------------------|-------------------|------------|
| 4.3 | 0.310 | 0.20 |
| 4.3 | 0.310 | 0.42 |
| 4.3 | 0.310 | 0.53 |
| 5.7 | 0.403 | 0.21 |
| 5.7 | 0.403 | 0.31 |
| 5.7 | 0.403 | 0.46 |
| 5.7 | 0.403 | 0.60 |
| 6.8 | 0.511 | 0.21 |
| 6.8 | 0.511 | 0.34 |
| 6.8 | 0.511 | 0.48 |
| 6.8 | 0.511 | 0.63 |
| 7.5 | 0.613 | 0.19 |
| 7.5 | 0.613 | 0.34 |
| 7.5 | 0.613 | 0.52 |
| 8.1 | 1.180 | 0.24 |
| 8.1 | 1.180 | 0.36 |
| 8.1 | 1.180 | 0.41 |
| 8.1 | 1.180 | 0.56 |

Appendix B: Dynamic Graphical User Interface Code


```

% Name: Dynamic GUI Code
% Author: Aradhana Agarwal
% DYNAMIC_GUI MATLAB code for Dynamic_GUI.fig
% DYNAMIC_GUI, by itself, creates a new DYNAMIC_GUI or raises the
% existing singleton*.
%
% H = DYNAMIC_GUI returns the handle to a new DYNAMIC_GUI or the handle
% to the existing singleton*.
%
% DYNAMIC_GUI('CALLBACK',hObject,eventData,handles,...) calls the
% local function named CALLBACK in DYNAMIC_GUI.M with the given
% input arguments.
% DYNAMIC_GUI('Property','Value',...) creates a new DYNAMIC_GUI or
% raises the existing singleton*. Starting from the left, property
% value pairs are applied to the GUI before Dynamic_GUI_OpeningFcn gets
% called. An unrecognized property name or invalid value makes
% property application stop. All inputs are passed to
% Dynamic_GUI_OpeningFcn via varargin.
%
% *See GUI Options on GUIDE's Tools menu. Choose "GUI allows only one
% instance to run (singleton)".
%
% See also: GUIDE, GUIDATA, GUIHANDLES

% Edit the above text to modify the response to help Dynamic_GUI

% Last Modified by GUIDE v2.5 24-Mar-2015 16:03:23

% Begin initialization code - DO NOT EDIT

function varargout = Dynamic_GUI(varargin)
gui_Singleton = 1;
gui_State = struct('gui_Name',       mfilename, ...
                  'gui_Singleton',   gui_Singleton, ...
                  'gui_OpeningFcn', @Dynamic_GUI_OpeningFcn, ...
                  'gui_OutputFcn',  @Dynamic_GUI_OutputFcn, ...
                  'gui_LayoutFcn',   [] , ...
                  'gui_Callback',    []);
if nargin && ischar(varargin{1})
    gui_State.gui_Callback = str2func(varargin{1});
end

if nargout
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
    gui_mainfcn(gui_State, varargin{:});
end
% End initialization code - DO NOT EDIT

% --- Executes just before Dynamic_GUI is made visible.
function Dynamic_GUI_OpeningFcn(hObject, eventdata, handles, varargin)
% This function has no output args, see OutputFcn.
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB

```

```

% handles      structure with handles and user data (see GUIDATA)
% varargin     command line arguments to Dynamic_GUI (see VARARGIN)

% Choose default command line output for Dynamic_GUI
handles.output = hObject;

% Update handles structure
guidata(hObject, handles);

% UIWAIT makes Dynamic_GUI wait for user response (see UIRESUME)
% uiwait(handles.figure1);

% --- Outputs from this function are returned to the command line.
function varargout = Dynamic_GUI_OutputFcn(hObject, eventdata, handles)
% varargout    cell array for returning output args (see VARARGOUT);
% hObject     handle to figure
% eventdata   reserved - to be defined in a future version of MATLAB
% handles     structure with handles and user data (see GUIDATA)

% Get default command line output from handles structure
varargout{1} = handles.output;

function StructureMass_Callback(hObject, eventdata, handles)
% hObject     handle to StructureMass (see GCBO)
% eventdata   reserved - to be defined in a future version of MATLAB
% handles     structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of StructureMass as
text
%           str2double(get(hObject,'String')) returns contents of
StructureMass as a double
struct_mass = str2double(get(hObject,'string'));
if isnan(struct_mass)
    errordlg('You must enter a numeric value','Invalid Input','modal')
    uicontrol(hObject)
    return
else
    struct_mass=get(handles.StructureMass,'String');
    assignin('base','struct_mass',str2double(struct_mass));
    handles.SM=struct_mass;
    guidata(hObject,handles);
end

% --- Executes during object creation, after setting all properties.
function StructureMass_CreateFcn(hObject, eventdata, handles)
% hObject     handle to StructureMass (see GCBO)
% eventdata   reserved - to be defined in a future version of MATLAB
% handles     empty - handles not created until after all CreateFcns
called

% Hint: edit controls usually have a white background on Windows.
%           See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
get(0,'defaultUicontrolBackgroundColor'))

```

```

        set(hObject, 'BackgroundColor', 'white');
    end

function StructureFrequency_Callback(hObject, eventdata, handles)
% hObject    handle to StructureFrequency (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject, 'String') returns contents of StructureFrequency
as text
%         str2double(get(hObject, 'String')) returns contents of
StructureFrequency as a double
struct_freq = str2double(get(hObject, 'string'));
if isnan(struct_freq)
    errordlg('You must enter a numeric value', 'Invalid Input', 'modal')
    uicontrol(hObject)
    return
else
    struct_freq=get(handles.StructureFrequency, 'String');
    assignin('base', 'struct_freq', str2double(struct_freq));
    handles.SF=struct_freq;
    guidata(hObject, handles);
end

% --- Executes during object creation, after setting all properties.
function StructureFrequency_CreateFcn(hObject, eventdata, handles)
% hObject    handle to StructureFrequency (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns
called

% Hint: edit controls usually have a white background on Windows.
%         See ISPC and COMPUTER.
if ispc && isequal(get(hObject, 'BackgroundColor'),
get(0, 'defaultUicontrolBackgroundColor'))
    set(hObject, 'BackgroundColor', 'white');
end

function StructureDamping_Callback(hObject, eventdata, handles)
% hObject    handle to StructureDamping (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject, 'String') returns contents of StructureDamping as
text
%         str2double(get(hObject, 'String')) returns contents of
StructureDamping as a double
struct_damp = str2double(get(hObject, 'string'));
if isnan(struct_damp)
    errordlg('You must enter a numeric value', 'Invalid Input', 'modal')
    uicontrol(hObject)
    return
end

```

```

else
    struct_damp=get(handles.StructureDamping,'String');
    assignin('base','struct_damp',str2double(struct_damp));
    handles.SD=struct_damp;
    guidata(hObject,handles);
end

% --- Executes during object creation, after setting all properties.
function StructureDamping_CreateFcn(hObject, eventdata, handles)
% hObject    handle to StructureDamping (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns
called

% Hint: edit controls usually have a white background on Windows.
%         See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
get(0,'defaultUiControlBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function OccupantMass_Callback(hObject, eventdata, handles)
% hObject    handle to OccupantMass (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of OccupantMass as text
%         str2double(get(hObject,'String')) returns contents of
OccupantMass as a double
occupant_mass = str2double(get(hObject,'string'));
if isnan(occupant_mass)
    errordlg('You must enter a numeric value','Invalid Input','modal')
    uicontrol(hObject)
    return
else
    occupant_mass=get(handles.OccupantMass,'String');
    assignin('base','occupant_mass',str2double(occupant_mass));
    handles.OM=occupant_mass;
    guidata(hObject,handles);
end

% --- Executes during object creation, after setting all properties.
function OccupantMass_CreateFcn(hObject, eventdata, handles)
% hObject    handle to OccupantMass (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns
called

% Hint: edit controls usually have a white background on Windows.
%         See ISPC and COMPUTER.

```

```

if ispc && isequal(get(hObject,'BackgroundColor'),
get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function OccupantFrequency_Callback(hObject, eventdata, handles)
% hObject    handle to OccupantFrequency (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of OccupantFrequency as
text
%          str2double(get(hObject,'String')) returns contents of
OccupantFrequency as a double
occupant_freq = str2double(get(hObject,'string'));
if isnan(occupant_freq)
    errordlg('You must enter a numeric value','Invalid Input','modal')
    uicontrol(hObject)
    return
else
    occupant_freq=get(handles.OccupantFrequency,'String');
    assignin('base','occupant_freq',str2double(occupant_freq));
    handles.OF=occupant_freq;
    guidata(hObject,handles);
end

% --- Executes during object creation, after setting all properties.
function OccupantFrequency_CreateFcn(hObject, eventdata, handles)
% hObject    handle to OccupantFrequency (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns
called

% Hint: edit controls usually have a white background on Windows.
%          See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function OccupantDamping_Callback(hObject, eventdata, handles)
% hObject    handle to OccupantDamping (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of OccupantDamping as
text
%          str2double(get(hObject,'String')) returns contents of
OccupantDamping as a double
occupant_damp = str2double(get(hObject,'string'));
if isnan(occupant_damp)
    errordlg('You must enter a numeric value','Invalid Input','modal')

```

```

    uicontrol(hObject)
    return
else
    occupant_damp=get(handles.OccupantDamping,'String');
    assignin('base','occupant_damp',str2double(occupant_damp));
    handles.OD=occupant_damp;
    guidata(hObject,handles);
end

% --- Executes during object creation, after setting all properties.
function OccupantDamping_CreateFcn(hObject, eventdata, handles)
% hObject    handle to OccupantDamping (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns
called

% Hint: edit controls usually have a white background on Windows.
%         See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

% --- Executes on button press in Calculate.
function Calculate_Callback(hObject, eventdata, handles)
% hObject    handle to Calculate (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

%Retrieving structure properties from handles structure
struct_mass=handles.SM;
struct_freq=handles.SF;
struct_damp=handles.SD;

%Retrieving occupant properties from handles structure
occupant_mass=handles.OM;
occupant_freq=handles.OF;
occupant_damp=handles.OD;

H=transfer(struct_mass,occupant_mass,struct_freq,occupant_freq,struct_d
amp,occupant_damp);
%ltiview({'bodemag'},H)
[mag,phase,w]=bode(H);
%Extracting magnitude data
mag11=reshape(mag(1,1,:),[],1);
mag12=reshape(mag(1,2,:),[],1);
mag21=reshape(mag(2,1,:),[],1);
mag22=reshape(mag(2,2,:),[],1);

%Extracting phase data
ph11=reshape(phase(1,1,:),[],1);
ph12=reshape(phase(1,2,:),[],1);
ph21=reshape(phase(2,1,:),[],1);

```

```

ph22=reshape(phase(2,2,:), [], 1);

%%Converting from circular (rad/sec)to natural frequency(Hz)
freq=w./(2*pi);

for i=1:1:size(freq)
    if freq(i,1)<10
        n=i;
    end
end

%%Creating imaginary and real portions for curve-fitting
%For 11
for i=1:1:size(mag11)
    imag_11(i,1)= mag11(i,1)*sind(ph11(i,1));
    real_11(i,1)=mag11(i,1)*cosd(ph11(i,1));
end

%For 12
for i=1:1:size(mag12)
    imag_12(i,1)= mag12(i,1)*sind(ph12(i,1));
    real_12(i,1)=mag11(i,1)*cosd(ph12(i,1));
end

%For 21
for i=1:1:size(mag21)
    imag_21(i,1)= mag21(i,1)*sind(ph21(i,1));
    real_21(i,1)=mag11(i,1)*cosd(ph21(i,1));
end

%For 22
for i=1:1:size(mag22)
    imag_22(i,1)= mag22(i,1)*sind(ph22(i,1));
    real_22(i,1)=mag11(i,1)*cosd(ph22(i,1));
end

%%Creating complex FRFs
%For 11
c11=complex(real_11,imag_11);
[B,A] = invfreqs(c11,w,2,4, [], 100, 0.0001);
h11=freqs(B,A,w);
hmag11=abs(h11);
[Residues,Poles,K] = residue(B,A);
Nat_Frequency = abs(Poles)*max(w)/(2*pi)
Damp_ratio = -real(Poles)./(abs(Poles))

%For 12
c12=complex(real_12,imag_12);
[B,A] = invfreqs(c12,w,2,4, [], 100, 0.0001);
h12=freqs(B,A,w);
hmag12=abs(h12);

%For 21
c21=complex(real_21,imag_21);

```

```

[B,A] = invfreqs(c21,w,2,4, [], 100, 0.0001);
h21=freqs(B,A,w);
hmag21=abs(h21);

%For 22
c22=complex(real_22,imag_22);
[B,A] = invfreqs(c22,w,2,4, [], 100, 0.0001);
h22=freqs(B,A,w);
hmag22=abs(h22);

%%Plotting all four responses
axes(handles.FrequencyDom);
plot(freq(1:n,1),mag11(1:n,1), 'b--',freq(1:n,1),hmag11(1:n,1), 'r')
xlabel('Frequency (Hz)')
ylabel('Magnitude')

axes(handles.axes6);
plot(freq(1:n,1),mag12(1:n,1), 'b--',freq(1:n,1),hmag12(1:n,1), 'r')
xlabel('Frequency (Hz)')
ylabel('Magnitude')

axes(handles.axes7);
plot(freq(1:n,1),mag21(1:n,1), 'b--',freq(1:n,1),hmag21(1:n,1), 'r')
xlabel('Frequency (Hz)')
ylabel('Magnitude')

axes(handles.axes8);
plot(freq(1:n,1),mag22(1:n,1), 'b--',freq(1:n,1),hmag22(1:n,1), 'r')
xlabel('Frequency (Hz)')
ylabel('Magnitude')

```


Appendix C: Matlab Script for Parametric Modeling

```

>Name: Parametric Model
Author: Aradhana Agarwal
Generates several occupant models within a range of properties and
outputs
dynamic properties of the occupied structure for several natural
frequency-mass ratio
configurations.

occu_prop=[5.2 33;5.2 34;5.2 35;5.2 36;5.2 37;5.2 38;5.2 39;
           5.3 33;5.3 34;5.3 35;5.3 36;5.3 37;5.3 38;5.3 39;
           5.4 33;5.4 34;5.4 35;5.4 36;5.4 37;5.4 38;5.4 39;
           5.5 33;5.5 34;5.5 35;5.5 36;5.5 37;5.5 38;5.5 39;
           5.6 33;5.6 34;5.6 35;5.6 36;5.6 37;5.6 38;5.6 39;
           5.7 33;5.7 34;5.7 35;5.7 36;5.7 37;5.7 38;5.7 39;
           5.8 33;5.8 34;5.8 35;5.8 36;5.8 37;5.8 38;5.8 39;
           5.9 33;5.9 34;5.9 35;5.9 36;5.9 37;5.9 38;5.9 39;];

config=[1 4.3 0.310 0.20;1 4.3 0.310 0.42;1 4.3 0.310 0.53;
        1 5.7 0.403 0.21;1 5.7 0.403 0.31;1 5.7 0.403 0.46;1 5.7 0.403
0.60;
        1 6.8 0.511 0.21;1 6.8 0.511 0.34;1 6.8 0.511 0.48;1 6.8 0.511
0.63;
        1 7.5 0.613 0.19;1 7.5 0.613 0.34;1 7.5 0.613 0.52;
        1 8.1 1.180 0.24;1 8.1 1.180 0.36;1 8.1 1.180 0.41;1 8.1 1.180
0.56;];

for i=1:18
    struct_mass=config(i,1);
    struct_freq=config(i,2);
    struct_damp=config(i,3);
    occu_mass=config(i,4);
    for j=1:56
        occu_freq=occu_prop(j,1);
        occu_damp=occu_prop(j,2);
        f=0.031089502; %to convert from pounds to slugs
        ms=struct_mass*f;
        mh=occu_mass*f;

        ks=4*pi*pi*(struct_freq)^2*ms;
        kh=4*pi*pi*(occu_freq)^2*mh;

        cs=2*(struct_damp)/100*sqrt(ks*ms);
        ch=2*(occu_damp)/100*sqrt(kh*mh);

        M=[ms 0;0 mh];
        C=[cs+ch -ch;-ch ch];
        K=[ks+kh -kh;-kh kh];

        s=tf('s'); %s discrete
        H=1/(M*s*s+C*s+K);

        %Creating bode response for H
        %w=0:1:315;
        [mag,phase,w]=bode(H);
        %Extracting magnitude data

```

```

mag11=reshape(mag(1,1,:), [], 1);

%Extracting phase data
ph11=reshape(phase(1,1,:), [], 1);

%Converting from circular (rad/sec) to natural
frequency (Hz)
freq=w./(2*pi);

for k=1:1:size(freq)
    if freq(k,1)<10
        n=k;
    end
end

%For 11
for m=1:1:size(mag11)
    imag_11(m,1)=mag11(m,1)*sind(ph11(m,1));
    real_11(m,1)=mag11(m,1)*cosd(ph11(m,1));
end

%%Creating complex FRFs
%For 11
c11=complex(real_11,imag_11);
c11=c11(1:length(w),1);
[B,A]=invfreqs(c11,w,2,4,[],100,0.0001);
h11=freqs(B,A,w);
hmag11=abs(h11);
[Residues,Poles,K]=residue(B,A);
results(i,j,1:4)=abs(Poles)*max(w)/(2*pi);
results(i,j,5:8)=-real(Poles)./(abs(Poles));

end %j loop (occupant model loop)
end %i loop (structural configurations loop)

RESULTS1=results(1:18,1,1:8);
RESULTS2=results(1:18,2,1:8);
RESULTS3=results(1:18,3,1:8);
RESULTS4=results(1:18,4,1:8);
RESULTS5=results(1:18,5,1:8);
RESULTS6=results(1:18,6,1:8);
RESULTS7=results(1:18,7,1:8);
RESULTS8=results(1:18,8,1:8);
RESULTS9=results(1:18,9,1:8);
RESULTS10=results(1:18,10,1:8);
RESULTS11=results(1:18,11,1:8);
RESULTS12=results(1:18,12,1:8);
RESULTS13=results(1:18,13,1:8);
RESULTS14=results(1:18,14,1:8);
RESULTS15=results(1:18,15,1:8);
RESULTS16=results(1:18,16,1:8);
RESULTS17=results(1:18,17,1:8);
RESULTS18=results(1:18,18,1:8);
RESULTS19=results(1:18,19,1:8);

```

```
RESULTS20=results(1:18,20,1:8);
```

```
RESULTS21=results(1:18,21,1:8);  
RESULTS22=results(1:18,22,1:8);  
RESULTS23=results(1:18,23,1:8);  
RESULTS24=results(1:18,24,1:8);  
RESULTS25=results(1:18,25,1:8);  
RESULTS26=results(1:18,26,1:8);  
RESULTS27=results(1:18,27,1:8);  
RESULTS28=results(1:18,28,1:8);  
RESULTS29=results(1:18,29,1:8);  
RESULTS30=results(1:18,30,1:8);  
RESULTS31=results(1:18,31,1:8);  
RESULTS32=results(1:18,32,1:8);  
RESULTS33=results(1:18,33,1:8);  
RESULTS34=results(1:18,34,1:8);  
RESULTS35=results(1:18,35,1:8);  
RESULTS36=results(1:18,36,1:8);  
RESULTS37=results(1:18,37,1:8);  
RESULTS38=results(1:18,38,1:8);  
RESULTS39=results(1:18,39,1:8);  
RESULTS40=results(1:18,40,1:8);
```

```
RESULTS41=results(1:18,41,1:8);  
RESULTS42=results(1:18,42,1:8);  
RESULTS43=results(1:18,43,1:8);  
RESULTS44=results(1:18,44,1:8);  
RESULTS45=results(1:18,45,1:8);  
RESULTS46=results(1:18,46,1:8);  
RESULTS47=results(1:18,47,1:8);  
RESULTS48=results(1:18,48,1:8);  
RESULTS49=results(1:18,49,1:8);  
RESULTS50=results(1:18,50,1:8);  
RESULTS51=results(1:18,51,1:8);  
RESULTS52=results(1:18,52,1:8);  
RESULTS53=results(1:18,53,1:8);  
RESULTS54=results(1:18,54,1:8);  
RESULTS55=results(1:18,55,1:8);  
RESULTS56=results(1:18,56,1:8);
```

Appendix D: Phase II Results for All Structural Configurations

Table D1. Summary of comparison for 4.3 Hz and 20% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 3.99 | 1.99 | 3.92 | 2.42 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 3.9065 | 4.55 | 5.801 | 0.304 | 2.1 | 128.6 | 0.3 | 88.0 |
| Wei and Griffin | 4.9 | 53 | 3.8005 | 5.06 | 5.541 | 0.53 | 4.7 | 154.3 | 3.0 | 109.1 |
| Zheng and Brownjohn | 5.24 | 39 | 3.7543 | 4.13 | 6.002 | 0.385 | 5.9 | 107.5 | 4.2 | 70.7 |
| Sachse et al | 5.9 | 33 | 3.771 | 2.6 | 6.728 | 0.335 | 5.5 | 30.7 | 3.8 | 7.4 |
| JWG | 5 | 40 | 3.7463 | 4.86 | 5.739 | 0.389 | 6.1 | 144.2 | 4.4 | 100.8 |

Table D2. Summary of comparison for 4.3 Hz and 42% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 3.96 | 2.69 | 3.61 | 3.77 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 3.3289 | 4.5 | 6.459 | 0.331 | 15.9 | 67.3 | 7.8 | 19.4 |
| Wei and Griffin | 4.9 | 53 | 3.4053 | 5.97 | 6.188 | 0.565 | 14.0 | 121.9 | 5.7 | 58.4 |
| Zheng and Brownjohn | 5.24 | 39 | 3.3759 | 4.4 | 6.674 | 0.415 | 14.8 | 63.6 | 6.5 | 16.7 |
| Sachse et al | 5.9 | 33 | 3.4092 | 2.8 | 7.442 | 0.361 | 13.9 | 4.1 | 5.6 | 25.7 |
| JWG | 5 | 40 | 3.3592 | 0.051 | 6.4 | 0.419 | 15.2 | 98.1 | 6.9 | 98.6 |

Table D3. Summary of comparison for 4.3 Hz and 53% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 3.83 | 3.16 | 3.68 | 4.53 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 3.1924 | 4.3 | 6.735 | 0.345 | 16.6 | 26.5 | 13.3 | 5.1 |
| Wei and Griffin | 4.9 | 53 | 3.2581 | 5.92 | 6.467 | 0.587 | 14.9 | 64.2 | 11.5 | 30.7 |
| Zheng and Brownjohn | 5.24 | 39 | 3.2363 | 4.28 | 6.962 | 0.431 | 15.5 | 26.0 | 12.1 | 5.5 |
| Sachse et al | 5.9 | 33 | 3.2724 | 2.74 | 7.753 | 0.374 | 14.6 | 9.8 | 11.1 | 39.5 |
| JWG | 5 | 40 | 3.2185 | 4.97 | 6.68 | 0.436 | 16.0 | 42.1 | 12.5 | 9.7 |

Table D4. Summary of comparison for 5.7 Hz and 21% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 5.06 | 4.97 | 5.45 | 9.85 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.6063 | 13.99 | 6.187 | 21.25 | 9.0 | 181.5 | 15.5 | 42.0 |
| Wei and Griffin | 4.9 | 53 | 5.1189 | 10.87 | 5.456 | 47.81 | 1.2 | 118.7 | 6.1 | 10.4 |
| Zheng and Brownjohn | 5.24 | 39 | 4.8478 | 11.95 | 6.161 | 31.11 | 4.2 | 140.4 | 11.0 | 21.3 |
| Sachse et al | 5.9 | 33 | 4.8153 | 7.22 | 6.984 | 29.08 | 4.8 | 45.3 | 11.6 | 26.7 |
| JWG | 5 | 40 | 4.8722 | 14.81 | 5.85 | 29.43 | 3.7 | 198.0 | 10.6 | 50.4 |

Table D5. Summary of comparison for 5.7 Hz and 31% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 5.16 | 4.13 | 5.29 | 8.86 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.3316 | 12.05 | 6.58 | 24.27 | 16.1 | 191.8 | 18.1 | 36.0 |
| Wei and Griffin | 4.9 | 53 | 4.7533 | 13.3 | 5.876 | 47.53 | 7.9 | 222.0 | 10.1 | 50.1 |
| Zheng and Brownjohn | 5.24 | 39 | 4.5139 | 11.37 | 6.617 | 33.05 | 12.5 | 175.3 | 14.7 | 28.3 |
| Sachse et al | 5.9 | 33 | 4.546 | 7.02 | 7.398 | 30.47 | 11.9 | 70.0 | 14.1 | 20.8 |
| JWG | 5 | 40 | 4.4886 | 13.59 | 6.349 | 32.02 | 13.0 | 229.1 | 15.1 | 53.4 |

Table D6. Summary of comparison for 5.7 Hz and 46% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 5.08 | 4.69 | 5.24 | 9.83 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.0365 | 10.23 | 7.061 | 27.79 | 20.5 | 118.1 | 23.0 | 4.1 |
| Wei and Griffin | 4.9 | 53 | 4.338 | 13.39 | 6.438 | 50.23 | 14.6 | 185.5 | 17.2 | 36.2 |
| Zheng and Brownjohn | 5.24 | 39 | 4.1911 | 10.21 | 7.127 | 36.19 | 17.5 | 117.7 | 20.0 | 3.9 |
| Sachse et al | 5.9 | 33 | 4.2519 | 6.48 | 7.909 | 32.75 | 16.3 | 38.2 | 18.9 | 34.1 |
| JWG | 5 | 40 | 4.1542 | 11.92 | 6.861 | 35.68 | 18.2 | 154.2 | 20.7 | 21.3 |

Table D7. Summary of comparison for 5.7 Hz and 60% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 4.88 | 5.10 | 5.06 | 11.30 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 3.8574 | 9.22 | 7.388 | 30.15 | 21.0 | 80.8 | 82.2 | 18.4 |
| Wei and Griffin | 4.9 | 53 | 4.0726 | 12.6 | 6.858 | 53.44 | 16.5 | 147.1 | 149.0 | 11.5 |
| Zheng and Brownjohn | 5.24 | 39 | 3.9735 | 9.3 | 7.517 | 38.89 | 18.6 | 82.4 | 83.8 | 17.7 |
| Sachse et al | 5.9 | 33 | 4.042 | 5.98 | 8.32 | 34.81 | 17.2 | 17.3 | 18.2 | 47.1 |
| JWG | 5 | 40 | 3.9353 | 10.75 | 7.242 | 38.66 | 19.4 | 110.8 | 112.5 | 4.9 |

Table D8. Summary of comparison for 6.8 Hz and 21% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 6.07 | 7.94 | 6.76 | 8.30 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.8299 | 22.8 | 7.04 | 12.35 | 16.0 | 55.5 | 4.1 | 48.8 |
| Wei and Griffin | 4.9 | 53 | 5.1633 | 46.81 | 6.453 | 11.78 | 6.3 | 48.4 | 4.5 | 41.9 |
| Zheng and Brownjohn | 5.24 | 39 | 5.2564 | 27.17 | 6.779 | 15.92 | 11.7 | 100.5 | 0.3 | 91.8 |
| Sachse et al | 5.9 | 33 | 5.4993 | 14.99 | 7.296 | 21.47 | 20.2 | 170.4 | 7.9 | 158.7 |
| JWG | 5 | 40 | 5.0181 | 30.32 | 6.776 | 13.77 | 11.6 | 73.4 | 0.2 | 65.9 |

Table D9. Summary of comparison for 6.8 Hz and 34% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 6.15 | 7.70 | 6.77 | 10.40 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.6262 | 18.71 | 7.349 | 17.87 | 19.5 | 132.1 | 8.6 | 71.8 |
| Wei and Griffin | 4.9 | 53 | 5.4538 | 39.61 | 6.11 | 22.17 | 0.7 | 187.9 | 9.8 | 113.2 |
| Zheng and Brownjohn | 5.24 | 39 | 4.9609 | 20.32 | 7.183 | 24.57 | 16.8 | 219.1 | 6.1 | 136.3 |
| Sachse et al | 5.9 | 33 | 5.0862 | 12.36 | 7.888 | 25.55 | 28.3 | 231.8 | 16.5 | 145.7 |
| JWG | 5 | 40 | 4.8351 | 23.71 | 7.032 | 22.31 | 14.3 | 189.7 | 3.9 | 114.5 |

Table D10. Summary of comparison for 6.8 Hz and 48% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 6.01 | 7.30 | 6.88 | 12.40 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.4217 | 15.93 | 7.689 | 22.09 | 27.9 | 202.6 | 11.8 | 78.1 |
| Wei and Griffin | 4.9 | 53 | 5.0364 | 24.32 | 6.616 | 40.11 | 10.1 | 449.5 | 3.8 | 223.5 |
| Zheng and Brownjohn | 5.24 | 39 | 4.6719 | 16.89 | 7.627 | 29.77 | 26.9 | 307.8 | 10.9 | 140.1 |
| Sachse et al | 5.9 | 33 | 4.7908 | 10.75 | 8.374 | 28.67 | 39.3 | 292.7 | 21.7 | 131.2 |
| JWG | 5 | 40 | 4.5857 | 19.51 | 7.414 | 28.32 | 23.4 | 287.9 | 7.8 | 128.4 |

Table D11. Summary of comparison for 6.8 Hz and 63% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 5.61 | 8.40 | 6.87 | 12.60 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.236 | 13.93 | 8.026 | 25.57 | 43.1 | 204.4 | 16.8 | 102.9 |
| Wei and Griffin | 4.9 | 53 | 4.6478 | 20.74 | 7.169 | 46.08 | 27.8 | 448.6 | 4.4 | 265.7 |
| Zheng and Brownjohn | 5.24 | 39 | 4.4356 | 14.69 | 8.033 | 33.79 | 43.2 | 302.3 | 16.9 | 168.2 |
| Sachse et al | 5.9 | 33 | 4.551 | 9.56 | 8.816 | 31.46 | 57.1 | 274.5 | 28.3 | 149.7 |
| JWG | 5 | 40 | 4.364 | 16.83 | 7.791 | 32.86 | 38.9 | 291.2 | 13.4 | 160.8 |

Table D12. Summary of comparison for 7.50 Hz and 19% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 7.31 | 13.67 | 7.53 | 9.22 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.8817 | 26.02 | 7.682 | 8.85 | 5.1 | 35.3 | 2.0 | 4.0 |
| Wei and Griffin | 4.9 | 53 | 5.0457 | 48.37 | 7.283 | 9.55 | 0.4 | 30.1 | 3.3 | 3.6 |
| Zheng and Brownjohn | 5.24 | 39 | 5.2255 | 31.76 | 7.521 | 10.88 | 2.9 | 20.4 | 0.1 | 18.0 |
| Sachse et al | 5.9 | 33 | 5.7302 | 21.84 | 7.722 | 14.39 | 5.6 | 5.3 | 2.6 | 56.1 |
| JWG | 5 | 40 | 4.9899 | 33.79 | 7.515 | 9.85 | 2.8 | 27.9 | 0.2 | 6.8 |

Table D13. Summary of comparison for 7.50 Hz and 34% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 7.29 | 10.60 | 7.55 | 9.48 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.7281 | 22.01 | 7.931 | 14.49 | 8.8 | 36.7 | 5.1 | 52.8 |
| Wei and Griffin | 4.9 | 53 | 5.1654 | 42.95 | 7.115 | 18.37 | 2.4 | 73.3 | 5.8 | 93.8 |
| Zheng and Brownjohn | 5.24 | 39 | 5.0768 | 25.64 | 7.741 | 19.16 | 6.2 | 80.8 | 2.5 | 102.1 |
| Sachse et al | 5.9 | 33 | 5.3526 | 16.5 | 8.267 | 21.43 | 13.4 | 102.2 | 9.5 | 126.1 |
| JWG | 5 | 40 | 4.8922 | 28.41 | 7.665 | 17.43 | 5.1 | 64.4 | 1.5 | 83.9 |

Table D14. Summary of comparison for 7.50 Hz and 52% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 7.41 | 14.40 | 7.97 | 15.57 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.5303 | 18.53 | 8.278 | 19.79 | 11.7 | 37.4 | 3.9 | 27.1 |
| Wei and Griffin | 4.9 | 53 | 5.1264 | 32.81 | 7.169 | 32.24 | 3.3 | 123.9 | 10.1 | 107.1 |
| Zheng and Brownjohn | 5.24 | 39 | 4.816 | 20.62 | 8.16 | 26.46 | 10.1 | 83.8 | 2.4 | 69.9 |
| Sachse et al | 5.9 | 33 | 5.0044 | 13.48 | 8.842 | 26.36 | 19.3 | 83.1 | 10.9 | 69.3 |
| JWG | 5 | 40 | 4.691 | 23.27 | 7.994 | 24.95 | 7.9 | 73.3 | 0.3 | 60.2 |

Table D15. Summary of comparison for 8.10 Hz and 24% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 8.27 | 15.63 | 8.05 | 8.00 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.8577 | 26.14 | 8.337 | 9.71 | 0.8 | 37.9 | 3.6 | 21.4 |
| Wei and Griffin | 4.9 | 53 | 5.0286 | 47.73 | 7.893 | 11.6 | 4.6 | 25.8 | 2.0 | 45.0 |
| Zheng and Brownjohn | 5.24 | 39 | 5.1812 | 31.69 | 8.192 | 12.06 | 0.9 | 22.8 | 1.8 | 50.8 |
| Sachse et al | 5.9 | 33 | 5.6674 | 22.83 | 8.432 | 14.42 | 2.0 | 7.7 | 4.8 | 80.3 |
| JWG | 5 | 40 | 4.9574 | 33.64 | 8.17 | 11.12 | 1.2 | 28.9 | 1.5 | 39.0 |

Table D16. Summary of comparison for 8.10 Hz and 36% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 8.38 | 17.13 | 8.06 | 7.18 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.7594 | 23.61 | 8.509 | 13.49 | 1.5 | 21.2 | 5.6 | 87.9 |
| Wei and Griffin | 4.9 | 53 | 5.0806 | 44.18 | 7.812 | 17.7 | 6.8 | 3.3 | 3.1 | 146.5 |
| Zheng and Brownjohn | 5.24 | 39 | 5.0884 | 28.01 | 8.341 | 17.38 | 0.5 | 1.5 | 3.5 | 142.1 |
| Sachse et al | 5.9 | 33 | 5.4508 | 19.28 | 8.768 | 19.31 | 4.6 | 12.7 | 8.8 | 168.9 |
| JWG | 5 | 40 | 4.892 | 30.35 | 8.279 | 16.08 | 1.2 | 6.1 | 2.7 | 124.0 |

Table D17. Summary of comparison for 8.10 Hz and 41% configuration.

| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | 8.27 | 14.67 | 8.05 | 8.28 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.7162 | 22.67 | 8.588 | 14.93 | 3.8 | 1.8 | 6.7 | 80.3 |
| Wei and Griffin | 4.9 | 53 | 5.0936 | 42.46 | 7.792 | 20.47 | 5.8 | 39.5 | 3.2 | 147.2 |
| Zheng and Brownjohn | 5.24 | 39 | 5.0396 | 26.62 | 8.422 | 19.42 | 1.8 | 32.4 | 4.6 | 134.5 |
| Sachse et al | 5.9 | 33 | 5.3634 | 18.17 | 8.19 | 20.94 | 1.0 | 42.7 | 1.7 | 152.9 |
| JWG | 5 | 40 | 4.8564 | 29.05 | 8.34 | 18.04 | 0.8 | 23.0 | 3.6 | 117.9 |

Table D18. Summary of comparison for 8.10 Hz and 56% configuration.

| | | | | | | | Forward | | Straight | |
|---------------------|------------|-----------|------------|-----------|------------|-----------|---------------------|---------------------|---------------------|---------------------|
| | | | | | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) |
| | | | | | | | 8.78 | 18.80 | 8.11 | 11.27 |
| Model | f_n (Hz) | ξ (%) | Mode 1 | | Mode 2 | | % difference, f_n | % difference, ξ | % difference, f_n | % difference, ξ |
| | | | f_n (Hz) | ξ (%) | f_n (Hz) | ξ (%) | | | | |
| Coermann | 5 | 32 | 4.585 | 20.23 | 8.833 | 18.84 | 0.6 | 0.2 | 8.9 | 67.2 |
| Wei and Griffin | 4.9 | 53 | 5.0654 | 36.62 | 7.836 | 29.25 | 10.8 | 55.6 | 3.4 | 159.5 |
| Zheng and Brownjohn | 5.24 | 39 | 4.8783 | 23.13 | 8.701 | 24.77 | 0.9 | 31.8 | 7.3 | 119.8 |
| Sachse et al. | 5.9 | 33 | 5.1271 | 15.68 | 9.321 | 24.99 | 6.2 | 32.9 | 14.9 | 121.7 |
| JWG | 5 | 40 | 4.731 | 25.6 | 8.561 | 23.42 | 2.5 | 24.6 | 5.6 | 107.8 |

