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The Influence of Network Effect on the Valuation of Online Networking Companies

Yi Xu
Bucknell University, yx002@bucknell.edu

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By

Yi (Sean) Xu

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Janet T Knoedler

Thesis Adviser: Prof. Janet Knoedler

Gregory A. Krohn

Second Reader: Prof. Gregory Krohn

Department Chairperson: Prof. Thomas Kinnaman

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Abstract:

Online networks are having an enormous economic impact, both at the micro and the macro level. The growth of the online networks involves the participation of users, advertisers, and platform providers. This paper argues that the optimal revenue is directly proportional to user population, which is a key factor driving the online networking firms forward. Also, different companies are able to generate different value from each individual user. These arguments are supported by our empirical tests analyzing the current online networking giants. Empirical models further suggest that our theoretical model performs best when we look at the relationship between growth rate of revenue and that of user population. In addition, higher speed of user population growth and larger initial user population have positive influences in a network's valuation.

Keywords: Network effects; network externalities; two-sided markets; advertising; pricing; social networks; valuation

Chapter 1. Introduction

In our daily lives, we are benefiting much from physical networks. We call and text our friends using mobile networks through different carriers; we travel to different places by accessing the networks established by various airlines and railroad companies; and we order pizza from Dominos by enjoying their stores and delivery networks. These physical networks yield a network effect: the larger the user population is, the more benefits any individual user will get. With the development of the Internet and computing technologies, some virtual online networks have developed at an amazing speed. The most popular one is Facebook, which connects around 1.3 billion people worldwide. As users of these virtual online networks, people easily realize that the bigger the user population, the more individuals can benefit from themselves being connected. This is the typical outcome of the network effect. The virtual network also expands to other parts of our lives such as selecting good places for dining based on other people's online evaluations and getting a group discount for certain stores. Thus, these online networks are becoming more important in our daily lives.

One of the key shortcomings for virtual networks is that it is extremely difficult to value them. On May 18, 2012, the largest social network, Facebook, became publicly traded at $38 per share. The stock's prices declined in the following trading days, and stayed below its IPO price for around a year. This means that when Facebook was first publicly listed, it was simply overvalued.

Traditional understanding for valuing a company is based on its financial performance: revenue and net income. These two key financial indicators are driven by companies' consumers. When we take a closer look at online networking firms, it is clear that they are serving two groups of people: advertisers and network users. Most of the online network firms provide free services to users and charge fees to advertisers for the advertising slots on the online networking platforms. This is a typical two-sided market. The size and make-up of the user population directly affects the number of the advertisers who want to advertise their products or services. On the other hand, because of people's natural tendency against advertisements, the user populations are affected by the amount of advertisement on the online networks based on whether the online network has any substitutes or "stickiness[1]." Intuitively, the advertising revenue of the network would increase with the increase in the size of the user population. This could be explained by the network effect. Advertising revenue is the most important income for most online networking firms. Given the fact that revenue is the key driver for companies' financial performance, the companies' valuation is thus closely related to user population.

In this thesis, my goal is to examine the theoretical relationship between network effect and companies' valuation by analyzing the behaviors of users, advertisers, and network providers in online networking industries. In addition, I will use data for publicly traded online networking firms such as Facebook, Twitter,

[1] The amount of time spent at a site over a given time period. http://www.marketingterms.com/dictionary/stickiness/

LinkedIn, Yelp, and others to test the validity of my theoretical results. By completing this analysis, I will be able to state how online network firms would be able to know how network effect would affect their valuation, and investors will be able to make better investment decisions.

The rest of the thesis is organized as follows. The next chapter reviews previous studies on valuation and network effects. Chapter 3 presents my theoretical model, which analyzes the behaviors for different agents of online networking platforms. Chapter 4 will describe empirical models testing my theory. Chapter 5 will discuss the results of the empirical models. Chapter 6 discusses the conclusion of the research.

Chapter 2. Literature Review

Scholars have identified numerous ways to measure a company's value. Alfred Rappaport (1986) utilized the concept of shareholder value, which is now adopted as a yardstick for measuring the performance of firms (Srivastava et al 1998). The shareholder value is increased because of an increase in the level of cash flows, a reduction in risk associated with cash flows, and a higher residual value of the business (Day and Fahey 1988). From these internal drivers for companies' valuation, we could see that cash flows play important roles in evaluating a company because returns are measured in terms of cash flows (Minchington and Francis 2002). Accordingly, investors expect regular and realistic cash flow potential of a company and its customers (Bayon et al 2002). For online social networking companies, they can use their network to generate higher revenues, which leads to an increasing level of cash flows. In other words, by operating online networks, firms are able to generate cash flows from the revenues of advertising and subscriptions. The underlying argument is that the network effect offers the opportunity for explosive shareholder returns because winners in the network competition can have accelerating sales growth. Moreover, when network effects are intense, shareholders can directly capture the benefit of network effects (Mauboussin et al 2000).

The concept of network effect has also been analyzed by economists. Before the proliferation of online social networks, most analyses by economists were based on tangible networks such as telephone and railroad networks. Katz and Shapiro (1985, 1986) provide the seminal definition for network effect. A direct network effect is the benefit of direct connection with the other users such as telephone networks. The indirect network effect stems from consumption externalities, which arise for a durable good when the quality and availability of post-purchase service for the good in question depends on the experience and size of the service network. This leads to one of the results that consumers will base their pre-purchase decisions on the post-purchase expected network sizes. Farrell and Saloner (1985) also argued that consumers would benefit from a direct "network externality" in the sense that one consumer's value for a good increases when another consumer has a compatible good.

Even though these results are derived from the analysis of physical networks, this can still be applied to intangible online networks. Jeffery Rohlfs (1974) points out that the utility that a subscriber derives from a communications service increases as others join the system. Since online network platforms can be regarded as communications services, user utility will increase as others join the network.

Typically, the formation of a network is the process of people joining the network. The growth of a network typically follows a logistic growth pattern (Mauboussin et al 2000). The logistic model was first introduced by Verhulst and used by Pearl to approximate population growth in the United States in 1920 (Tsoularis and Wallace 2002). The online social networking user population growth resembles a population growth pattern. At the beginning, the growth of a user population is slow and the whole population is small. However, with the continuation of providing beneficial services, more users start to use the online platform. This growth would continue until the population size reaches the limit, which is defined as the network's carrying capacity. Then, the user population would remain at that level for the foreseeable future, as long as no drastic events happen. This shows that strong network effects would keep the limited entrants dominating the industry.

The result of these network effects is intuitive. The companies who build and maintain a solid network have the potential to become a strong monopoly. This is caused by demand-side economies of scale, which means that it is the consumer who voluntarily enables these companies to become a monopoly. With a network effect, consumer's expectations will cause consumption externalities to give rise to demand-side economies of scale because consumers will base their purchase decisions on expected network sizes (Katz and Shapiro 1985). In social networking contexts, it is easy to apply this theory. People tend to adopt a social network platform that their friends are on so they can maximize the utility of connecting to as many friends as possible on the same platform. Moreover, they would expect the domino network such effect that more users in the network increase the likelihood of connecting to more people in the future (MIT 2006).

In addition, this monopoly is hard to reverse. Firms with good reputations or large existing networks are typically not willing to make their products compatible to other systems; thus, their products or services are very different from their competitors (Katz and Shapiro 1986). Switching costs are a prime barrier to entry in appreciating a network's sustainable competitive advantage (Mauboussin et al

2000). Users have to learn new skills in order to switch to other network providers. In the context of online social networks, this problem becomes more obvious. When one thinks to switch to a new social network, the user will consider whether his or her friends on the same platform would make the same change. This switching problem inevitably increases the cost and difficulty of changing to a new online social networking platform.

As stated previously, there are different forms of network effect, including direct and indirect ones. Among these forms, the two-sided market model can be applied to online networking firms smoothly. Online networking platforms serve two groups of customers: advertisers and platform users. Rosse (1978) defined "demand interdependence" as the fact that demand for advertising is linked to the demand of platform users. A change in either the volume of advertising or in the population of platform users will not only directly affect the other variable, but will possibly have several rounds of effects as the resultant change in one variable brings about a further change in the other. Chaudhri (1998) defined a circulation industry as a two sided market: "any industry in which two sets of consumers, of different goods, are being serviced by the same proprietor, and the demand from at least one good depends upon the demand for the other, is termed a circulation industry." Online networking platforms clearly meet this definition for circulation industry. Thus, a two-sided market analysis can be applied.

The two-sided market model is used in analyzing the decision-making process for the pricing and advertising levels of the platform providers. For online networking firms, the pricing and advertising level decisions directly affect their financial performance. Parker and Van Alstyne (2005) use a two-sided market model to explain the reason why firms are willing to give away free products or services. First, they argue that low marginal costs would encourage firms to discount price to zero so that firms can subsidize an arbitrarily large market based solely on fixed initial costs. Second, they distinguish intermarket and intramarket network externalities. The former takes place among several kinds of agent, such as between consumers and advertisers, while the latter takes place inside one kind of agent. The study shows that free-goods markets can exist whenever a profit-maximizing price of zero or less generates cross-market network externality benefits greater than intramarket losses. Their two-sided market setup is important to explain the phenomenon that most online networking firms are offering free services.

Scholars have also analyzed the role of monopoly and competitive markets in the two-sided market realm. Chaudhri (1998) analyzed the Australian newspaper industry, which is a monopoly market. After categorizing the newspaper industry as a circulation industry, he argues that "the profit maximizing output of a firm in a circulation industry with monopoly power in one product dimension is greater than that of a monopolist without the circulation aspect." Thus, network effects in a circulation industry with a monopoly power are very intense. Social networks are a circulation industry because they fit the definitions made by Chaudhri. Given the fact that they also enjoy monopoly powers because of demand-side economies of scale, social networks have a strong network effect.

Furthermore, a two-tier market analysis based on the two-sided market model has also been presented. Scholars use this model to simulate the optimal advertising level on the platform. This strategy can be used to explain some online networking firms, which are offering different subscription options for their users. Riggins (2002) defines high type consumers who are willing to pay for high quality service but have less tolerance for online advertising, and low type consumers who are not willing to pay for services, but may tolerate some degree of banner advertising. In the end, platform providers may be forced to lower the quality of the low type product to make it less attractive to the high type consumers. At the same time, all users will be forced to tolerate more advertisements. Also, when advertising slots getting cheaper, the platform providers decrease the quality of content on the sponsored portion of the website. Prasad, Mahajan, and Bronnenberg (2003) examine another possibility in the market segmentation theory: pooling strategy. Under this strategy, a single version of a product or service is offered, which is targeted for consumption by both high and low consumers. In reality, most online network giants, such as Facebook, Twitter, and Yelp, are providing free services to users. This shows that a substantial proportion of online networking firms have been adopting Prasad et al’s pooling strategy so they obtain all their revenue from advertising.

With the proper decision about advertising levels and the prices for their network services to platform users, platform providers could generate the highest possible revenue, which will lead in turn to higher cash flows. The company’s value is then raised. The synthesis of various economic and financial research confirms the influence of network effects on the companies' valuation. My model is different from all above analysis. Unlike Katz and Shapiro, I no longer analyze the characteristics and effects of network externality.

I apply the network effect as the underlying mechanism to approximate the growth in the user population for online network platforms. In addition, I apply the two-sided market model to examine the demand for advertisement slots. My model extends the Parker and Van Alstyne model by considering inter-and intra-market externalities as well as taking the cost of different firms into consideration. In order to determine the optimal level of advertisement for networking firms purely in terms of advertiser strategy, I extend Prasad, Mahajan, and Bronnenberg's analysis of pooling strategy. I will not consider high and low type of consumers, but will regard every consumer as identical. Most importantly, by building upon the setup as well as the findings from the previous literature, my model takes user population growth as given, analyzes advertiser's decisions and platform providers' decisions, and apply these result to study the valuation of online networking companies. The uniqueness of my research is shown in the following figure.

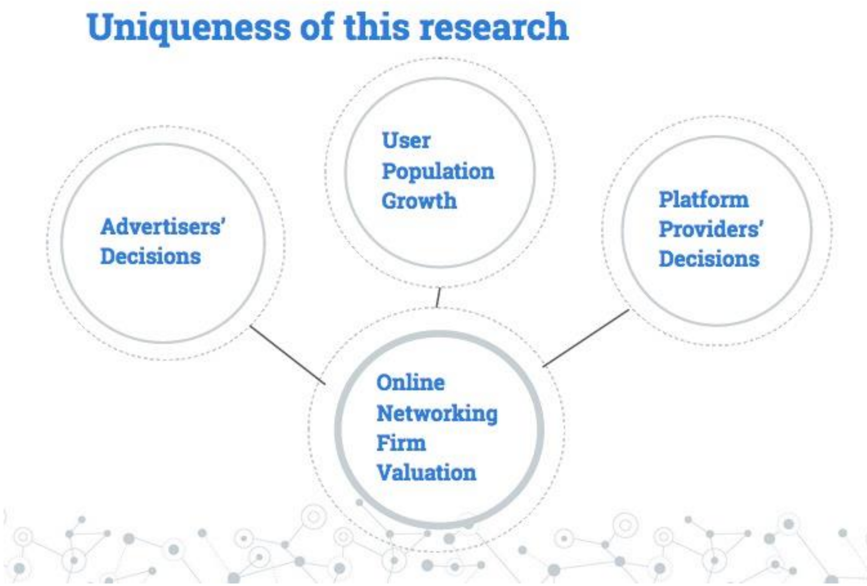

Figure 1. Uniqueness of the research

Chapter 3. Theoretical Model

The economic model consists of two major parts. The first part models the behavior and decision making process for online networking firm users, advertisers, as well as the platform providers. After obtaining the optimal decisions made by these agents, the second part of the economic model looks at the valuation of online networking firms and finds the effects of different parameters on the company values.

The first part of the model focuses on three agents: platform users, advertisers, and platform providers. The behaviors of one agent have an impact on the others. The platform user population has an effect on advertisers. The advertisers' decisions for placing advertisements on the online networks affects both users' decisions about using the platform and platform providers' decisions about pricing. Thus, I can analyze the behaviors of these three agents one by one.

Platform Users

Platform user population growth is the driving force of the growth of the online networking firms. Intuitively, more users on a platform will attract more advertisers to the platform to advertise their products simply because these products can be seen by more people. The growth of the user population is more like an adoption process of a product or technology, which involves a strong network effect during the process. The larger the user population is, the more benefits any individual user will obtain.

One key assumption in my model is that the number of platform users, Q_c, follows a logistic growth model over time. This assumption is based on the following important observations. First, the spread of information of a network is very similar to the spread of a disease, which is commonly modeled by a logistic growth model. When more people know the existence of an online network and more people use the network, the information will be spread more rapidly and the network will be more rapidly adopted by new users. Second, there is an upper limit for the user population. This observation shares the same characteristics of the original logistic model, which takes carrying capacity into consideration. This assumption is illustrated by the following figure showing user population growth pattern of Groupon.

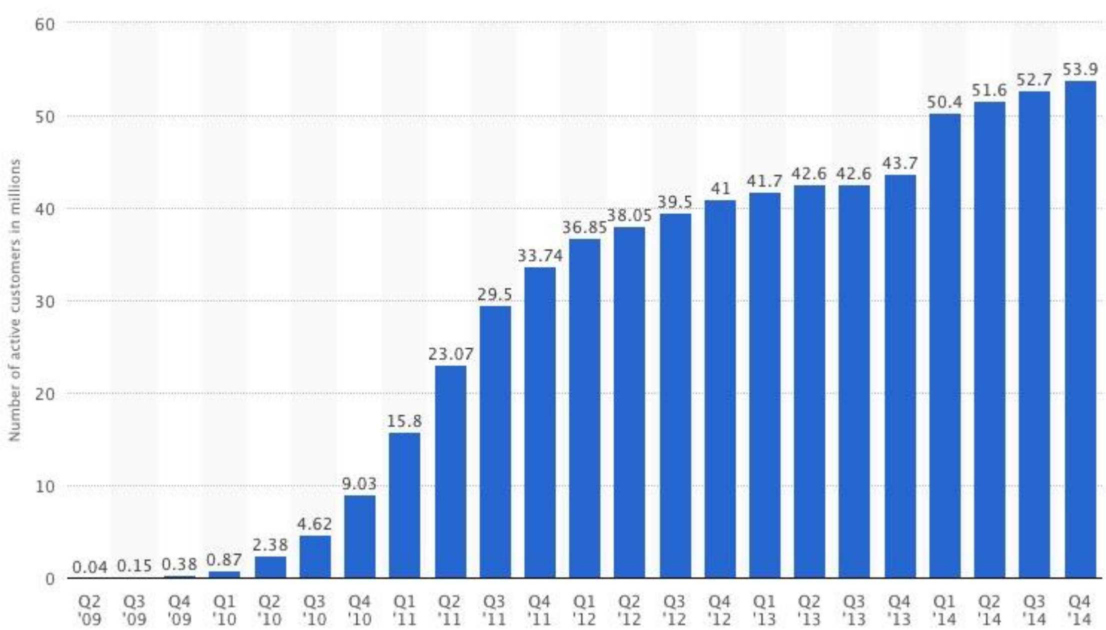

Figure 2. Groupon User Population Growth[2]

In theory, the total number of people who have absolute free access to the

[2] http://www.statista.com/statistics/273245/cumulative-active-customers-of-groupon/

Internet is the upper limit for the user population for any network. However, this upper limit is subject to change. For example, China is a large market for Internet services but the government bans Facebook at the moment. If this restriction were to be abandoned, the upper limit would rise substantially. Also, different functions of online networks will form different upper limits for user populations. Let K be the upper limit for the user population and t be the time. We assume the user population growth follows the following rate: $\frac{dQ_C}{dt} = rQ_C \left(1 - \frac{Q_C}{K}\right)$.

This differential equation has the following solution:

(1)

$$Q_c(t) = \frac{K\ Q_0}{(K - Q_0)\ e^{-rt}\ + Q_0}$$

with parameter r determining the speed of approaching the upper limit and Q_0 as the initial user population when the network is launched.

Unlike Peitz and Valletti's (2008) analysis in their comparison of Pay-tv and free-to-air services, my model is not going to analyze consumer welfare or social welfare issues in general. Thus, there is no optimization problem for users to solve.

<u>Advertisers</u>

Advertisers in my model are those agents who have products or services to be advertised; thus, there will be more people purchasing their products or using their services. Let M represent the number of firms with goods or services that need to be advertised. For all the advertisements that advertisers put on online networks, not everyone who sees an advertisement would remember it and make a purchasing decision. Different goods and services are appealing to different groups of people. Online network users who see the ads will only remember those ads that seem useful to them and make the purchasing decision accordingly. This ends up with the extra profit that advertisers earn from releasing the ads on the online networks. Thus, suppose θ to be a fixed extra profit whenever a viewer happens to see and to remember the ad from the advertisers and purchase the goods or services in the end, and α to be the proportion of the population of overall viewers who see an ad, remember it, and make the purchasing decision.

Since α is a proportion, its value is greater than zero but less than one. Assume θ is derived from a uniform distribution on $[0, \sigma]$. It has the density function $f(\theta) = \frac{1}{\sigma}$, which means the distribution function is $F(\theta) = \int_0^\theta f(x)dx = \frac{\theta}{\sigma}$.

Suppose the advertising slots have the same price, which is denoted by P_a. For advertisers, the major reason behind their advertising decision is to earn profit from the advertisements they put on online networks. They are rational, so they will only decide to put their ads on these online networks when the profit they earn from these ads is equal or larger than the cost of advertising. Let θ^* denote the fixed profit earned by the firms when the expected return from advertising equals the cost of advertising. Thus we get the following equation when advertisers can break even when advertising their products on online networks: $\theta^* \alpha Q_C = P_a$.

Thus, the firm will be profitable when the expected return by advertising is higher than the cost of advertising: $\theta^* \alpha Q_C > P_a$. Under this condition, the number of firms for whom it is profitable to advertise is $M[1 - F(\theta^*)] = M[1 - \frac{P_a}{\sigma\alpha Q_C}]$. Further assume that one advertiser will only purchase one unit of advertisement slot, so the quantity of ads demanded, Q_j, is equal to $M[1 - \frac{P_a}{\sigma\alpha Q_C}]$. I solve the equation for P_a to get the willingness to pay for a unit of advertisement, and I get:

(2)

$$P_a = \sigma\alpha Q_C \, [1 - \frac{Q_j}{M}]$$

This expression integrates the advertisers' decision-making criteria and provides information for platform providers to decide the optimal price and quantity.

Platform Providers

Our goal for the economic model is to find the valuation of the online network firms, so their decisions made as an economic agent are important in the analysis. For online networks, they have to make various decisions to optimize their profits. In theory, platform providers can generate their revenue from two channels, subscription and advertising revenue, which correspond to the two groups of customers they serve. The first group of customers is platform users. Platforms can choose to charge users to access the online networks or simply to provide free services. The second group of customers is advertisers. They charge advertisers for the advertisement slots on their networks. Typically, platforms can charge both users and advertisers or they can decide only to charge advertisers. Thus, platform providers' revenue can come from subscription and advertising revenue or only from the latter.

Given prominent examples in real life such as Facebook and Twitter, I assume in our model that all revenue that online networks generate is solely from advertising revenue. In other words, online networking platform providers are providing free services to network users. This is theoretically and practically viable according to Parker and Van Alstyne's (2005) argument. From the current online network industries, the services different firms provide are distinct. The S-1 filings from the U.S. Securities and Exchange Commission show that different online networking companies have distinguished themselves. For example, Facebook claims that the platform enables users to "connect with your friends, discover and learn, express yourself, experience Facebook Across the Web, and stay connected with you friends on mobile devices" by sharing photos, expressing what matters to users, or even playing games, listening to music, watching movies, reading news, and engaging in other activities[3] (Facebook S-1).

Twitter is also helping users to build their own social network similarly to Facebook. However, Twitter argues that its

> platform is unique in its simplicity: Tweets are limited to 140 characters of text. This constraint makes it easy for anyone to quickly create, distribute, and discover content that is consistent across our platform and optimized for mobile devices. As a result, Tweets drive a high velocity of information exchange that make Twitter uniquely 'live.' [Twitter] aim[s] to become an

[3] http://www.sec.gov/Archives/edgar/data/1326801/000119312512034517/d287954ds1.htm

indispensable daily companion to live human experience.[4] (Twitter S-1)

LinkedIn also indicates that its platform is special. It defines itself as:

> the world's largest professional network on the Internet with more than 90 million members in over 200 countries and territories. Through [its] proprietary platform, members are able to create, manage and share their professional identity online, build and engage with their professional network, access shared knowledge and insights, and find business opportunities, enabling them to be more productive and successful.[5] (LinkedIn S-1)

From these descriptions, we can easily see that these platforms have different functions so they are fundamentally different. Moreover, their services cannot be easily substituted. This distinction is strengthened by the network effect. Katz and Shapiro (1985, 1986) have argued that established networks tend to decrease the compatibility and increase the cost if their customers choose to switch to different networks. This is applicable in the use of online networks because users not only need to consider their own choices, but also have to think about their friends' choices. Thus, users tend to choose to be on the same network so they are able to maximize the possibility of connecting to people they know on the specific network and benefiting from the network size. In this case, users prefer services provided by monopolies. Even though there are several companies providing similar products, it is inevitable that users would help one of the companies to establish its monopoly in

[4] http://www.sec.gov/Archives/edgar/data/1418091/000119312513390321/d564001ds1.htm
[5] http://www.sec.gov/Archives/edgar/data/1271024/000119312511016022/ds1.htm

the end based on their own interests. As a result, online networking companies are monopolies, both because of their distinguishable products and because of users' choices.

Platform providers have an optimizing problem to solve. The goal is the maximize $P_a Q_j$, which is the revenue for the network, by choosing appropriate the Q_j level, given Q_c and inverse demand function $P_a(Q_j)$. By applying the results from analyzing the advertiser behaviors, we get:

(3)

$$R = P_a Q_j = \sigma \alpha Q_C \, [Q_j - \frac{{Q_j}^2}{M}]$$

Then I maximize the revenue by taking the derivative and set marginal revenue equal to zero, which yields the optimal level for both price and quantity of the advertising slots they are going to offer.

Denote the optimal price for advertising slot as ${P_a}^*$ and the optimal advertising slot quantity offered as ${Q_j}^*$.

(4)

$${P_a}^* = \frac{1}{2} \ \sigma \alpha Q_C$$

(5)

$${Q_j}^* = \frac{M}{2}$$

(Derivation in appendix A)

Corollary 1

The optimal price is half of the maximum profit can be extracted by an advertiser in the networks. The optimal quantity of advertising slots offered is half of the number of firms that want their goods and services to be advertised.

The optimal price for an advertising slot is proportional to the platform user population at a particular time. $\sigma\alpha Q_C$ is the maximum profit an advertiser can obtain from putting an advertisement on online networks. The optimal price for a unit of advertisement slot is half of this value. I also notice that the optimal quantity of advertising slots offered only relates to the number of firms that want their goods and services to be advertised, which means the optimal quantity is independent of the user population. This makes sense because I have assumed the platform providers are monopolies in their specific service category. With the optimal price as well as the optimal quantity of advertisement level, we will be able to calculate the maximum value for revenue in order to find the valuation of the online networking firms.

Valuation

The second part of the economic model is to find the company's value by obtaining its revenue each year and discounting the future value to the present one. The goal is to find which parameters of user population growth and other possible factors will affect firms' valuations.

Since all revenue is generated by charging advertisers with the optimal price and for the optimal quantity of advertisement slots, we are able to obtain the optimal revenue that platform providers can obtain at any point of time. Thus,

$$R^* = P_a{}^* \, Q_j{}^*$$

By plugging in the optimal level, the revenue for platform providers is:

(6)

$$R^* = \frac{1}{4}\, \sigma \alpha Q_C M$$

For platform providers, their maximum net profit equals the difference between optimal revenue and cost. I assume that for online networking firms, the only cost that occurs is the fixed cost, so there is no marginal cost associated with each additional user or each additional advertisement slot. Due to the character of the online networking industry, their cost components are different from other business industries. They have high upfront fixed costs such as setting up the network and hiring software engineers. However, the marginal cost for each individual is almost zero. There is nothing extra they need to invest and the initial fixed cost can cover all the additional user and advertisement slots. Denote FC as the fixed cost incurred every year for online networking firms and π^* as the optimal company profit. Thus,

$$\pi^* = R^* - FC$$

$$= \frac{1}{4}\, \sigma \alpha Q_C M - FC$$

Corollary 2:

The optimal revenue is directly proportional to user population. The optimal net income has a close but not directly proportional relationship with user population.

By discounting a company's profit, we are able to find the present value of the online networking firms. Let PV be the present company value and i be the interest rate. The interest rate is a value larger than zero. In the discounting model, we choose a continuous compound rate in order to match the continuous user population growth modeled in the continuous logistic function. Thus, the discount factor is e^{it}. Accordingly, we get present value as:

$$PV = \sum_{t=0}^{\infty} \frac{\pi^*}{e^{it}}$$

(7)

$$PV = \sum_{t=0}^{\infty} \frac{\sigma\alpha Q_C M}{4e^{it}} - \sum_{t=0}^{\infty} \frac{FC}{e^{it}}$$

Denote the first summation term as PV_R, which is the present value revenue term, and the second summation term as PV_C, which is the present value cost term. Since the interest rate i is less than one, the PV_C series converges, which yields the fact that PV_C is a fixed value:

(8)

$$\sum_{t=0}^{\infty} \frac{FC}{e^{it}} = \frac{FC}{1 - e^{-i}}$$

Thus, change in valuation all depends on the first summation term. We only need to analyze the first summation term PV_R in order to know how different factors will affect a company's present value.

Plugging in the logistic function solution equation for PV_C, we get:

$$PV_R = \sum_{t=0}^{\infty} \frac{\sigma\alpha MKQ_0}{4e^{(i-r)t}(K-Q_0) + 4e^{it}Q_0}$$

In order to simplify this expression, let $A = \sigma\alpha MKQ_0$ $B = 4(K-Q_0)$, and $C = 4Q_0$. Thus:

(9)

$$PV_R = \sum_{t=0}^{\infty} \frac{A}{Be^{(i-r)t} + Ce^{it}}$$

By applying the ratio test, we know this series converges.

The subsequent trivial result is that PV is bounded and is equal to:

$$PV = \frac{Ae^{i}}{C(e^{it}-1)} - \frac{FC}{1-e^{-i}}$$

(Proof and calculation see appendix B.)

Then, the convergence allows us to use derivatives to find the relationship between each parameter and present value.

First, we can find the relationship between parameter r in the logistic function and the present value.

$$\frac{dPV_R}{dr} = \frac{d}{dr}\sum_{t=0}^{\infty} \frac{A}{Be^{(i-r)t} + Ce^{it}}$$
$$= \sum_{t=0}^{\infty} \frac{d}{dr}\frac{A}{Be^{(i-r)t} + Ce^{it}}$$

(10)

$$\frac{dPV_R}{dr} = \sum_{t=0}^{\infty} \frac{ABe^{(i-r)t}t}{(Be^{(i-r)t} + Ce^{it})^2} > 0$$

Corollary 3

The more rapidly the user population approaches the user upper limit, the higher the company valuation will be.

I take the derivative of PV_R on r, and obtain a positive result from expression (10). I know that r is a parameter in a logistic model influencing the rate of user population approaching the upper limit. The larger the value for r, the more rapidly the user population approaches the theoretical upper limit. Thus, (10) yields the positive relationship between the parameter r and the present value for online networking companies.

Furthermore, in order to find the relationship between other factors and an online networking firm's present value, I have to analyze the original form of PV_R, which can be rewritten as following:

$$PV_R = \frac{1}{4} \sum_{t=0}^{\infty} e^{-it} \frac{\sigma \alpha M K Q_0}{e^{-rt}(K - Q_0) + Q_0}$$

Then I can explore the relationship between PV_R respect to $\sigma, \alpha, M, K,$ and Q_0.

For $\sigma, M,$ and α,

(11)

$$PV_R = \frac{\sigma \alpha M}{4} \sum_{t=0}^{\infty} \frac{K Q_0}{e^{(i-r)t}(K - Q_0) + e^{it} Q_0}$$

Corollary 4

$\boldsymbol{\sigma}, \boldsymbol{M},$ and $\boldsymbol{\alpha}$ have positive linear relationships with $\boldsymbol{PV_R}$.

From (11), we see that σ, M and α have a positive linear relationship with the company's present value. The higher each parameter is, the higher the company value is. This result is intuitively correct. First, recall that σ is the maximum fixed extra profit that advertisers are able to get when they put an advertisement on the online network platform. If an online network would enable advertisers to extract more profit from an advertisement slot, the network value would be higher. Second, recall that α is the proportion of the population of overall viewers who see an ad, remember it, and make the purchasing decision. If an online network can persuade more users to view the ads released by various advertisers and increase the effectiveness of those ads, the value of online networks will increase accordingly. Third, recall that M is the number of firms with goods or services that need to be advertised. If there is an increase in the number of firms that want to place their ads on the network, the online networking firms will have an increase in their valuation because the optimal level of advertising slots is always $\frac{M}{2}$.

Finally, I rewrite the PV_R function as the following to see the relationship between K and Q_0 with respect to the value for online networking firms.

$$PV_R = \frac{1}{4}\sum_{t=0}^{\infty} e^{-it}\,\sigma\alpha Q_C M$$

Since Q_C is the function containing K and Q_0, by taking the derivatives we get:

$$\frac{\partial PV_R}{\partial K} = \frac{1}{4}\sum_{t=0}^{\infty} e^{-it}\,\sigma\alpha M\frac{\partial Q_C}{\partial K}$$

and

$$\frac{\partial PV_R}{\partial Q_0} = \frac{1}{4}\sum_{t=0}^{\infty} e^{-it}\,\sigma\alpha M\frac{\partial Q_C}{\partial Q_0}$$

we know that:

$$\frac{\partial Q_C}{\partial K} = \frac{{Q_0}^2 e^{rt}(e^{rt} - 1)}{(K + Q_0(e^{rt} - 1))^2} \geq 0$$

(The equal sign is valid when $r = 0$ or $t = 0$. In our case r is strictly larger than 0, so the derivative is strictly larger than 0 when $t > 0$.)

Thus,

$$\frac{\partial Q_C}{\partial Q_0} = \frac{K^2 e^{rt}}{(K + Q_0(e^{rt} - 1))^2} > 0$$

Therefore, we have

(12)

$$\frac{\partial PV_R}{\partial K} > 0;\ \frac{\partial PV_R}{\partial Q_0} > 0$$

Corollary 5

The value of online networking companies increases with the increase of K and Q_0.

Recall that K is the upper limit for the user population and Q_0 is the initial user population when the network is launched. Our economic model predicts that the company valuation will increase if the upper limit for user population grows. In addition, the valuation of online networking firms will increase if the initial population grows.

Chapter 4. Empirical Model and Data

My empirical models are designed to test the two main results in the previous section:

1. $R^* = \frac{1}{4}\,\sigma\alpha Q_C M$
2. $\pi^* = \frac{1}{4}\,\sigma\alpha Q_C M - FC$

There are two groups of empirical tests in this thesis. The first group of tests will look at relationship between user population and revenue as well as net income among all the companies. The second group of tests will examine the same relationship but for particular companies.

The first group of models is shown in the following equations:

(1) $Rev\ =\ \beta_0 + \beta_1\ *\ UserPop\ +\ \mu$

(2) $NetInc\ =\ \beta_0 + \beta_1\ *\ UserPop\ +\ \mu$

(3) $Log\_Rev\ =\ \beta_0 + \beta_1\ *\ log\_UserPop\ +\ \mu$

The Rev is the real revenue for any particular online networking companies generated in a particular quarter. $NetInc$ is the real net income for any networking companies at the end of a particular quarter. This term can be positive when the company has profit, and can be negative when the company actually loses money under the situation that cost is higher than revenue. Both Rev and $NetInc$ are adjusted for inflation instead of using the nominal value reported by companies. $UserPop$ is the user population on any particular network in any particular time period. For the third model, the model introduces two new variables: Log_Rev and $log\_UserPop$. The reason behind this is that I would now be able to see how the percentage change in user population affects the percentage change in revenue. I do not include the model analyzing the percentage change of user population and percentage change of net income because I will not obtain a linear relationship if I take the log for the theoretical equation for the net income for companies.

The predicted outcome of the model is intuitive. According to the theoretical model, a greater user population will increase the revenue that is generated by the online networking firms. Thus, I expect that β_1 in equation (1) to be positive. Moreover, β_1 in equation (2) is also expected to be positive because I assume that online networking firms only have to deal with fixed cost. However, if there are some variable costs for online networking firms, the relationship between user population and net income, estimated by β_1 in equation (2), will be uncertain because the relative size of cost and revenue will then be ambiguous. Also, strategies among companies are different. Some technology companies are well funded by hedge funds and private equities, so they invest heavily in hardware and software, while others try to break even and make a profit if they are able to do so. Since the theory predicts a linear relationship between revenue and user population, the coefficient is expected to be 1 in equation (3).

The second group of models will examine the effect of user population on a particular online networking company. Thus, I added dummy variables for different companies from D2- D7 based on equation (1)-(3), given there are seven companies I am studying in our data set. Also, I introduced interaction terms between dummy variables and the user population variable. By doing this, I am able to use regression analysis to obtain the predicted effect of user population on revenue or net income for the different companies we are analyzing. The models have the following equations:

(4) $Rev = \beta_0 + \beta_1\, UserPop + \beta_2 D2 + \beta_3 D3 + \beta_4 D4 + \beta_5 D5 + \beta_6 D6 + \beta_7 D7 + \beta_8 D2\_User + \beta_9 D3\_User + \beta_{10} D4\_User + \beta_{11} D5\_User + \beta_{12} D6\_User + \beta_{13} D7\_User + \mu$

(5) $NetInc = \beta_0 + \beta_1\, UserPop + \beta_2 D2 + \beta_3 D3 + \beta_4 D4 + \beta_5 D5 + \beta_6 D6 + \beta_7 D7 + \beta_8 D2\_User + \beta_9 D3\_User + \beta_{10} D4\_User + \beta_{11} D5\_User + \beta_{12} D6\_User + \beta_{13} D7\_User + \mu$

(6) $Log\_Rev = \beta_0 + \beta_1\, log\_UserPop + \beta_2 D2 + \beta_3 D3 + \beta_4 D4 + \beta_5 D5 + \beta_6 D6 + \beta_7 D7 + \beta_8 D2\_log\_UserPop + \beta_9 D3\_log\_UserPop + \beta_{10} D4\_log\_UserPop + \beta_{11} D5\_log\_UserPop + \beta_{12} D6\_log\_UserPop + \beta_{13} D7\_log\_UserPop + \mu$

Based on the theoretical model, we expect that the coefficient estimates for different companies are positive, which indicates that over time, the company will generate more revenue with an increasing user population. In addition, similar to the estimates in the first group, the effect of user population on net income is expected to be positive for any particular company. For equation (6), I also expect the coefficient of user population for different companies equal to one due to the linear relationship predicted by the theoretical model.

The data set used to estimate the model is gathered from the legal filings for online networking firms. Every quarter, publicly traded online networking firms need to file a legal document indicating the financial performance and operational situation of the company. Also, historical financial performance and operational data can be found in the initial public offering legal documents from the online networking firms. All these documents can be accessed at the U.S. Securities and Exchange Commission, www.sec.gov.

The nominal revenue and net income information can be accessed from the legal documents easily. However, in my models, I adjust nominal revenue and net income to real revenue and income. In order to predict the effect of user population more precisely, the model excludes the impact of inflation, which drives up the nominal revenue and net income over time. The adjustment factor is $\frac{CPI_{base}}{CPI_{current}}$. The CPI_{base} is the CPI in February 2009, which is the mid-month of the first quarter in 2009. I start from 2009 because our first data point for revenue and net income for online networking companies comes from that time. Since revenue and net income are posted quarterly, we choose the mid-month for every quarter for $CPI_{current}$ term for all the companies. All the inflation-adjuested revenue and net income will be measured in millions of dollars. Both CPI_{base} and $CPI_{current}$ used are the CPI for all urban consumers and can be accessed at Bureau of Labor Statistics, www.bls.gov.

The key independent variable in our model is the user population. The user population can be understood as the size of online networks. Online networks have different measurements for the user population. The two most common measurements are monthly active users (MAU) and daily active users (DAU). We choose the monthly active users (MAU) as the relevant measure for the majority of the companies we are analyzing here. For those online networking firms who do not use monthly active users as measurement, we tend to choose the closest measurement for the user population variable for that particular online network. For example, Yelp uses monthly unique visitors as their measurement for the popularity of their network. Thus, I use this measurement instead of MAU in Yelp's case.

Table 1. Table of Key Variables

Variable	**Description/Definition**	**Source**	**Mean**
User Population	The user population is the number of users on particular online networks in specific period of time, usually reported quarterly. The user population can be understood as the size of the network. The unit for the user population is million.	www.sec.gov (from company legal filings, such as S-1, 10-K, 10-Q, etc)	281.12
Net Income	The net income is the online networking firm's profit or loss when revenue minus the costs. The unit for the net income is in	www.sec.gov (from company legal filings, such as S-1, 10-K, 10-Q, etc)	36.05

	million dollars. This data is reported by firms quarterly.		
Revenue	The revenue is the money online networking firms generate from providing services including subscription and advertising. This data is reported by firms quarterly.	www.sec.gov (from company legal filings, such as S-1, 10-K, 10-Q, etc)	413.55

The companies I include in the study are Facebook, Twitter, LinkedIn, RenRen, Weibo, Yelp, and Groupon. Most of the companies are already household names. RenRen is simply a copy of Facebook operating in mainland China. Weibo is simply a copy of Twitter operating in mainland China. It is legitimate to include these two companies because they are publicly listed in the U.S., so they closely follow the U.S. accounting and legal reporting system. Two other firms that go beyond the traditional understanding of social networks are Yelp and Groupon. They are not typical social networking firm, but according to their own description in their initial public offering legal documents, they have strong network effect characteristics. According to Yelp, it

> connects people with great local businesses. [The] platform features more than 22 million reviews of almost every type of local business, from restaurants, boutiques and salons to dentists, mechanics, plumbers and more. These reviews

are written by people using Yelp to share their everyday local business experiences, giving voice to consumers and bringing "word of mouth" online. The information these reviews provide is valuable for consumers and businesses alike.[6] (Yelp S-1)

When people are reading others' review, they are automatically benefiting from the existence of the user who wrote the review for any particular local business. For Groupon, it "is a local e-commerce marketplace that connects merchants to consumers by offering goods and services at a discount" (Groupon S-1). The more people using the platform, the more benefit individual users will have.

There are some limitations for the empirical test. The total observations are limited. Since online networks have only been thriving for just the past several years, as well as the limited number of companies, I am not able to get large numbers of observations. Furthermore, all these companies are publicly listed, so the data is not randomly selected. However, analysts and researchers are only able to access the financial performance and user population data when companies are public.

In addition, only selecting publicly listed companies tends to create the problem that selected companies are successful companies. In reality, not all the companies are publicly traded. The choice of being listed on the stock market is a self-selection process. However, companies involving networks have demonstrated a tendency for making the choice of going public. Stoughton et al (2001) points out that the propensity to go public depends on the growth in market size generated by network externality effects. In their model, consumers infer quality from the stock prices, and

[6] http://www.sec.gov/Archives/edgar/data/1345016/000119312511315562/d245328ds1.htm

the stock market anticipates profits generated by the quality perception of the consumers. High stock prices could increase perceptions of quality, which would strengthen the reputation of the firm. For online networking firms, going public can be beneficial because it could attract huge public attention and expand their services and network more easily. Being publicly listed is an essential step for successful online networking firms.

It is true that publicly listed companies in general are successful, but the situation for the online network industry is different. Almost every large online networking firm is a monopoly because of demand-side economies of scale, which means that the entry barrier is extremely high. New firms are not able to attract enough user population to grow, so I am not able to see the relationship between their financial performance and user population. Most importantly, publicly listed online networking firms can be regarded as representatives of online networking firms. Online networking companies have similar operational models and for every online network, the key for growth is always their users. Thus, the relationship between users and revenue as well as net income is similar among public and private companies.

People may also argue that spurious variables that we do not include here could affect the financial performance instead of the key variable user population in my model because I do not include the time variable. However, the time variable is already embedded in the user population term because the theoretical model has assumed that user population follows a logistic growth pattern, in which time is a key parameter. In addition, I have tried to adjust the revenue and net income in order to eliminate the effect of inflation, which is the most likely factor that will drive the nominal revenue and income over time.

Breusch-Pagan tests for heteroskedasticity are conducted on all six models. Heteroskedasticity exists in model 1, 2, 4 and 5. Model 6's heteroskedasticity test statistics is significant at the 10% level but not significant at the 5% level. Thus, regression estimates with robust standard errors are presented for model 1, 2, 4, and 5 as the final regression result in the following result section. In addition, I also conducted RESET tests for model 4 and 6. Neither of these two models passed the RESET test, which means that adding square terms or cube terms could potentially increase the R-squared value of our econometric models. This suggests that even though my theory predicts that revenue for online networking firms is directly proportional to user population, the relationship between the revenue and user population in reality could be more complicated than what I modeled in my theory.

Chapter 5. Empirical Results

Table 2. Regressions Result for Group 1 Models

Variable	Model 1	Model 2	Model 3
UserPopulation	0.182***	0.053***	
Log_UserPopulation			0.514***
Constant	-0.116	-10.673***	0.383
Observations	113	113	113
R-Squared	0.591	0.527	0.193
F-statistics	160.06***	123.74***	26.52***

(*,**,*** indicate that the coefficient is statistically significant at the 10%, 5%, 1% levels, respectively)

Table 2 presents the results of estimating equations (1) –(3) using data from the first quarter of online networking companies from early 2009 to the end of 2014. In model 1 and model 2, we find statistically significant coefficient estimates so we reject the hypothesis that user population has no impact on the revenue or income among companies. The dependent variables are real revenue for model 1, real net income for model 2, and log term of revenue for model 3. In model 1, the coefficient is 0.182, which is economically significant. Adopting the theoretical model described in the previous section, the coefficient is actually estimating the $\frac{1}{4}\sigma\alpha M$ part. User population can be understood as the size of online networks. Thus, each additional user would bring in about 18 cents of real revenue for online networking firms in general. In addition, in model 1, I obtain a statistically insignificant estimate for the constant so I cannot reject the hypothesis that the starting revenue is zero. This is intuitively correct and consistent with my theory because when there is no user using the platform, there is no way for online networking firms to generate revenue. However, the true relation in reality could be non-linear. In this case, the intercept is not necessarily zero.

When I interpret the model 2, I can follow a similar procedure. For model 2, the coefficient is 0.053, and it is statistically significant. In other words, each user would bring in about 5 cents of additional real profit for online networking firm. The bigger the network, the more profit the company will get. In model 2, the constant is -10.673, which is statistically significant. We reject the hypothesis that the net income for online networking firms is zero when there is no user using the platform. Even though this is not intuitive, this can be explained by the special characteristics of online networking industry. Online networking firms have extremely high upfront costs. They have to invest in servers as well as software before providing any services for any users. Also, companies tend to spend whatever money is needed to attract as many new users as possible. All these factors can explain the statistically and economically significant negative constant value in model 2 when we analyzing the relationship between the user population and net income for the companies.

Model 3 estimates the relationship between the log of user population and the log of real revenue. Among the online networking firms we are looking at, a 1% increase in user population will lead to a 0.51% increase in the revenue. According to the theoretical model, there is a linear relationship between user population and revenue. Thus, we expect the coefficient of log term of user population to be equal to one. However, we reject this hypothesis by conducting an F-test, which yields the F-statistic equal to 23.66 at the 1% significance level. This means that it is possible that, in reality, the relationship between user population and revenue among companies has a more complex relationship than a linear relationship.

Table 3 presents the estimates for different firms included in our data set. The dependent variables are the same as the first three models examined previously. When including the dummy variables and interaction terms between dummy variables and user population, the models are able to estimate the impact of user population on the revenue or net income for any particular company.

Table 3. Regression Result for Group 2 Models

Variable	**Model 4**	**Model 5**	**Model 6**
UserPopulation	0.315***	0.087***	
Log_UserPopulation			1.443***
D2	84.831	71.867***	-10.05***
D3	112.009***	46.039***	-0.99

D4	137.723***	33.884**	5.742**
D5	128.778**	43.418**	-1.523
D6	127.886***	30.626**	5.80***
D7	124.255	40.307	-6.245
D2_USER	0.034	-0.281**	1.908**
D3_USER	-0.015	-0.091**	0.323
D4_USER	-0.309	-0.012	-1.356***
D5_USER	-0.183	-0.069	0.359
D6_USER	1.72***	0.172	-0.543*
D7_USER	-0.174	-0.066	1.252
Constant	-134.82***	-45.07	-4.949**
Observations	113	113	113
R-squared	0.768	0.652	0.916
F-Statistics	25.15***	14.3***	82.91***

(*,**,*** indicate that the coefficient is statistically significant at the 10%, 5%, 1% levels respectively)

Model 4 estimates the relationship between user population and real revenue for any particular company over different periods of time. D2 to D7 correspond to Twitter, Linkedin, Renren, Yelp, Groupon, and Weibo respectively. The coefficient 0.315 on the user population term estimates the impact of user population on revenue for Facebook. One extra user would bring in approximately 3 cents of real revenue for Facebook. The coefficient for Twitter can be calculated as the sum of the coefficient for the user population term and the coefficient of the interaction term between the Twitter dummy variable and user population. Thus, the coefficient for Twitter equals the sum of 0.315 and 0.034, which is 0.349. Using this method, we are able to calculate the coefficient for all remaining companies. For all the companies we are analyzing, user population contributes positively towards revenue growth.

An F-test is conducted on model 4. The null hypothesis is that the impact of user population on revenues is the same across all companies. The F-statistic is insignificant at 5% level but is significant at 10% level. Even though I cannot reject the hypothesis at 5% significant level, when I calculate the coefficient for different firms, I see relatively significant variation among them. For different companies, their ability to generate revenue from individual users should be different. The coefficients here are estimating the $\frac{1}{4}\sigma\alpha M$ term for different companies. Recall that M is the number of firms with goods and services that need to be advertised; α is the proportion of overall viewers' population who watch an ad, remember it, and purchase the product in the end; and σ is the maximum fixed extra profit that advertisers are able to get when they put an advertisement on the online network platform. By adopting different advertising strategies and different pricing schemes, α and σ should vary across different companies.

Furthermore, the constant in model 4 is economically and statistically significant. This constant is for Facebook. However, when we calculate the constant for the remaining six companies and apply the F-test, none of the remaining constants is statistically significant even at 10% level. As a result, we can still use the model to predict that there is no revenue for online networking firms when there is no user in general.

Model 5 shows that, for most of the online networks we are analyzing, user population contributes an increase in net income. For example, Facebook would gain 8 cents of real profit for an additional user. There are two companies for which user population has a negative impact on its net income: Twitter and LinkedIn. This could be explained by the fact that the additional expenditures that these two companies have to spend in order to attract and maintain more users are higher than the benefits they get from the additional user population.

The F-test for all interaction terms is significant at the 5% level. Thus, we are able to reject the hypothesis that user population has the same impact across companies to their net income. Different online networking firms, they are focusing on providing different functions. Also, various companies have different strategies to grow. Thus, they will have different financial and operational structures so their spending differs sharply. This can explain the uncertain impact of user population on different online networking firms' net income.

Model 6 examines the relationship between user population growth and revenue growth for a particular firm. The coefficient for the log user population is 1.44. This means that 1% growth of user population will contribute to about 1.44% increase in real revenue for Facebook. The coefficient for the other companies can be calculated by adding the coefficient of the log term of user population and the interaction term between the dummy variable and the log user population for any specific company. The hypothesis is that all coefficients equal one. Some of the coefficients are significant at 1% level to reject the hypothesis that coefficient equals one, while others cannot reject the hypothesis even at 10% significant level. As a result, for certain companies, the user population has a linear relationship with their revenues over different time periods. For some other companies, the relationship between user population and revenue is more complicated than a linear one in reality.

Model 6 also shows that the relationship between user population growth and revenue growth are different among companies. When we conduct an F-test for interaction terms in this model, we reject the hypothesis that coefficients for interaction terms are zeros. This suggests that user population growth has different impacts on the revenue growth for different companies.

When comparing results among models, we find two additional results regarding the theoretical model and the empirical model. First, empirical tests show that my theoretical model makes better predictions when dealing with particular online networking companies over time than when dealing with all online networking firms together as an industry. Results in Table 3 are more consistent with the theoretical model and R-squared values show that Table 3 models fit data better. Empirical models in Table 3 show that firms have different abilities in generating revenue from individual users. Furthermore, when we compare model 3 for the industry and model 6 for particular companies, we see that the coefficients in model 6, unlike model 3, show that revenue of proportional to user population.

Second, the theoretical model we presented can explain the relationship between log of user population and log of revenue quite well. The R-squared value in model 6 is 0.916. This means that about 91% of the dependent variable, log of revenue, can be explained by the independent variable, log of user population. As a result, my theoretical model performs best in predicting the relationship between user population growth and revenue growth for a particular company over different time periods.

Chapter 6. Conclusion

The theoretical models in this paper examine the impact of user population on the financial performance of online networking firms, which directly affect valuation of these companies. There are two forms of network effect in the operating process of online networks. The network effect will increase the rate of adoption of a particular network among Internet users. The process is assumed to follow a logistic pattern. In addition, online networking firms serve two groups of customers: advertisers and users. This is a typical two-sided market in which behaviors of one group of customers have a positive or negative impact on the other. Our model is distinct because previous literature either looks at two-sided markets or the network effects during the growth of user population. Our model takes both network externalities into consideration, and analyzes the dynamics of online networking firms. Also, since online networks have only been thriving for a few years, my research is quite current and captures the changing social norms in people's lives as they use these networks.

Our theoretical models have argued several simple and intuitive results. First, the optimal price for an advertising slot on any particular network is directly proportional to the user population. At the same time, the optimal quantity of available advertising slots only relates to the number of firms that want their goods and services to be advertised, which is independent of the user population. These results lead to the conclusion that optimal revenue is directly proportional to user population. This gives important suggestions for online networking firms on their pricing strategy. According to the theory, they should not offer as many advertising slots as they wish and should not charge the highest rate possible.

Second, online networking firms will have higher valuation if their user population approaches the upper limit more rapidly and they have higher upper limits for user populations and larger initial user populations. Our theoretical model is able to analyze the relationship between different parameters of changes in user population and the online networking firms' valuation. By discounting the profits from future periods, we can see the current valuations of online networking firms are influenced by key parameters driving the user population.

Third, online networking firms would be able to generate varying revenues and profits from individual users based on different strategies. By adopting special advertising methods or operational strategies, online networking firms are able to increase the percentage of users who view the advertisement, remember it, and finally purchase the products. Also, different networks have different functions, which appeal to different user demographics. In addition, online networking firms can also boost their revenue by increasing the fixed profit that advertisers are getting from individual users. Thus, for online networks, the ability of monetizing their individual users is key for good financial performance.

Our empirical models test the first and last results presented by the theoretical model. First, the empirical models indicate that user population has a statistically and economically significant positive impact on both real revenue and net income for online networking firms. With the increasing user population size over time, online networking companies are able to increase their revenue and net income. When we compare different online networking companies, firms are able to have higher profit and revenue with larger networks. All these results are intuitively correct and in line with the prediction by theoretical models.

Second, for some firms, optimal revenue is directly proportional to user population when I am analyzing a particular firm over different time periods. The predicted model performs quite well when I analyze the relationship between the log of user population and log of revenue for a particular company. For some of the companies I included in the empirical tests, the results are consistent with my theoretical model that revenue has a proportional relationship with user population. At the same time, this also suggests that relationship between user population and real revenue would be more complicated than a linear one for certain companies in reality.

Third, different companies are able to generate different revenue and net income from individual users. This result is intuitively correct and consistent with my theoretical model because different companies can have different strategies and their ability to monetize their user population also varies because of the different functions of their networks as well as their philosophy about users. Thus, my empirical results strengthen the predictions and conclusions made in the theoretical part.

Those undertaking further theoretical research may wish to take variable costs into consideration. This would be a complicated but meaningful next step in order to develop a more detailed analysis of online network values. In addition, future research may choose to include other revenue channels within economic model and use more complicated profit distribution. One of the most obvious channels is subscription revenue. Then we would be able to analyze how companies' valuation and pricing strategies should change when online networks begin charging users to use the network. Moreover, monopoly assumption could also be eliminated when we analyze the valuation of online networking startups or when we take people's disutility of advertisements into consideration.

Finally, further empirical research may choose to estimate the impact of parameters affecting user population growth, which are expected to influence the financial performance for online networking firms. Due to the lack of public data, I was not able to get estimated value for key parameters in the assumed logistic models, let along conducting these tests. But with more online networking firms going public and current publicly traded online networking companies experiencing business cycles, more data will become available in the future, so the results of empirical tests will better reflect the true relationship between user population and companies' valuation based on their financial performances.

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Appendix A:

From (3) we know:

$$R = P_a Q_j = \sigma\alpha Q_C \, [Q_j - \frac{{Q_j}^2}{M}]$$

then,

$$MR = \sigma\alpha Q_C \, [1 - \frac{2Q_j}{M}] = 0$$

$${Q_j}^* = \frac{M}{2}$$

and,

$${P_a}^* = \frac{1}{2} \; \sigma\alpha Q_C$$

Appendix B:

PV bound calculation:

$$PV \ = \sum_{t=0}^{\infty} \frac{\pi^*}{e^{it}}$$

$$PV \ = \sum_{t=0}^{\infty} \frac{\frac{1}{4}\ \sigma\alpha Q_C M \ \ - \ FC}{e^{it}}$$

$$PV \ = \sum_{t=0}^{\infty} \frac{\sigma\alpha Q_C M}{4e^{it}} - \sum_{t=0}^{\infty} \frac{FC}{e^{it}}$$

$$\sum_{t=0}^{\infty} \frac{FC}{e^{it}}$$

converges, thus:

$$\sum_{t=0}^{\infty} \frac{FC}{e^{it}} = \frac{FC}{1 - e^{-i}}$$

Proof of convergence:

$$PV_R = \sum_{t=0}^{\infty} \frac{\sigma\alpha Q_C M}{4e^{it}} = \sum_{t=0}^{\infty} \frac{\sigma\alpha M K Q_0}{4e^{it}((K - Q_0)\ e^{-rt}\ + Q_0)}$$

$$PV_R = \sum_{t=0}^{\infty} \frac{\sigma\alpha M K Q_0}{4e^{(i-r)t}(K - Q_0)\ + 4e^{it}Q_0}$$

let $A \ = \ \sigma\alpha M K Q_0$

$B = \ 4(K - Q_0)$,

and $C = 4Q_0$

Thus,

$$PV_R = \sum_{t=0}^{\infty} \frac{A}{Be^{(i-r)t}\ + Ce^{it}}$$

We conduct a ratio test on PV_R

$$L = \lim_{t\to\infty}\left|\frac{a_{t+1}}{a_t}\right| = \lim_{t\to\infty}\left|\frac{A}{Be^{(i-r)(t+1)} + Ce^{i(t+1)}} * \frac{Be^{(i-r)t} + Ce^{it}}{A}\right|$$

$$= \lim_{t\to\infty}\left|\frac{Be^{(i-r)t} + Ce^{it}}{Be^{(i-r)t}e^{(i-r)} + Ce^{it}e^{i}}\right|$$

Since $e^{(i-r)}$ and e^{i} are positive, L is less than one when t approaches infinity. Thus, PV_R is absolutely converges.

Implementing the convergence result to examine parameter *r*:

$$f_n(r) = \sum_{t=0}^{n}\frac{A}{Be^{(i-r)t} + Ce^{it}}$$

$$PV_R = f(r) = \lim_{n\to\infty}\sum_{t=0}^{n}\frac{A}{Be^{(i-r)t} + Ce^{it}} = \sum_{t=0}^{\infty}\frac{A}{Be^{(i-r)t} + Ce^{it}}$$

$$f'(r) = \lim_{n\to\infty} f_n{}'(r)$$

Thus,

$$\frac{dPV_R}{dr} = \frac{d}{dr}\sum_{t=0}^{\infty}\frac{A}{Be^{(i-r)t} + Ce^{it}}$$

$$= \sum_{t=0}^{\infty}\frac{d}{dr}\frac{A}{Be^{(i-r)t} + Ce^{it}}$$

$$= \sum_{t=0}^{\infty}\frac{ABe^{(i-r)t}t}{(Be^{(i-r)t} + Ce^{it})^2} > 0$$