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The Modified Direct Analysis Method: an Extension of the Direct Analysis Method

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THE MODIFIED DIRECT ANALYSIS METHOD:
AN EXTENSION OF THE DIRECT ANALYSIS METHOD

By

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A Thesis

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May Thu Nwe Nwe

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ABSTRACT

The purpose of this research project is to study an innovative method for the stability assessment of structural steel systems, namely the *Modified Direct Analysis Method* (MDM). This method is intended to simplify an existing design method, the *Direct Analysis Method* (DM), by assuming a sophisticated second-order elastic structural analysis will be employed that can account for member and system instability, and thereby allow the design process to be reduced to confirming the capacity of member cross-sections. This last check can be easily completed by substituting an effective length of $KL = 0$ into existing member design equations. This simplification will be particularly useful for structural systems in which it is not clear how to define the member slenderness L/r when the laterally unbraced length L is not apparent, such as arches and the compression chord of an unbraced truss. To study the feasibility and accuracy of this new method, a set of 12 benchmark steel structural systems previously designed and analyzed by former Bucknell graduate student Jose Martinez-Garcia and a single column were modeled and analyzed using the nonlinear structural analysis software MASTAN2. A series of MATLAB-based programs were prepared by the author to provide the code checking requirements for investigating the MDM. By comparing MDM and DM results against the more advanced distributed plasticity analysis results, it is concluded that the stability of structural systems can be adequately assessed in most cases using MDM, and that MDM often appears to be a more accurate but less conservative method in assessing stability.

CHAPTER 1: INTRODUCTION

1.1.Thesis Statement

By employing a rigorous second-order elastic analysis that accounts for the destabilizing effects of imperfections and inelasticity, the stability of structural steel systems can be adequately assessed with only the need to check the cross section strength of members.

1.2. Structural System Stability

A structural system is considered stable when the load effects acting on each of its members are less than or equal to its strength to resist them. This basic concept of comparing demand to capacity is applied in designing structural members to make sure that each member has enough capacity to support its demand.

As for demand on a structural member, load effects from both applied axial forces and bending moments need to be considered. In a pure axial case, with no bending moment, the force being resisted by the member should be equal to or less than its axial strength required for stability. Similarly, in a pure bending case with no axial force, the bending moment resisted by the member should be equal to or less than its bending moment strength. However, in structural systems, both axial and bending load effects tend to be present in each member (beam-column), and thus it becomes necessary to understand how the interaction between these two load effects and their corresponding strengths impact the stability of the member.

The interaction between axial force and bending moment effects on a member follows the concept that one effect will reduce the member's ability to resist the other effect. In the absence of one load effect, the member would have its largest possible strength to resist the other load effect. The AISC (American Institute of Steel Construction) interaction equations to represent this concept were derived, following the process of determining axial strength in the presence of a given bending moment, or determining bending moment strength in the presence of a given axial load (Geschwindner, 2012, p. 256). The resulting interaction equations are specified in the AISC Specification 2010 (Eq. H1-1a and Eq. H1-1b) as follows:

For $\frac{P_u}{\emptyset P_n} \geq 0.2$,

$$\frac{P_u}{\emptyset P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\emptyset M_{nx}} + \frac{M_{uy}}{\emptyset M_{ny}}\right) \leq 1.0$$

For $\frac{P_u}{\emptyset P_n} < 0.2$,

$$\frac{P_u}{2\emptyset P_n} + \left(\frac{M_{ux}}{\emptyset M_{nx}} + \frac{M_{uy}}{\emptyset M_{ny}}\right) \leq 1.0 ,$$

where

P_u = applied axial load,

P_n = nominal axial strength,

M_u = applied bending moment,

M_n = nominal bending strength,

Φ = factor of safety for design according to Load and Resistance Factor Design (LRFD)

x = subscript related to major axis bending, and

y = subscript related to minor axis bending.

A structural member subjected to both axial load and bending moment is considered stable if its load effects and corresponding strengths satisfy the AISC interaction equation.

The demand components of the AISC interaction equation include the axial load effect P_u and bending moment effect M_u. At the given loading condition, these load effects in each structural member can be determined using structural analysis software, such as MASTAN2 (Ziemian & McGuire, 1999).

The capacity components of the AISC interaction equation include axial strength, P_n, and bending strength, M_n. P_n is calculated using equations Eq. D2-1 (tension) and Eq. E3-1 to Eq. E3-4 (compression) as specified in AISC Specification (2010). It is determined by accounting for both cross-section strength, P_y (yielding of cross-section), and member length strength, P_{cr} (elastic or inelastic buckling of member along its length). M_n is calculated using AISC equations Eq. F2-1 to Eq. F2-4, and also is determined considering both cross-section strength, M_p (plastic yielding of cross-section), and member length strength, M_{LTB} (elastic or inelastic lateral torsional buckling of member along its length). For the case studies used in this thesis, the structural systems are assumed fully braced out of plane, and thus the systems essentially become two-dimensional structures, and only in-plane strengths need to be considered. Therefore, in calculating P_n, only the in-plane P_{cr} will be considered. In calculating M_n, only M_p, will be considered.

The buckling strength of a structural member subjected to an axial force, P_{cr}, is determined by first finding the Euler buckling strength, P_e, when the member is assumed as a perfect column using the following equation (Ziemian, 2010, Eq. 3.1):

$$P_e = \frac{\pi^2 \mathrm{EI}}{L^2},$$

where

P_e = Euler buckling strength of a column,

E = elastic modulus of material,

I = moment of inertia of cross-section, and

L = actual length of the column.

The Euler buckling strength equation is derived assuming frictionless pinned end restraint conditions and buckling shape of a half sine wave (Euler curve) (Geschwindner, 2012, p. 114-116). To account for the actual end restraint conditions of the member, the length of the member used in this equation should be the length that makes up the Euler elastic curve when buckled, not the actual length of the member. The concept of effective length, KL, is then introduced to represent the length that makes up the Euler elastic curve when a member is buckled. The effective length can be visualized as the length between two inflection points when the member is buckled (Geschwindner, 2012, p. 118). It is achieved by multiplying the effective length factor K by the actual length L of the member. Therefore, the Euler buckling strength of a member is then determined using the following modified equation that takes into account of its effective length depending on its actual end restraint conditions (AISC Specification 2010, Eq. E3-4):

$$P_e = \frac{\pi^2 \mathrm{EI}}{(KL)^2}\,,$$

where

KL = member effective length,

K = effective length factor based on member end restraint conditions, and

L = member actual length.

This Euler buckling strength equation also makes the assumptions that the member is perfectly straight, and that the material behaves elastically. The actual buckling strength, P_{cr}, of the member can then be estimated from the Euler buckling strength, P_e, using the following equations that account for the effects of geometric imperfections and material inelasticity (AISC Specification 2010, Eq. E3-2 and Eq. E3-3):

For $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}}$

$$P_{cr} = \left[0.658^{\frac{P_y}{P_e}}\right] P_y$$

For $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$

$$P_{cr} = 0.877 P_e\,,$$

where

P_{cr} = actual buckling strength of member accounting for geometric imperfections and material inelasticity,

P_e = Euler buckling strength of member, assuming perfect member straightness and elastic material,

P_y= cross-section yield strength,

KL = member effective length,

K= effective length factor,

L = member actual length,

r= radius of gyration of member cross section

E = material elastic modulus, and

F_y = material yield strength.

It is important to emphasize the role of effective length factor K introduced in determining the buckling strength of the member, P_{cr}. The accuracy of determining effective length factor K based on end restraint conditions of a member will impact the axial strength calculation, and thus will ultimately impact the AISC interaction equation. To illustrate, a miscalculated, lower K value will result in estimated higher buckling strength of the member than it actually has. This will ultimately result in lower interaction equation value, and will tend to overestimate the strength of the member. Therefore, the accuracy of determining K based on given end restraint conditions is critical in assessing structural system stability.

1.3. Stability Analysis Methods

For stability assessment of a structural system, interaction equations are used to evaluate whether each member of the system has adequate strength to resist the estimated load effects. AISC recognizes two existing methods, including the effective length method (ELM) and the direct analysis method (DM), for evaluating structural stability by means of interaction equations.

1.3.1 Effective Length Method (ELM)

The effective length method (ELM) evaluates the stability of a structural system by means of interaction equations. The distinguishing characteristic of this method is the use of non-unity effective length factors K based on end restraint conditions.

Effective Length Factor K

As previously mentioned, the actual length L of a member is multiplied by the effective length factor K. The resulting effective length KL represents the length of the member that would make the Euler curve when buckled. This effective length KL, not the actual length, is used in calculating the buckling strength P_{cr} of a member because the derivation of buckling strength comes from the buckling strength of a perfect column which makes the Euler curve when buckled.

The determination of K for each member depends on its end restraint conditions because these conditions impact the length to form the Euler curve when buckled. The larger the end restraint, the shorter the portion of the member that would make the Euler curve. Similarly, the more flexible the end restraint, the longer the portion of the member that would make the Euler curve. The effective length factors for six idealized end restraint conditions are provided in Figure 1.

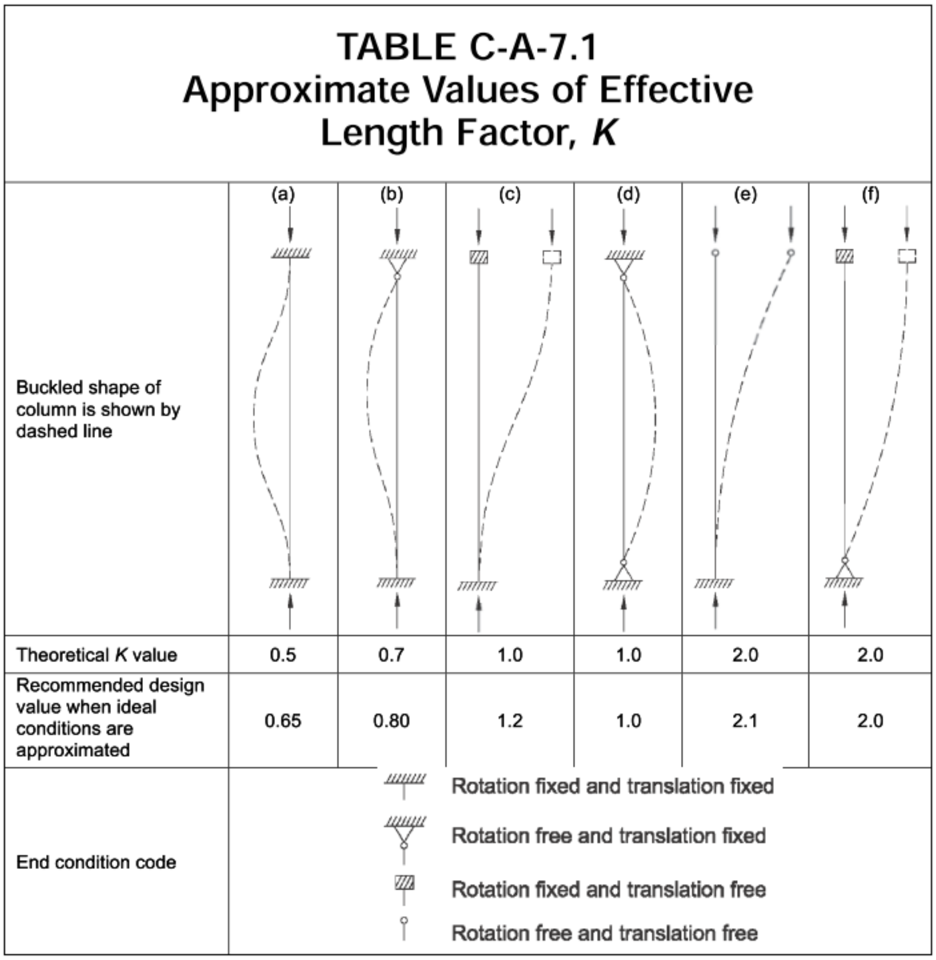


TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0

End condition code

Rotation fixed and translation fixed

Rotation free and translation fixed

Rotation fixed and translation free

Rotation free and translation free

Figure1. Approximate Values of Effective Length Factor, K, for Six Idealized End Restraint Conditions (AISC Commentary 2010, Table C-A-7.1)

For members with end restraint conditions that are not included in any of the basic cases in Figure 1, the alignment charts shown in Figures 2 and 3 for sidesway inhibited and sidesway uninhibited frames, respectively are used to determine their effective length factors.

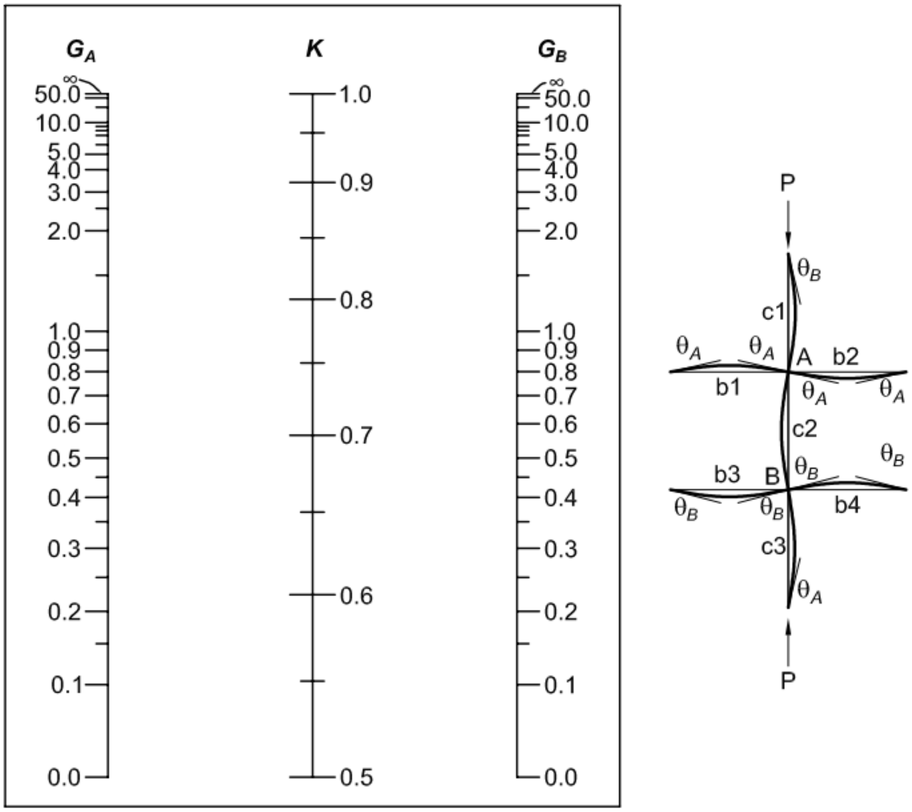


Figure2. Alignment Chart- Sidesway Inhibited (Braced Frame)
(AISC Commentary 2010, Figure C-A-7.1)

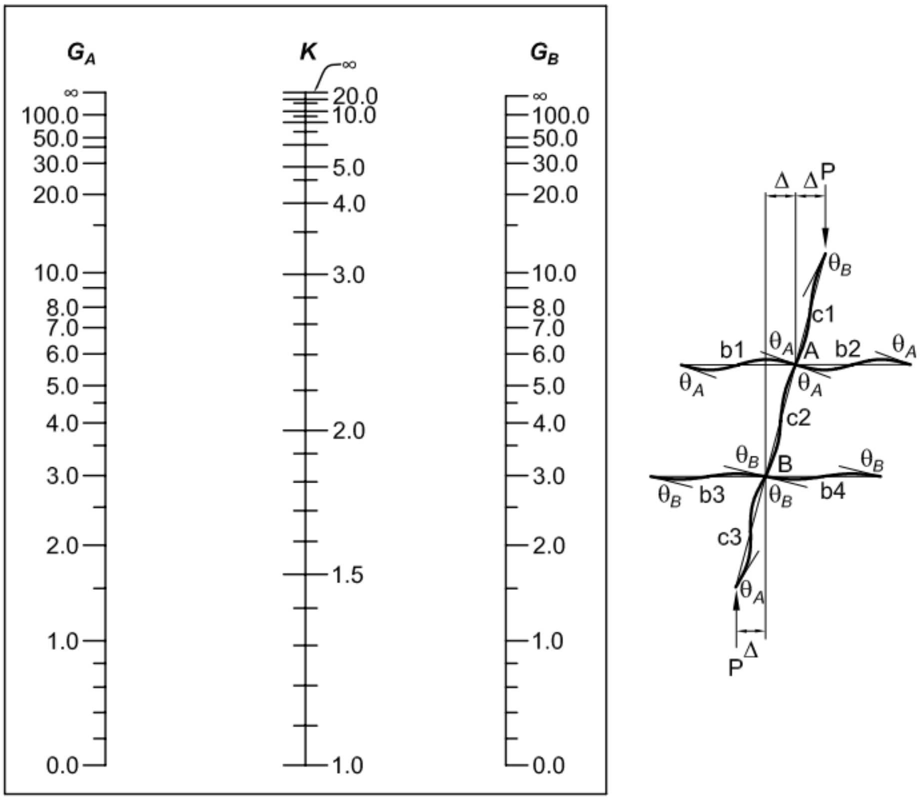


Figure 3. Alignment Chart- Sidesway Uninhibited (Moment Frame)
(AISC Commentary 2010, Figure C-A-7.2)

The alignment chart determines the end restraint conditions of a compression member based on its stiffness relative to the beam members connected at its ends. In other words, the measure of restraint at one end of a member is determined by the ratio of its stiffness to the stiffness of the other member connected at that end (Martinez-Garcia, 2002, p. 63). If the ratio is high, this means that the member is restrained by a less stiff member, and that the degree of restraint at that end is considered relatively small. Similarly, if the ratio is low, this means that the compression member is restrained by a stiffer member, and that the resulting degree of restraint at that end is considered relatively high. The ratio of member stiffness to that of its connecting member is determined by the following equation (AISC Commentary 2010, Eq. C-A-7.3):

$$G = \frac{\sum(E_c I_c / L_c)}{\sum(E_g I_g / L_g)},$$

where

E = elastic modulus

I = moment of inertia of member cross section,

L = member length,

c = subscript standing for column (member),

g = subscript standing for girder (connecting member), and

G = the relative stiffness of a member to its connecting member at one end.

Once the degrees of restraint at both ends of the member (G_A and G_B in the alignment charts) are obtained, the effective length factor K of the member can then be determined from the intersection point of the connecting line between the two G values in the corresponding alignment charts.

There are several assumptions made in developing alignment charts (AISC Commentary 2010, Appendix 7). It is assumed that behavior is purely elastic, and that members have constant cross sections. All joints are assumed rigid. Certain deformation patterns for braced frames and moment frames are *assumed*. For columns in braced frames, single curvature bending is assumed, and for columns in moment frames, double curvature bending is assumed. The stiffness parameter $L\sqrt{P/EI}$ is assumed equal for all columns in a story. All columns are assumed to buckle simultaneously, and no significant axial compression force in the girders is assumed. Since the alignment charts are developed under idealized conditions, and these conditions seldom exist in real structures, adjustments are often required to come up with accurate K factors, as explained in AISC Commentary 2010, Appendix 7.

Since the determination of K depends on several assumptions of idealized conditions and corresponding adjustments, there exists the potential for inaccuracies and errors in their calculation. As noted at the end of the previous section (Section 1.2), the accuracy of K factor impacts the axial capacity of the member, and ultimately impacts the assessment of its stability. Therefore, the potential for inaccuracies of K factors involved in the Effective Length Method makes it a less desirable method in assessing structural stability.

Analysis Procedure for ELM

The structural system and its loading conditions are modeled in structural analysis software, such as MASTAN2. A second-order elastic analysis is performed to obtain the load effects on each member (P_u and M_u). There is no reduction in material elasticity (i.e. E=1.0E). No initial imperfection or material inelasticity effects are included in the modeling.

The axial compressive capacities (P_n) are determined from axial strength equations Eq. E3-1 to Eq. E3-4, as specified in AISC Specification 2010, are based on the effective lengths, KL. The effective length factors K are determined as explained above, depending on member end restraint conditions. The moment capacities (M_n) are obtained from moment strength equations Eq. F2-1 to Eq. F2-4 as specified in AISC Specification 2010 Chapter F. (For the structural systems studied in this thesis, since the structures are assumed to be fully braced out of plane, lateral torsional buckling failure modes are not allowed, and only M_p is considered for moment strengths.)

The calculation of interaction equations using load effects and capacities determined are used to evaluate the stability of the structural system.

It is important to note that when employing the ELM design procedure, the effects of initial geometric imperfections, such as frame sway and member out-of-straightness, and material inelasticity are taken into account ONLY in capacity components (P_n and M_n) of the AISC interaction equation. Both AISC axial and moment capacity equations are developed taking into account of geometric imperfections and material inelasticity. These effects are not included in the analysis model and, hence, do not impact the calculation of P_u and M_u.

1.3.2. Direct Analysis Method (DM)

The direct analysis method (DM) is an alternative method to the effective length method in evaluating the stability of structural systems. The essential characteristic of this method is the absence of effective length factors (effective length factor of unity K =1.0 are assumed for all members).

Unit Effective Length Factor (K=1.0)

The concept of a unit effective length (assuming K=1.0 for all members) was developed to avoid the complexity and inaccuracy of determining K factors for every single member based on their end restraint conditions (as is the case in the previous ELM design procedure).

The consequences of assuming unit effective length factors (K=1.0) are offset as follows. Given that a unit effective length factor would tend to overestimate the axial capacity (P_n) of the members whose effective length factors are actually greater than 1.0, this potential increase in P_n is offset by intentionally increasing the moment load effect term (M_u), so as to achieve the interaction equation results that are quite similar to those when actual effective length factors are used. The increase in M_u is obtained by including initial imperfection and material inelasticity effects in the analysis model. Specifically, how these effects are included in the analysis model will be explained in the following sub-section, Analysis Procedure for DM.

In the absence of effective length factors (K=1.0), DM relies on an accurate second-order analysis that includes the destabilizing effects of initial imperfections and material inelasticity in the modeling, to adequately assess the stability of a structural system.

Analysis Procedure for DM

The structural system and its loading conditions are modeled in structural analysis software, such as MASTAN2. A rigorous second-order elastic analysis is performed to obtain load effects on each member (P_u and M_u). To offset the potential increase in P_n from unit effective length assumption, M_u will be intentionally increased. This will be achieved by including the destabilizing effects of initial imperfections and material inelasticity in the analysis model, either by direct modeling or through the use of equivalent notional loads (AISC Specification 2010).

The destabilizing effects of initial geometric imperfections can be modeled using two different approaches (AISC Specification 2010). In the first approach, namely the Direct Modeling Approach, the geometry of the structural model is includes an initial story out-of-plumbness of H/500 (H is the height measured from story level to story level). In the second approach, namely the Notional Load Approach, equivalent artificial lateral loads of 0.2% of the gravity load on each story level ($0.002Y_i$) are applied to the structure. Both approaches result in nearly the same internal load effects.

The destabilizing effects of material inelasticity can also be modeled using the two approaches (AISC Specification 2010). In both approaches, the elastic modulus of the steel is reduced by 20% (i.e., E= 0.8E). To further capture the stiffness reduction due to material inelasticity in Direct Modeling Approach, an additional stiffness reduction factor (τ_b) is applied to the flexural stiffness term (EI/L). In Notional Load Approach, equivalent notional loads of 0.1%

of gravity load values at each story are applied to the structure and replace the need for the use of a stiffness reduction factor (τ_b).

As for calculating the capacity components, the axial compressive strengths (P_n) are obtained from AISC Eq. E3-1 to Eq. E3-4, but now with the assumption of unit effective length factors (K=1.0) for all members. Using the same procedure as with ELM, the moment capacities (M_n) are obtained from the AISC moment strength equations Eq. F2-1 to Eq. F2-4. For the structural systems studied in this thesis, all the structures are assumed to be fully braced out of plane, and hence, lateral torsional buckling failure modes are suppressed and only the plastic moment strength M_p is considered.

The calculation of interaction equations using the load effects and capacities determined as are then used to evaluate the stability of the structural system.

It is important to note that, unlike ELM, DM takes into account the effects of initial geometric imperfections and material inelasticity in capacity components (P_n and M_n) as well as in demand components (P_u and M_u).

Comparison of Analysis Procedure for ELM and DM

The differences between the two methods, ELM and DM, can be summarized as follows. The first major difference is that ELM uses K factors based on member end restraint conditions, whereas DM uses unit effective length factors (K=1.0) for all members. Secondly, ELM applies no reduction in material modulus (E=1.0E) in the analysis model, whereas the DM requires a reduction of material modulus by 20% (E =0.8E). Again, the material modulus is reduced in the DM analysis models to intentionally increase the moment load effects (M_u). Thirdly, ELM considers the destabilizing effects of geometric imperfections and material inelasticity only in the capacity components (P_n and M_n), whereas the DM incorporates these effects in the analysis models as well to intentionally increase the moment load effects (M_u).

Of the two methods, the direct analysis method (DM) simplifies the stability assessment procedure by eliminating the need to determine K factors for all compression members, a process that can be inaccurate and prone to errors due to several idealized assumptions and adjustments involved. With the assumption of unit effective length factors, the direct analysis method relies on an accurate second-order analysis that takes into account of geometric imperfections and material inelasticity in the analysis models, to adequately assess the stability of structural systems.

1.3.3. Modified Direct Analysis Method (MDM)

Different from the two existing methods (Effective Length Method, ELM, and Direct Analysis Method, DM), a third alternative method for stability assessment will be proposed and studied for its feasibility in this thesis. Given that DM simplifies the stability assessment procedure by the assumption of a unit effective length factor (K=1.0) for all members regardless of end restraint conditions, the next logical question raised in this thesis is whether or not all member and system stability can be assessed by the analysis, and thereby permit the use of the cross-section capacity in computing P_n. In other words, allow for effective length factors equal to zero (K=0) in all design calculations that compute and employ P_n. This modification would imply the analysis would assess the stability of a structural member and thereby permitting the need to only check cross-section strength, which for a compact section would be the axial yield strength (P_y). This modification would further simplify the existing direct analysis method, and would be especially useful for assessing the stability of members in which the unbraced length is difficult to define, such as an arch or the top-chord of an unbraced truss. Therefore, Modified

Direct Analysis Method (MDM) with only checking member cross-section strengths will be studied in this thesis to determine whether this new method can adequately assess stability.

Checking Only Member Cross-section Axial Strength (P_y)

The concept of checking only member cross-section axial strength is developed from the logical quest to further simplify the unit effective length factor (K=1.0) assumption in DM. With analysis that can capture both frame and member instabilities, one would need to check only the cross-section yielding failure of the member ($P_y = A_gF_y$).

Of course, the consequence of checking only the member cross-section axial strength will be the potential overestimation of the axial capacity of a member (P_n). This is because design equations will ignore the possibility of a buckling failure mode, and rely exclusively on the analysis. To adequately access the stability of a member using this assumption, this overestimation in the member axial capacity will need to be offset in the AISC interaction equation. In this thesis, the proposed methods to offset the axial capacity increase will be the same methods as in DM; the overestimation of the axial capacity will be offset by using a rigorous second-order elastic analysis and intentionally increase in moment demands (M_u) by including the destabilizing effects of geometric imperfections and material inelasticity in the analysis model.

With only the need to check the member cross-section axial strength, MDM intends to rely on a rigorous second-order elastic analysis that includes the destabilizing effects of imperfections and inelasticity to adequately access the stability of a structural system.

Proposed Analysis Procedure for MDM

The analysis procedure for MDM is proposed in this thesis as follows. The structural system and its loading conditions are modeled with nonlinear structural analysis software, such as MASTAN2. A rigorous second-order elastic analysis is performed to obtain load effects on each member (P_u and M_u). To offset the potential increase in P_n by checking only the member cross-section strength, M_u will be intentionally increased. As in DM, this will be achieved by including the destabilizing effects of initial imperfections and material inelasticity in the analysis model, either by direct modeling or by the use of notional loads.

The destabilizing effects of initial geometric imperfections will be modeled using the same two approaches defined previously for DM. In the first approach, namely Direct Modeling Approach, the geometry of the structure includes an initial out-of-plumbness of H/500 (H is the height measured from the story level to story level). In the second approach, namely Notional Load Approach, notional loads of 0.2% of the gravity load on each story level ($0.002Y_i$) are applied to the structure. Both approaches are expected to produce similar internal load effects.

The destabilizing effects of material inelasticity will also be modeled using the two approaches defined for the Direct Analysis Method. In both approaches, the elastic modulus will be reduced by 20% (i.e., E= 0.8E). In Direct Modeling Approach, an additional stiffness reduction factor (τ_b) will be applied to the flexural stiffness term (EI/L). In Notional Load Approach, equivalent notional lateral loads of 0.1% of the gravity load will be included.

As for calculating the capacity components, P_n will be taken as equal to the member cross-section axial yield strength ($P_y= A_gF_y$). The moment capacities (M_n) will be obtained from the same AISC moment strength equations Eq. F2-1 to Eq. F2-4 used in DM and ELM. Again, for the structural systems studied in this thesis, the system is assumed fully braced out of plane resulting in all flexural strengths equaling the plastic moment capacity M_p.

The calculation of interaction equations using load effects and capacities determined as will continue to be used to evaluate the stability of the structural system.

It is important to note that, similar to DM, MDM will take into account the effects of initial geometric imperfections and material inelasticity in capacity components (P_n and M_n) as well as in demand components (P_u and M_u).

Comparison of Analysis Procedures for DM and MDM

The only difference between the analysis procedures of the two methods, DM and MDM, will be the calculation of P_n. The former considers both P_{cr} (with K=1.0) and P_y in calculating P_n, whereas the latter will only consider P_y in calculating P_n.

1.4. Purpose and Objectives

The primary purpose of this research project is to determine whether the newly-proposed method, Modified Direct Analysis Method (MDM), can be used to adequately assess the stability of steel structural systems. This will be accomplished by studying a set of 12 benchmark frames.

The specific objectives of this research project include:

1. To compare the existing Direct Analysis Method (DM) and the newly proposed Modified Direct Analysis Method (MDM) by performing stability assessments using both methods on a set of 12 benchmark structural systems and a single column
2. To compare the results of the two methods against more advanced non-linear analysis results

3. To ultimately determine whether MDM with the proposed modifications (only checking cross-section axial strengths) will be sufficient to assess the stability of structural steel systems

1.5. Thesis Overview

The chapters in this thesis will be outlined as follows.

Chapter 1 first claims the thesis statement, introduces the concept of structural system stability, explains the background of stability analysis methods, and then defines the purpose and objectives of this research study.

Chapter 2 will first provide background information of case studies used in this thesis, continue by explaining the detailed methods and steps used to perform the stability assessment of the case studies using DM and MDM, and then define how the adequacy and the accuracy of each method in assessing the stability of these case studies will be determined in this thesis.

Chapter 3 will discuss results from the case studies, in regards to whether MDM will adequately assess structural stability compared to the advanced analysis and the DM procedure.

Chapter 4 will summarize overall conclusions from the case studies.

Chapter 5 will provide a summary of this research, will emphasize the overall conclusions from this study, and then will make recommendations for further research.

CHAPTER 2: CASE STUDIES AND METHODS

This section will first explain the background of case studies, describe the stability assessment methods used in this thesis, and then define how the adequacy and the accuracy of each method in assessing the stability of structural systems will be determined in this thesis.

2.1. Background of Case Studies

The structural systems that will be used in this thesis are taken from a study on the feasibility of Direct Analysis Method conducted by Jose Martinez-Garcia and his research advisor, Dr. Ronald Ziemian (Professor at Bucknell University) in 2002. Their study involved a set of 11 benchmark structural systems that were designed to satisfy AISC LRFD Specification strength and serviceability requirements (Martinez-Garcia, 2002, p. 29). The first six structural systems were taken from other research reports, and the last two structural systems (one with four variants) were conceived especially for the research on the feasibility of Direct Analysis Method and designed accordingly (Martinez-Garcia, 2002, p. 33).

These same structural systems will be used in this study, with the addition of one structural system (Structural System 1b) as well as one single column (Column Study), and with modifications to one structural system (Structural System 8). The rationales for these additions and modifications are explained later in corresponding structural system descriptions.

2.1.1. Design of Structural Systems

The structural systems involved in this thesis study were designed according to the following general design procedure (Martinez-Garcia, 2002, p. 29-35).

The geometry and initial loading conditions of a structural system are determined based on representative conditions for a certain type of structure. Preliminary sections are chosen for each member in the structural system.

The preliminary model is analyzed using one of the following three different approaches: elastic analysis with a first-hinge limit point, elastic-perfectly plastic hinge analysis, or inelastic distributed plasticity analysis.

If the first-hinge approach is used, a second-order elastic analysis is performed on the preliminary model, and sections are considered satisfactory if the first hinge forms at a load ratio greater than 1.0 (Martinez-Garcia, 2002, p. 33). Then all strength requirements (in-plane, out-of-plane, local buckling, etc) are checked using the equations provided in the AISC LRFD specifications.

If the second or third approach is used, an elastic-perfectly plastic hinge analysis or inelastic distributed plasticity analysis is performed on the preliminary model respectively, and sections are considered satisfactory if the ultimate load is reached at a load ratio greater than 1.0 (p. 33). Behavioral effects due to inelasticity should be captured by the analysis, and no AISC equations are needed.

In addition to satisfying strength requirements, the structural system is checked to satisfy the following serviceability requirements (Martinez-Garcia, 2002, p. 34):

1. total lateral drift and interstory drift due to the unfactored wind load are limited to H/250, where H is either the height of the structure or the story height. (Code of Standard Practice for Steel Buildings and Bridges)

2. beam deflections under unfactored live loads are limited to L/360, where L is the beam span. (Code of Standard Practice for Steel Buildings and Bridges)

3. plastic hinges are prohibited from forming under service loads.

In addition to strength and serviceability requirements, other design considerations were included in designing the structural systems. These considerations include the feasibility and cost of connections based on member sizes, the economy of member sizes, and their local availability. (Martinez-Garcia, 2002, p. 33-35)

2.1.2. Load Calibration

The original load magnitudes applied to each structural system studied were determined based on representative values, and member sizes were then chosen to satisfy stability requirements under these load conditions.

However, to perform benchmark studies on the accuracy of different methods for stability assessment, failure loads should be applied to the structural system so that it is easy to see whether the method predicts failure at a lower or higher load than the applied load.

To calibrate the original loads to failure loads, an advanced spread-of-plasticity analysis by structural analysis software NIFA (Clarke & Zablotskii, 1995) was performed on each structural system to obtain the ultimate load ratio (Martinez-Garcia, 2002, p. 54). The second-order inelasticity analysis with the reduced elastic modulus of 0.9E, and reduced material yield strength of 0.9 F_y was used. The original loads were then scaled by the ultimate load ratio to obtain the ultimate failure loads. In doing so, the applied load ratio at the strength limit states of the frames will always equal 1.0. These calibrated ultimate failure loads were applied to the structural systems in the benchmark study of Direct Analysis Method against Effective Length Method, and other advanced analyses by Martinez-Garcia. This benchmark study of Modified Direct Analysis against the prior methods will also be used calibrated failure loads in the structural systems.

Because all given factored loads are purposely defined so that an advanced second-order inelastic analysis with 0.9E, 0.9Fy and Δ_0 = H/500 will result in system limit at an applied load ALR = 1.0 (i.e. satisfy AISC's Appendix 1 – Design by Inelastic Analysis), an adequate stability assessment of these structural systems should provide interaction equation values close to 1.0 at the given applied loads. Therefore, the adequacy of different methods in accessing stability of these structural systems will be determined based on their interaction values. If the maximum interaction equation provides a value larger than 1.0, then the method can be considered conservative when compared to the design procedure using inelastic analysis. On the other hand, a maximum interaction equation value less than unity indicates that the design can resist additional load, thereby making the design procedure unconservtiave.

2.1.3. Brief Description of Structural Systems

Structural System 1a – Unsymmetrical Frame

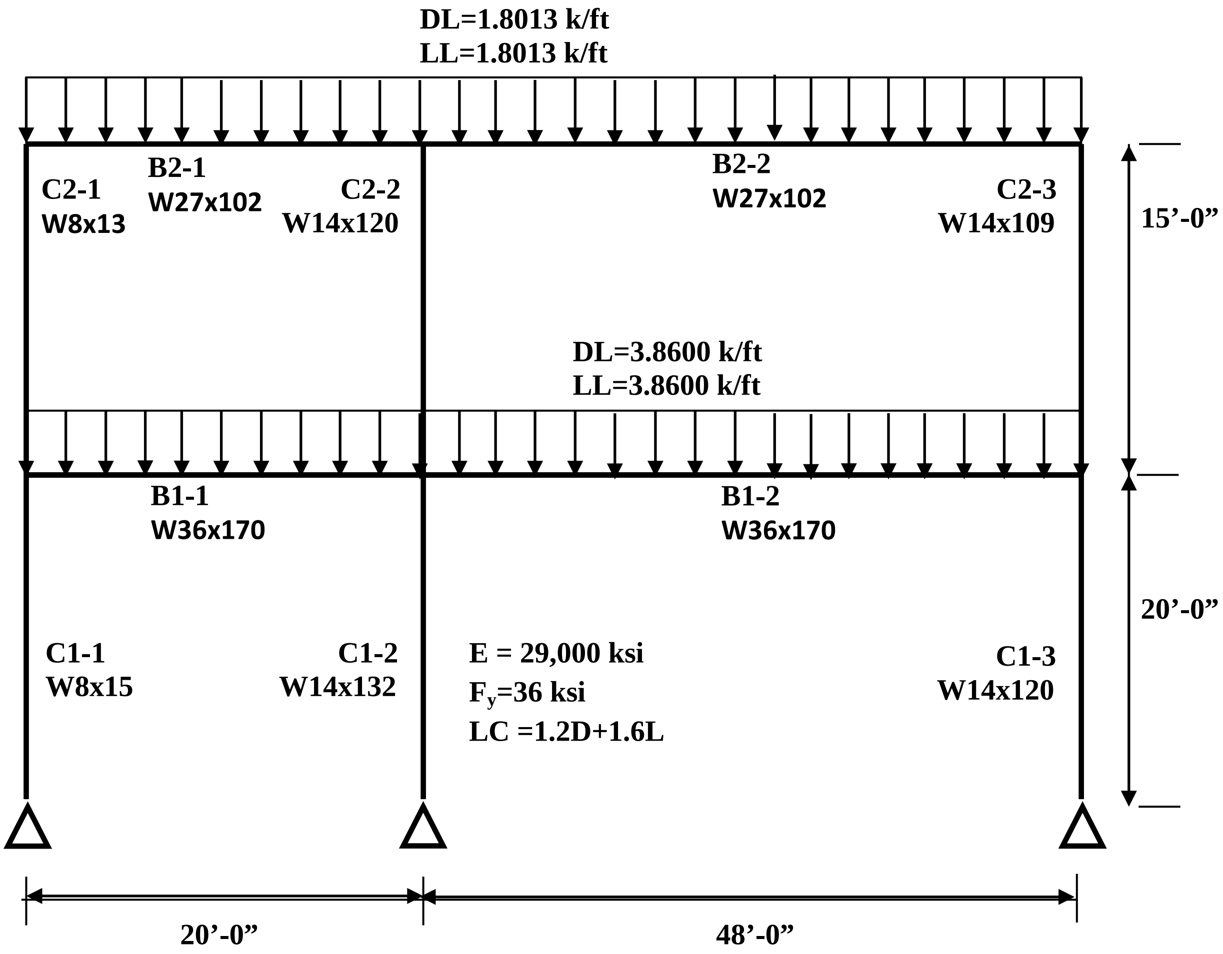


Figure4. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 1a
(Credit: Jose Martinez-Garcia, 2002)

This structural system is a representative of a two-story industrial frame. Specifically, in its original studies by Iffland and Birnstiel (1982) and Ziemian et al. (1992), its geometry and high ratio of gravity to lateral load ratio were intended to represent typical low-rise industrial buildings (Martinez-Garcia, 2002, p. 86).

In the prior study on this structural system by Martinez-Garcia for assessing the feasibility of Direct Analysis Method, four possible cases with two load combinations (gravity and lateral load combinations) and two initial imperfection and wind directions (to the left and to the right) were initially considered to determine one controlling case (Martinez-Garcia, 2002, p. 88). In this thesis, however, only the controlling case as determined in Martinez-Garcia's thesis (gravity load combination with initial out-of-plumb imperfection to the left) will be studied.

There are a few noteworthy characteristics of the system:

The left-most W8 columns of the frame were designed smaller than the other W14 columns to act as leaning columns (Martinez-Garcia, 2002, p. 86).

The comparatively large gravity load in this structural system was intended to produce significant second-order effects in the presence of a small lateral initial imperfection. The presence of the leaning columns was also intended to accentuate this second order effect (Martinez-Garcia, 2002, p. 88).

All sections are oriented for bending about their major axis, and the structure is assumed to be fully braced out of plane.

Structural System 1b – Unsymmetrical Frame

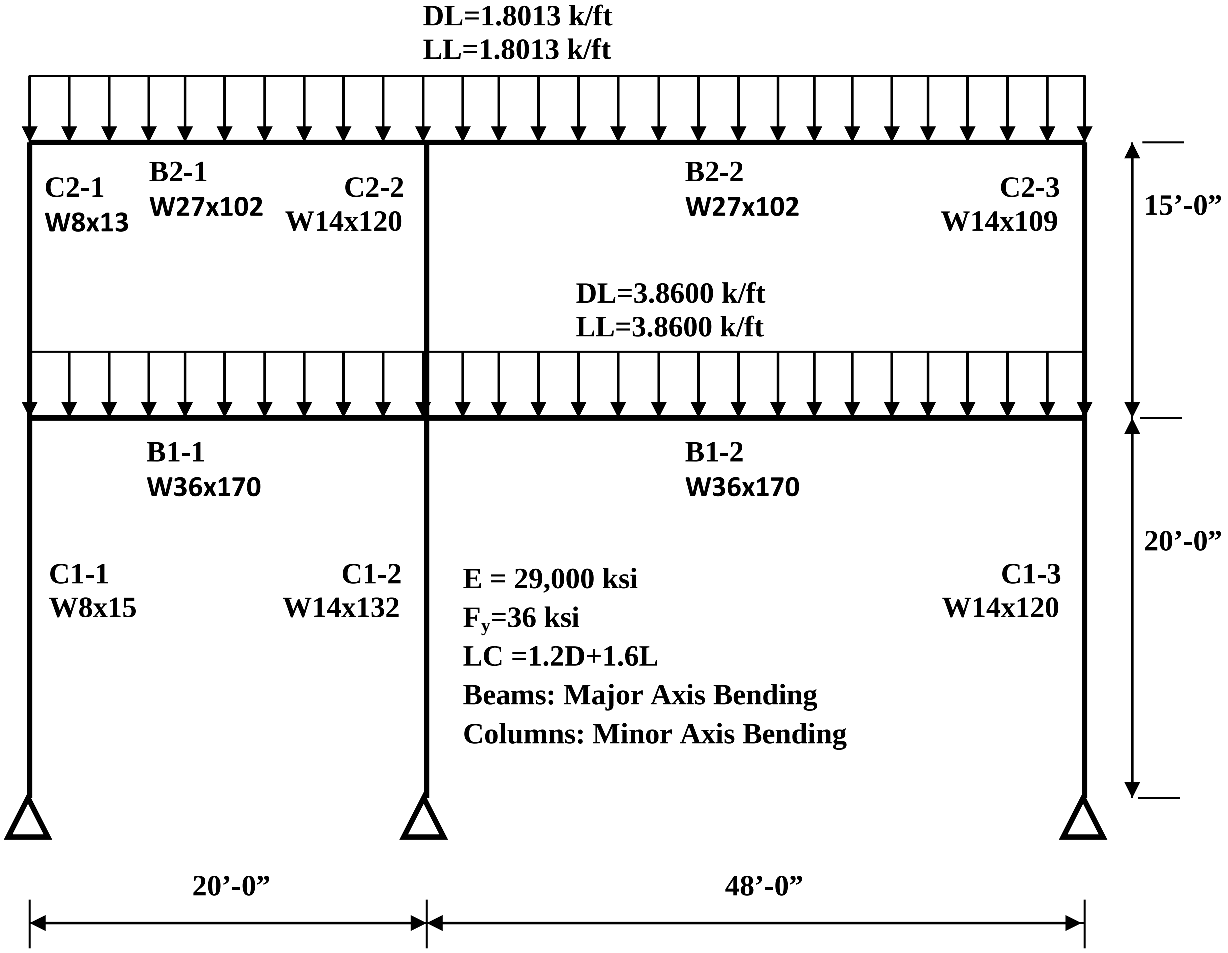


Figure 5. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 1b (Credit: Jose Martinez-Garcia, 2002)

This structural system is the same frame as the previous structural system (Structural System 1a), except that in this system the columns will be oriented for bending about their minor axis. The purpose of studying this structural system is to observe whether the Modified Direct Analysis Method is adequate to assess the stability of structural system with minor axis column orientation.

Similar to Structural System 1a, only the controlling case as determined in Martinez-Garcia's thesis (gravity load combination with initial out-of-plumb imperfection to the left) will be studied in this thesis.

Structural System 2 – Industrial Frame

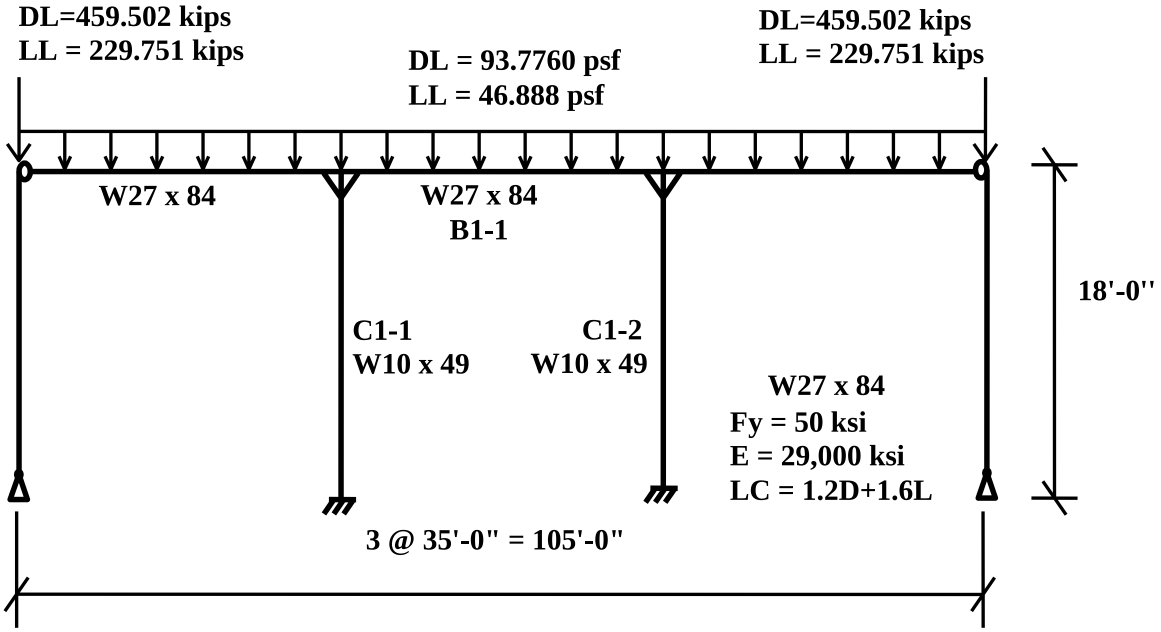


Figure 6. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 2 (Credit: Jose Martinez-Garcia, 2002)

This structural system is a representative model of a multi-bay single-story industrial frame. This model was proposed by AISC TC10 in the early stages of the development of the Direct Analysis Method to compare it against Effective Length Method and more advanced analyses (Martinez-Garcia, 2002, p. 105). In the prior study by Martinez-Garcia for assessing the feasibility of Direct Analysis Method, the model was simplified from eleven-bay to three-bay, with two exterior leaning columns each representing the equivalent of five leaning columns (Martinez-Garcia, 2002, p. 105-106).

In the study by Martinez-Garcia, two load combinations (gravity and lateral load combinations) were initially considered to determine one controlling case (Martinez-Garcia, 2002, p. 107). In this thesis, however, only the controlling case as determined in Martinez-Garcia's thesis (gravity load combination) will be studied.

There are a few noteworthy characteristics of the system:

All the exterior columns are pinned at both ends to act as leaning columns (Martinez-Garcia, 2002, p. 106). Therefore, only a frame comprised of the two central columns and a 3-span continuous beam resist the lateral loads applied to the system.

The comparatively large gravity load in this structural system was intended to produce significant second-order effects in the presence of a small lateral initial imperfection. The presence of leaning columns was also intended to accentuate this second order effect (Martinez-Garcia, 2002, p. 106).

Because the five columns have been represented by one exterior column, the distributed gravity load along the omitted four bays is included in the model as a concentrated load on the given exterior column (Martinez-Garcia, 2002, p. 106). Moreover, to represent the equivalent axial stiffness of five columns, the exterior columns are made five times more rigid by increasing their modulus of elasticity by five times (Martinez-Garcia, 2002, p.106).

All sections are oriented for bending about their major axis, and the structure is assumed to be fully braced out of plane.

Structural System 3 – Grain Storage Bin

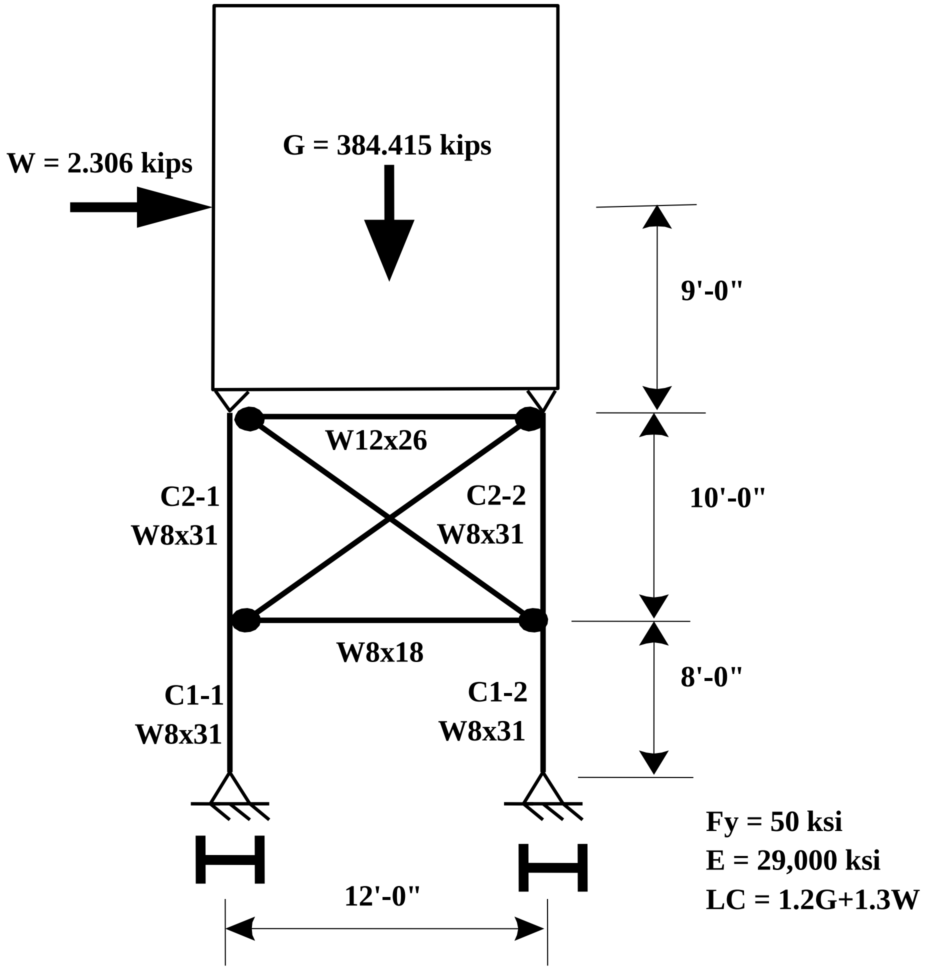


Figure 7. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 3 (Credit: Jose Martinez-Garcia, 2002)

This structural system is a representative model of an elevated structure where stability effects are accentuated by the position of most of the weight at an elevation the ground (Martinez-Garcia, 2002, p. 121). This model was also proposed by AISC TC10, specifically LeRoy Lutz, an engineer at Computerized Structural Design, in the early stages of the development of the Direct Analysis Method to compare this design method with the Effective Length Method and more advanced analyses (Martinez-Garcia, 2002, p. 121).

In the prior study on this structural system by Martinez-Garcia for assessing the feasibility of Direct Analysis Method, two load combinations (gravity and lateral load combinations) were initially considered to determine one controlling case (Martinez-Garcia, 2002, p. 123). In this thesis, however, only the controlling case as determined in Martinez-Garcia's thesis (lateral load combination) will be studied.

There are a few noteworthy characteristics of the system:

The columns are braced in-plane in their upper section as seen in the figure. The function of this bracing is mainly to provide stability against lateral loads (Martinez-Garcia, 2002, p. 122). The W4x13, lightest W section included in LRFD Manual, is used for the bracing. In modeling the bracing, only the bracing in tension is included in the analyses, and the bracing in compression is omitted, given that its buckling load is very low (Martinez-Garcia, 2002, p. 122).

The cross-beams and the bracing are pin-connected to the column so that they cannot resist any moment (Martinez-Garcia, 2002, p. 122).

The comparatively large gravity load in this structural system is intended to produce significant second-order effects in the presence of a small lateral initial imperfection or the deflection caused by a small lateral wind load (Martinez-Garcia, 2002, p. 122).

In modeling wind and gravity loads, these applied loads are converted into equivalent horizontal and vertical forces at the upper ends of the columns (Martinez-Garcia, 2002, p. 122).

All sections are oriented for bending about their major axis, and the structure is assumed fully braced out of plane.

Structural System 4 – Multi-story Frame

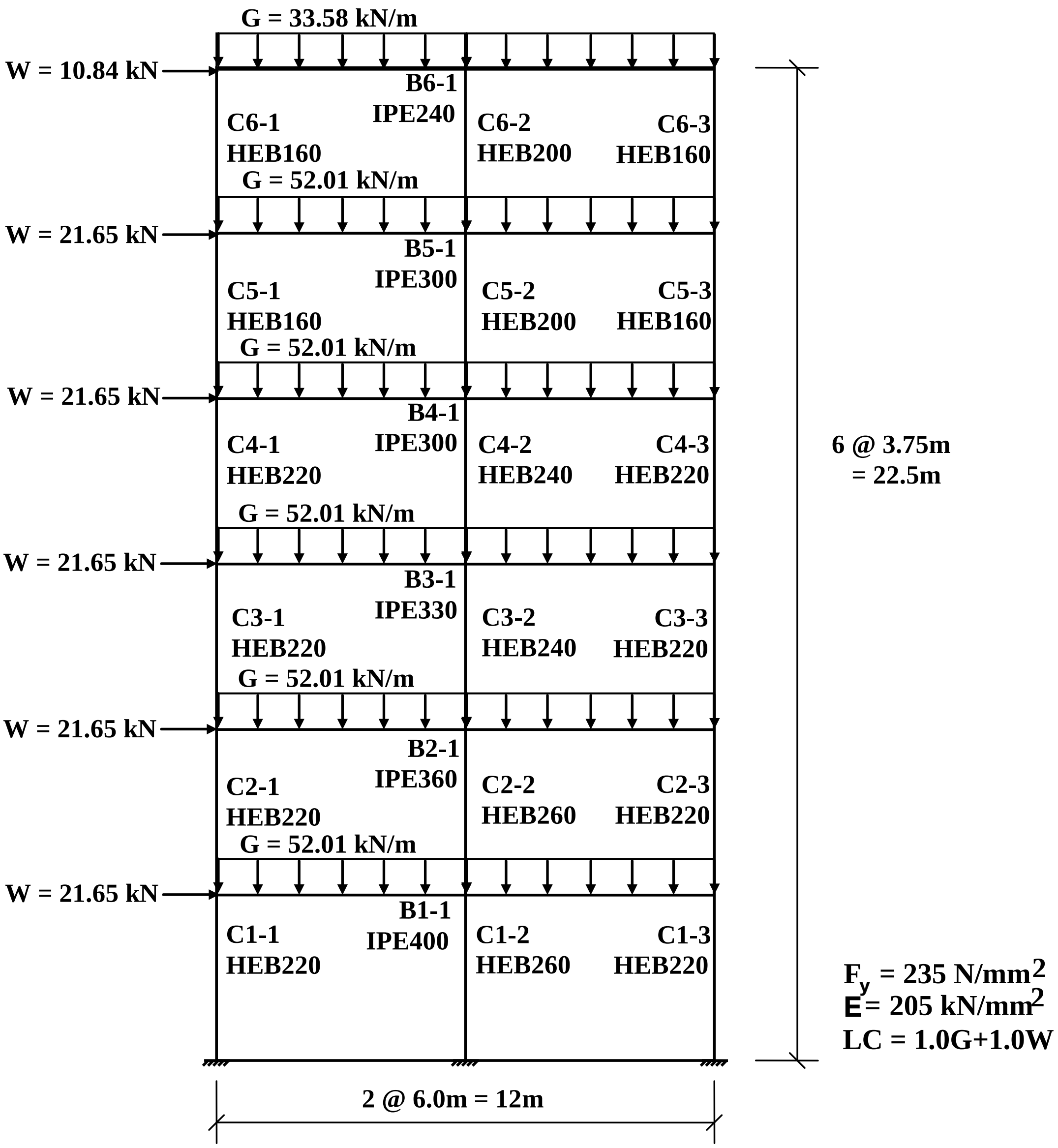


Figure 8. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 4 (Credit: Jose Martinez-Garcia, 2002)

This structural system is a representative model of a multi-story residential or office building. This frame was originally proposed by Vogel (Vogel U. Calibrating Frames. Berlin: Stahlbau, 1985, 1–7, 10) and AISC TC 10 investigated it in the early stages of the development of Direct Analysis Method to compare it against Effective Length Method and more advanced analyses (Martinez-Garcia, 2002, p. 140-141).

In the prior study on this structural system by Martinez-Garcia for assessing the feasibility of Direct Analysis Method, since both gravity and wind loads had already been factored in prior to his study, only one load combination (LC1 = 1.0G+1.0W) was investigated (Martinez-Garcia, 2002, p. 141). In this thesis, the same load combination will be studied as the controlling case.

There are a few noteworthy characteristics of the system:

All the sections used are European sections: HEB sections (European standard wide-flange sections) are used for the columns, and IPE sections (European standard I-shaped sections) are used for the beams.

All connections are assumed rigid, and the bases of the first story columns are fixed to the foundation.

All sections are oriented for bending about their major axis, and the structure is assumed fully braced out of plane.

Structural System 5 – Gabled Frame

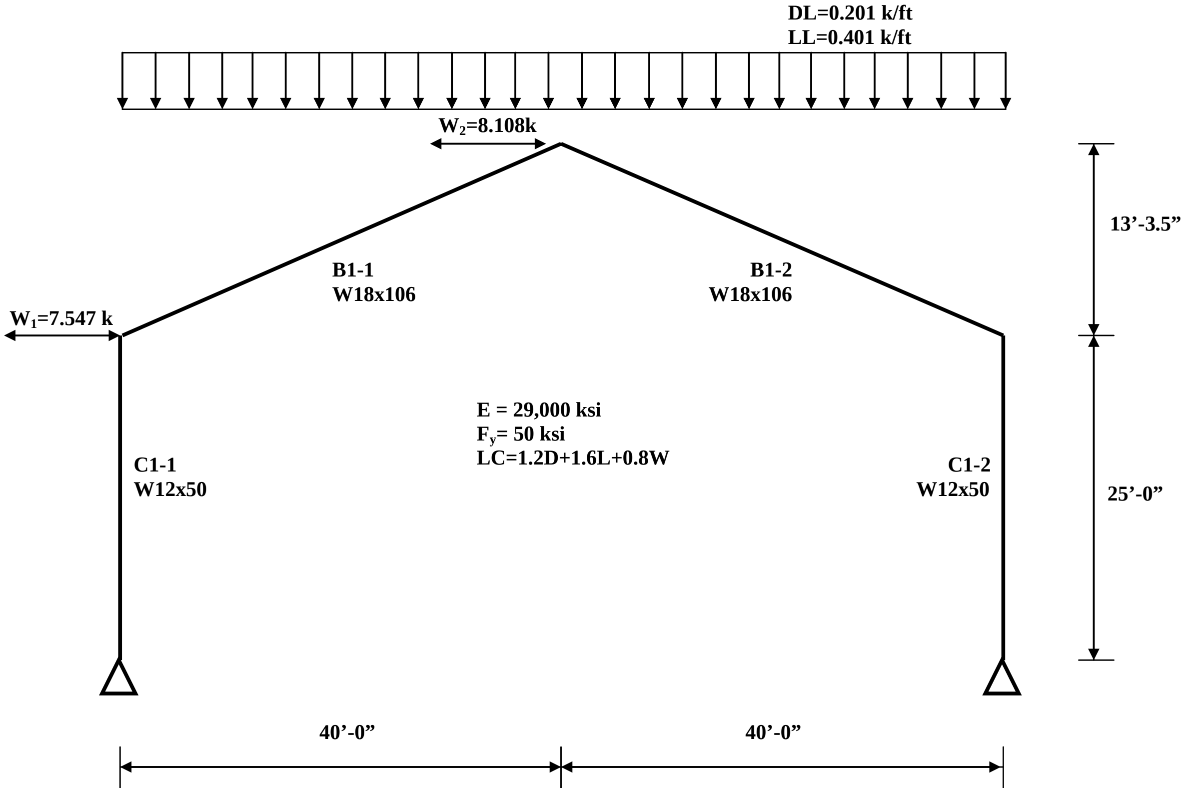


Figure 9. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 5 (Credit: Jose Martinez-Garcia, 2002)

This structural system is a representative model of an industrial gabled frame. This frame was originally taken from *Description of Frames*, Section 3.1, Internal Report, by Murray, T.M. (2001), Virginia Polytechnic Institute, Blacksburg, Virginia. In the study on this frame by Martinez-Garcia for the feasibility assessment of Direct Analysis Method, the loads were modified to act directly downward at all points (Martinez-Garcia, 2002, p. 153).

In the study by Martinez-Garcia, only one load combination (LC1 = 1.2D+1.6L+0.8W) was considered (Martinez-Garcia, 2002, p. 154). In this thesis, the same load combination will be studied as the controlling case.

There are a few noteworthy characteristics of the system:

The structure is statically indeterminate to the second degree.

Wind load is applied at the edge and at the ridge of the roof, with a higher value at the ridge because the effect of wind increases with elevation.

All sections are oriented for bending about their major axis, and the structure is assumed fully braced out of plane.

Structural System 6 – Two-bay Frame with Irregular Geometry

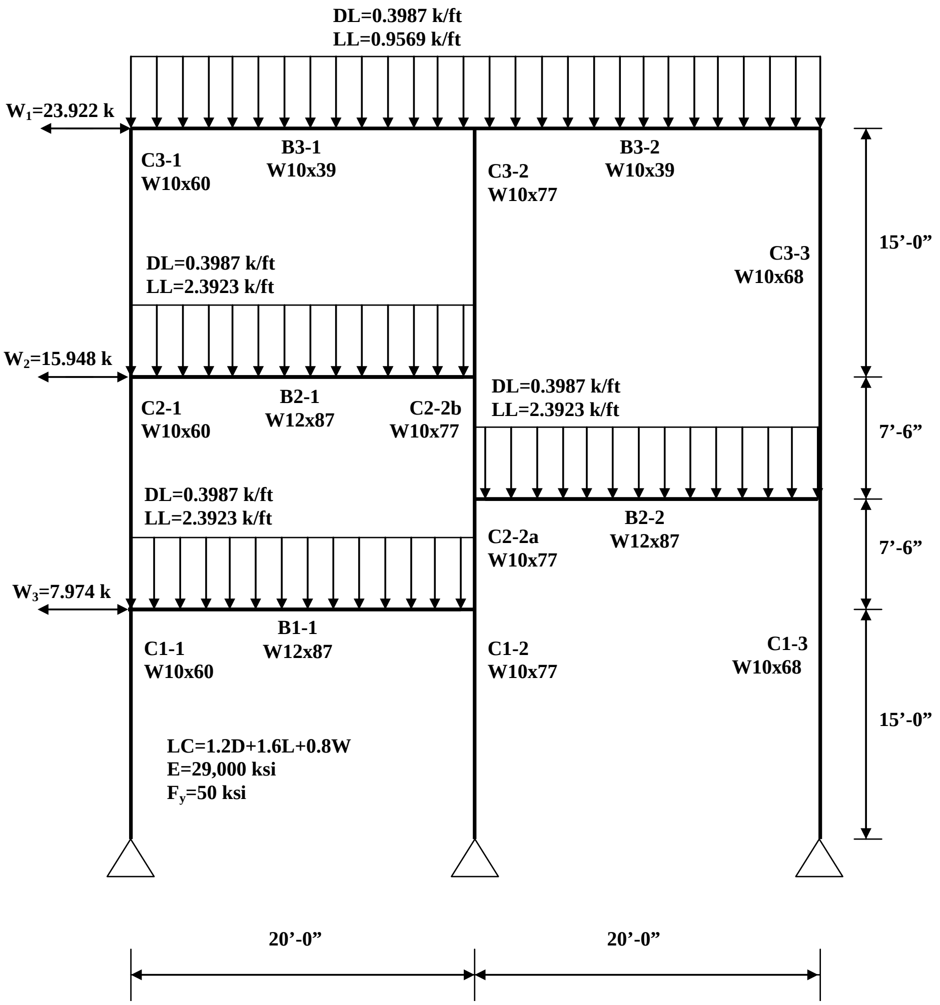


Figure10. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 6
(Credit: Jose Martinez-Garcia, 2002)

This structural system is a representative model of an irregular two-bay frame, where circumstances impose certain requirements about its geometry. This frame was taken from *Description of Frames*, Section 3.1, Internal Report, by Murray, T.M. (2001), Virginia Polytechnic Institute, Blacksburg, Virginia (Martinez-Garcia, 2002, p. 167).

In the prior study on this structural system by Martinez-Garcia for assessing the feasibility of Direct Analysis Method, only one load combination (LC1 = 1.2D+1.6L+0.8W) with both the wind load and initial imperfection acting to the right was considered (Martinez-Garcia, 2002, p.168). Although the asymmetry of the frame suggests the study of initial imperfection to both sides, the original problem defined the wind load to act to the right, and provided only the point load values. Therefore, it was assumed that the case for the initial imperfection and the wind load acting to the right will fail the frame at a lower ultimate load ratio than the case for the left (Martinez-Garcia, 2002, p. 168). In this thesis, the same load combination with the initial imperfection and wind load acting to the right will be studied as the controlling case.

There only noteworthy characteristics of the system is that all sections are oriented for bending about their major axis, and the structure is assumed fully braced out of plane.

Structural System 7a, 7b, 7c and 7d – Two-bay Frame with Unequal Heights

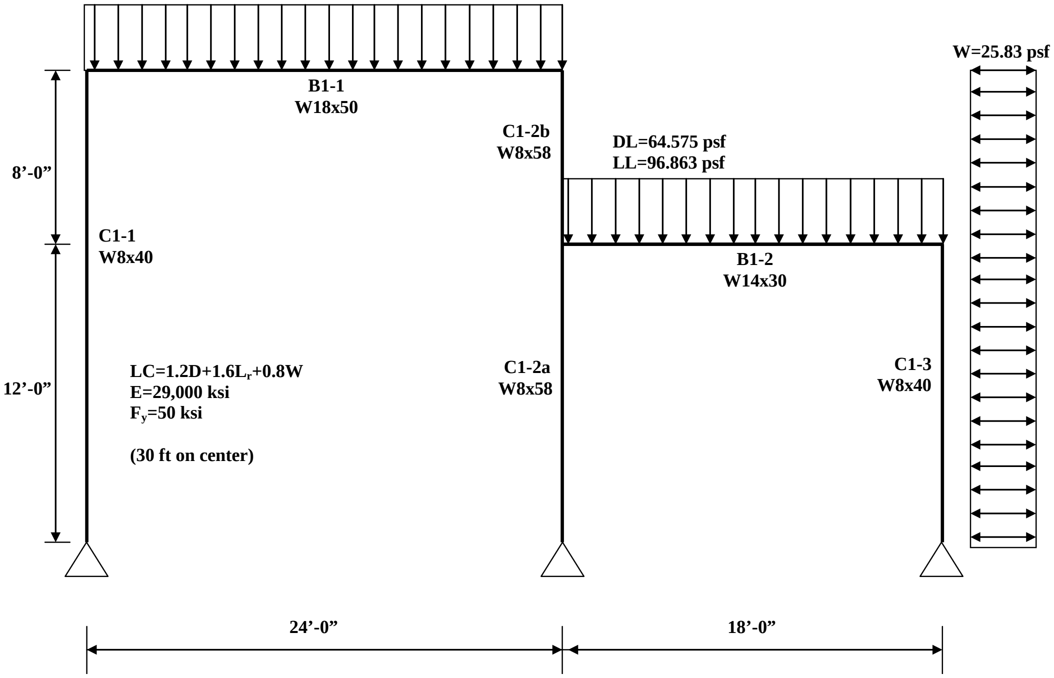


Figure11. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 7a
(Credit: Jose Martinez-Garcia, 2002)

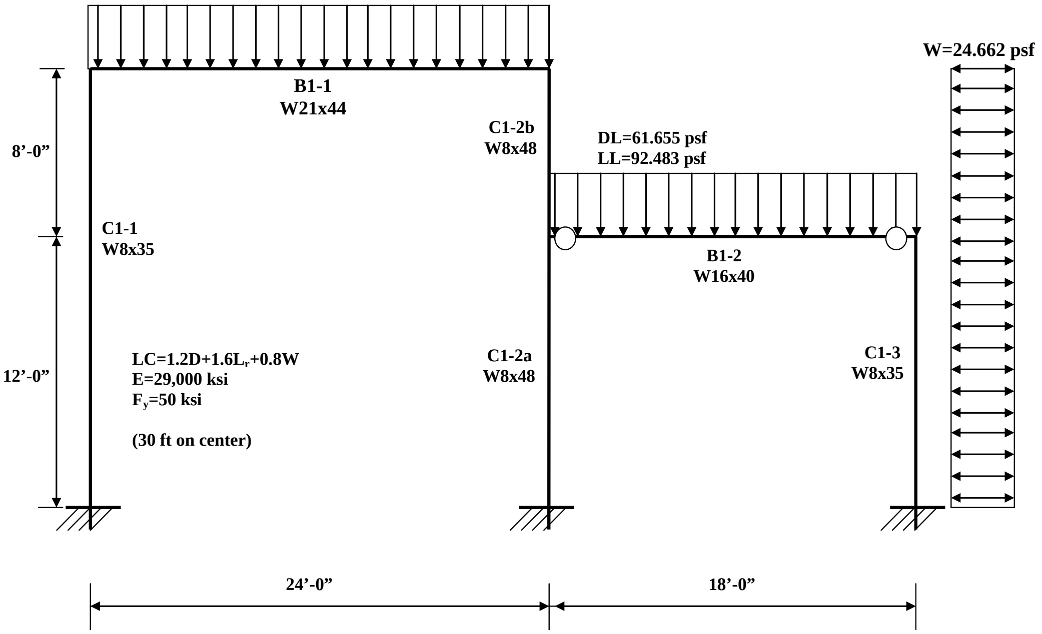


Figure12. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 7b (Credit: Jose Martinez-Garcia, 2002)

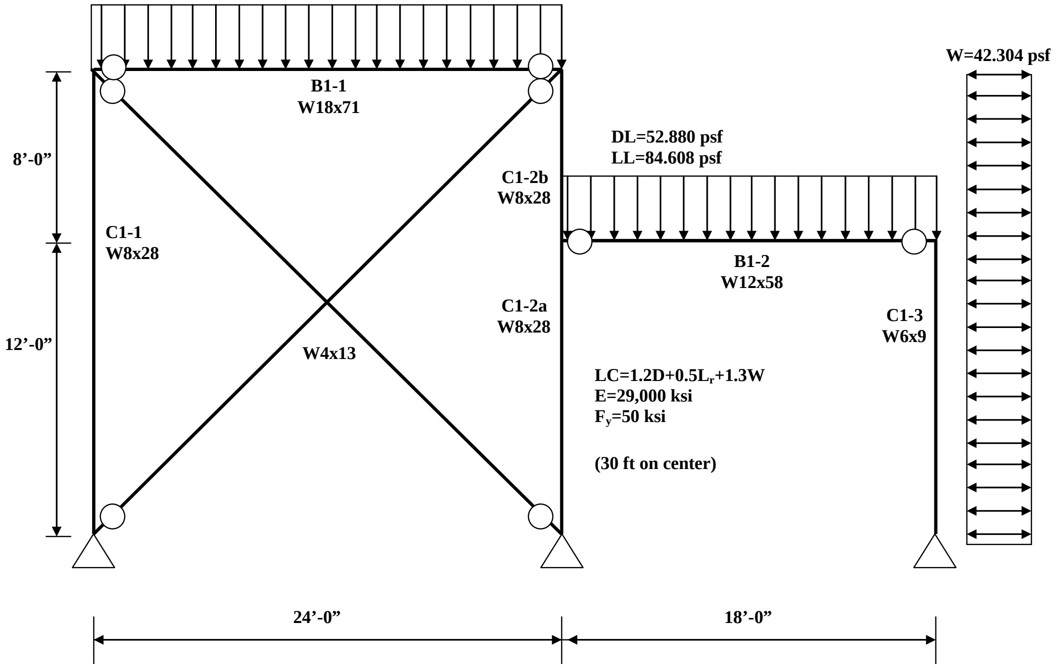


Figure 13. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 7c (Credit: Jose Martinez-Garcia, 2002)

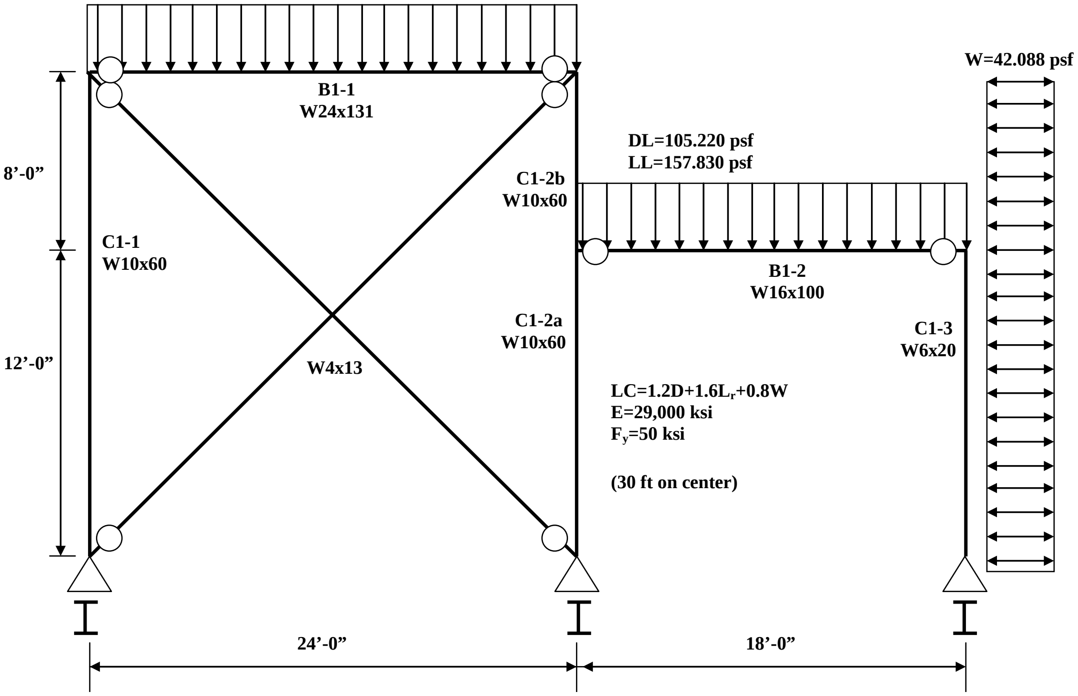


Figure 14. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 7d
(Credit: Jose Martinez-Garcia, 2002)

These four structural systems have the same basic geometry with variations defined by bracing and available connection restraint. They are representative models of simple and irregular geometry with different boundary conditions and interactions between the two parallel frames. Defined by Martinez-Garcia (with input from Joseph Yura, Professor Emeritus at the University of Texas at Austin), members of AISC TC10 proposed these structural systems to compare the Direct Analysis Approach with Effective Length Method and advanced analyses (Martinez-Garcia, 2002, p. 186).

In the prior study by Martinez-Garcia for assessing the feasibility of Direct Analysis Method, four possible cases with two load combinations (gravity and lateral load combinations) and two initial imperfection and wind directions (to the left and to the right) were initially considered to determine one controlling case for each structural system (Martinez-Garcia, 2002, p. 190). In this thesis, however, only the controlling case as determined in Martinez-Garcia's thesis will be studied for each structural system.

There are a few noteworthy characteristics of these structural systems:

The gravity loads are the same for structural systems 7a, 7b and 7c, except 7d where the loads are decreased to permit the use of smaller beams that are compatible with its smaller columns (Martinez-Garcia, 2002, p. 186).

Support conditions and connections of all four systems vary. System 7a has pinned supports at the bases, and all connections are fully restrained (rigid). System 7b has fixed supports at the bases, and the right bay beam is pinned at both ends. System 7c and 7d have pinned supports at the bases, and all member ends are simply supported (pinned). In addition, their left bays are braced against sway with light W4x13 sections. The difference between Systems 7c and 7d, except obvious differences in their sections and loading conditions, is that the columns are oriented for major axis bending in 7c, whereas the columns are oriented for minor axis bending in 7d (Martinez-Garcia, 2002, p. 186).

The rightmost columns in 7c and 7d can be assumed as leaning columns because they do not provide resistance to against lateral effects (Martinez-Garcia, 2002, p. 189).

All sections in all systems, except the columns in System 7d, are oriented for bending about their major axis, and the structure is assumed fully braced out of plane.

Structural System 8 – Vierendeel Truss

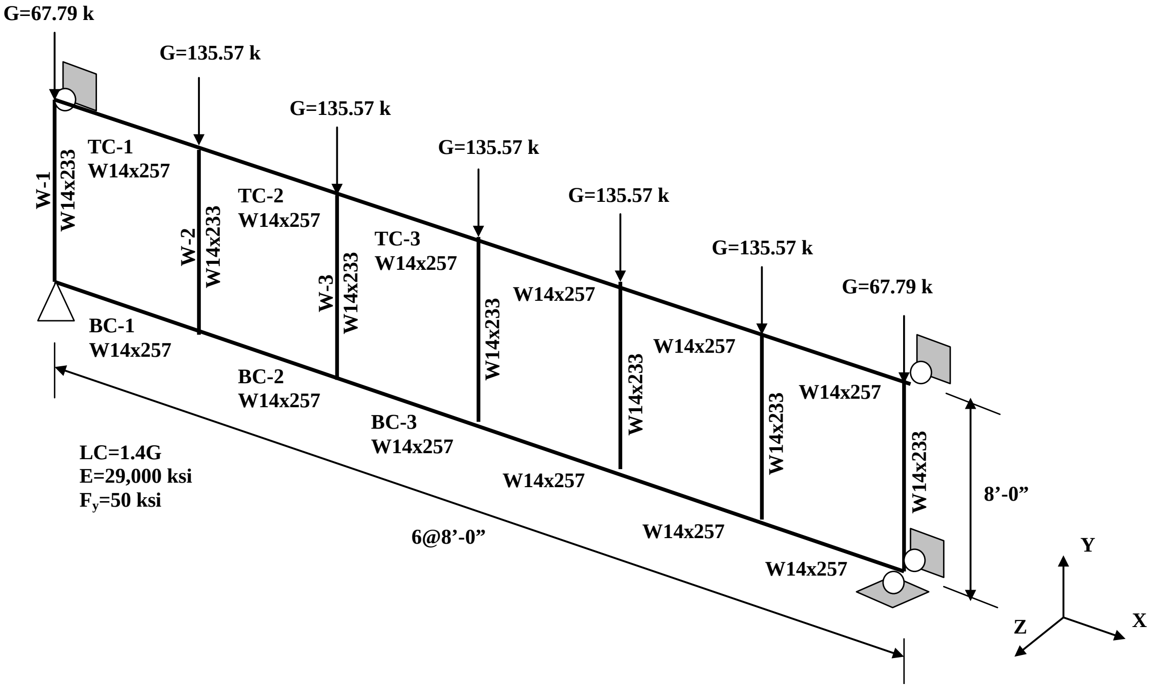


Figure15. Geometry, Section Properties, Material Properties, and Loading Conditions of Structural System 8
(Credit: Jose Martinez-Garcia, 2002)

This structural system is a representative model of a Vierendeel Truss, commonly used to support pedestrian walkways. This truss was originally designed by Martinez-Garcia for comparing the Direct Analysis Method with the Effective Length Method for a three-dimensional system.

In the prior study by Martinez-Garcia for assessing the feasibility of Direct Analysis Method, only one load combination (LC1 = 1.4G) was considered (Martinez-Garcia, 2002, p. 222). In this thesis, the same load combination will be studied as the controlling case. However, the loadings and sections of the original design have been modified in this study to ensure that the plastic yielding controls for moment strength. In addition, warping effects included in the prior study will be omitted in this study.

There are a few noteworthy characteristics of the system:

This structural system is not assumed fully braced out of plane, in contrast to all other structural systems in this study. Therefore, the system will be modeled as three-dimensional frame, and it will fail in a lateral-torsional buckling mode.

All sections are oriented for bending about their major axis.

Given the symmetry of the system, only the three leftmost top chord members will be studied as representative of compressive members in sway frames.

Column Study

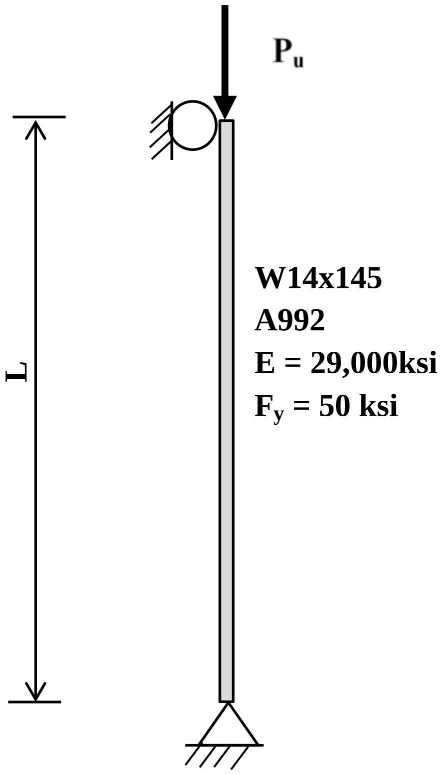


Figure 16. Geometry, Section Properties, Material Properties, and Loading Conditions of A Single Column Study

A single column is chosen for study in this thesis to compare the stability analysis results obtained by the newly proposed MDM method on the simplest structure against those obtained by both the advanced inelastic method and the existing direct analysis method (DM).

The section of the column is W14x145. The applied load to the column will be its ultimate strength. The L/r of the column will vary from 0 to 200.

Two separate studies will be conducted, one for bending of the column about its major axis and the other for bending about its minor axis.

Different from the other case studies, P_u/P_y values of the column obtained by the two methods (DM and MDM) will be compared against those obtained by the advanced inelastic method (Appendix 1) to evaluate which method is a more accurate method.

2.2. Conducting Stability Assessment Procedures

In this thesis, the twelve structural systems and single column study will be evaluated using two different stability assessment methods, Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM). Detailed steps for conducting stability assessment using each method will be explained in the following sections.

2.2.1. Direct Analysis Method

The Direct Analysis Method uses interaction equations to assess stability of a structural system.

To obtain demand components (P_u and M_u) for use in AISC's interaction equation, the structural system will be modeled and a second-order analysis will be performed in MASTAN2 using the following steps:

Nodal coordinates will be defined based on the given geometry of the system. Elements will be defined, and corresponding sections will be attached.

Material properties will be defined. In all DM and MDM studies, the modulus of elasticity E will be reduced to 0.8E, as required by Direct Analysis Method to include material inelasticity effects in the modeling.

All columns and beams will be subdivided into four elements. The purpose of subdividing columns is to best capture second-order P-δ effects and also allow for member buckling, which is essential to the MDM method. The purpose of subdividing the beams is to have distributed loads act as point loads.

The calibrated failure loads as determined in Martinez-Garcia's study by an advanced spread-of-plasticity analysis using NIFA will be applied to the system. The given distributed loads will be converted to point loads based on tributary length.

Connections and fixities will be defined as given for each case study.

As required by the Direct Analysis Method, the destabilizing effects of initial imperfections and material inelasticity will be included in the modeling to account for unit effective length assumption employed when defining P_n in the AISC interaction equation. For modeling these destabilizing effects, two different approaches, by direct modeling or by the use of notional loads, will be used in this study. Observations will be made to validate the equivalency of these two approaches.

When using the direct modeling approach, the destabilizing effects of initial imperfection will be included by distorting the geometry by H/500 (H = height measured from the specific story-to-story level) using Move Node option available in MASTAN2. In addition, the destabilizing effects of material inelasticity will be included by making use of the second-order inelastic analysis, which has the option of directly including the flexural stiffness reduction factor (τ_b) in the analysis. To prevent any plastic hinge formation while using this inelastic analysis option (all DM and MDM analyses are to be elastic), the yield surface of each element will be enlarged 10 times using Yield Surface Control option in MASTAN2.

As just indicated, the direct inclusion of the flexural stiffness reduction factor τ_b will be achieved by the use of the second-order inelastic analysis with Et option in MASTAN2. This analysis is programmed to automatically calculate τ_b factors for each element and reduce the corresponding flexural stiffness term in the analysis model using the following equation (AISC Specification 2010, Eq. C2-2a and Eq. C2-2b):

When $\alpha P_u/P_y \leq 0.5$

$$\tau_b = 1.0$$

When $\alpha P_u/P_y > 0.5$

$$\tau_b = 4(\alpha P_u/P_y)\left[1 - \left(\alpha P_u/P_y\right)\right],$$

where

τ_b = flexural stiffness reduction factor,

α = 1.0 (LRFD); α = 1.6 (ASD),

P_u = axial load effect on the member, kips (N), and

P_y = axial yield strength (=F_yA_g), kips (N).

When using the equivalent notional load approach, the destabilizing effects of initial imperfections will be included by applying notional lateral loads of $0.002Y_i$ at each story level (which is considered equivalent to distorting the geometry by H/500). The destabilizing effects of material inelasticity will be included by applying a notional load of $0.001Y_i$ at each story level (which is considered equivalent to the inclusion of flexural stiffness reduction factor τ_b).

After preparing the model, the corresponding analysis, either second-order inelastic analysis for direct modeling approach or second-order elastic analysis for notional load approach, will be performed. For each approach, two different analyses (one at ALR = 1.0 and another up to ultimate failure load ratio) will be performed. This is done because this thesis will compare DM and MDM considering these two different cases. Details for how DM and MDM will be compared are explained in Section 2.3. Each MASTAN2 analysis will provide the resulting axial and moment load effects in each element (P_u and M_u) that are necessary to calculate the AISC interaction equations.

Incorporating the results from the MASTAN2 analysis as inputs, a MATLAB program is written to assess the stability of each element in the system by computing the AISC interaction equation values. The program will first input P_u and M_u in each element obtained by the MASTAN2 analysis. Second, the program will input material properties (E and F_y), section properties (A, Z, I, r) and geometry (L) of the elements, and calculate the axial and moment strengths (P_n and M_n) of each element using AISC equations as specified in Chapter E and Chapter F (In this thesis, calculation of M_n will be simplified to $M_p = Z F_y$ for all systems, since all systems except System 8 are assumed fully braced out of plane, and, in System 8, the loads and sections are modified so that the controlling moment strength will be M_p). Third, after obtaining both P_u and M_u and P_n and M_n, the program will then assess the stability of each element in the system using its corresponding interaction equation (H1-1a or H1-1b as mentioned in Section 1.2). In accessing the stability of the system using the AISC interaction equations, the program will use two different MATLAB functions for the two different MASTAN2 analyses performed. For the first analysis, the program will calculate the AISC interaction equation value of each element using the MASTAN2 results at ALR= 1.0. For the second analysis, the program will determine the applied load ratio at which the AISC interaction equation value for each element becomes 1.0.

2.2.2. Modified Direct Analysis Method

The proposed stability assessment method, Modified Direct Analysis Method, will also use the AISC interaction equations to assess the stability of the structural systems.

As with Direct Analysis Method, demand components (P_u and M_u) of each interaction equation will be obtained by modeling the structural system, and performing a second-order analysis in MASTAN2. The modulus of elasticity E will still be reduced to 0.8E to include material inelasticity effects in the modeling. The destabilizing effects of initial imperfections and material inelasticity will be included in the modeling using two different approaches, Direct Modeling Approach and Notional Load Approach, in the same ways as in Direct Analysis Method.

The results from the MASTAN2 analyses, P_u and M_u, as well as material properties, section properties and geometries of each element in the system will then be input into the MATLAB programs to assess the stability of the structural system. As in Direct Analysis Method, two different MATLAB codes will be used to calculate the AISC interaction equation value of each element using MASTAN2 results at ALR=1.0, and to achieve the applied load ratio at which the AISC interaction equation value for each element becomes 1.0. Similar to the Direct Analysis Method, the programs will calculate the bending moment strength of each member using AISC equations as specified in Chapter F. However, unlike Direct Analysis Method, this new method will calculate the axial strength of every member using the cross-section yield strength ($P_n = P_y$).

Overall, the steps in this new stability assessment procedure are the same as those of Direct Analysis Method, except that the axial strength (P_n) of each member in the new method will be calculated using the cross-section yield strength ($P_y = A_g F_y$).

2.3. Determining Adequacy and Accuracy of Methods in Assessing Stability

In determining the adequacy and the accuracy of a stability assessment method, two different comparisons are made in this thesis.

The first comparison will be made based on assessing the stability of the system at the given applied loads. For this comparison, interaction equation values at the applied load ratio of 1.0 (H1-1 when ALR =1.0) using DM and MDM will be calculated. Because the given applied loads are calibrated failure loads using the advanced inelastic analysis, a method will be considered adequate to assess the stability of the structural system if it results in an interaction equation value of 1.0 or greater. Moreover, the method that results in the AISC interaction equation value of closer to 1.0 will be considered a more accurate method.

The second comparison will be made based on assessing the stability of the system at the corresponding failure loads by each method. For this comparison, the applied load ratios at which the failure of the system occurs will be obtained (ALR when H1-1 =1.0). A method will be considered adequate to assess the stability of the structural system if it results in an applied load ratio of 1.0 or smaller. Moreover, the method that results in an applied load ratio closer to 1.0 for failure will be considered a more accurate method.

CHAPTER 3: CASE STUDIES RESULTS

As mentioned earlier (Section 1.3.2 and Section 1.3.3), whether employing unit length factors in with Direct Analysis Method (DM) or only checking the member cross-section strength (P_y) in the Modified Direct Analysis Method (MDM), both require employing a rigorous second-order elastic analysis to obtain the load effects. These analyses account for member imperfections and material inelasticity in the modeling by using either of the two approaches - Direct Modeling Approach or Notional Load Approach. As a side, the studies in this thesis will also confirm the equivalency of these two approaches, given that both of these approaches will be employed in conducting DM and MDM stability assessments**.**

Structural System 1a – Unsymmetrical Frame

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 1a, Tables 1 and 2 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 1 analysis results were obtained using Direct Modeling Approach, whereas Table 2 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Column C2-3 (or beam B1-1) has the largest H1-1 value at ALR of 1.0, and thus this member is the most crucial member in determining the stability of the entire structural system. For this member, it is observed that

- Eq. H1-1 values by DM and MDM are greater than 1.0 by 27%,

- Eq. H1-1 value by MDM is closer to 1.0 than that of DM, or at least the same as that of DM,
- Eq. H1-1 value by MDM is less than that of DM, or the same as that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is less than 4.5%.

Table 1: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 1a – Unsymmetrical Frame)

Imperfection	Direct Modeling	**Stiffness Adjustment**	0.8E and tau_b
	second-order elastic; P-C;increment 0.01		

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.899	0.113	0.999	0.679	0.113	0.779
C1-2	0.540	0.542	1.021	0.500	0.542	0.981
C1-3	0.333	0.369	0.660	0.308	0.369	0.636
C2-1	0.327	0.304	0.598	0.277	0.304	0.548
C2-2	0.177	1.095	1.184	0.170	1.095	1.180
C2-3	0.116	1.209	**1.267**	0.111	1.209	**1.265**
B1-1	0.002	1.379	**1.379**	0.002	1.379	**1.379**
B1-2	0.047	1.041	1.065	0.044	1.041	1.063
B2-1	0.002	1.275	1.276	0.002	1.275	1.276
B2-2	0.096	1.115	1.162	0.083	1.115	1.156

Table 2: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 1a – Unsymmetrical Frame)

Imperfection NL ($0.002Y_i$) **Stiffness Adjustment** 0.8E and NL ($0.001Y_i$)
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.902	0.128	1.015	0.681	0.128	0.795
C1-2	0.540	0.570	1.047	0.500	0.570	1.007
C1-3	0.332	0.341	0.635	0.307	0.341	0.610
C2-1	0.327	0.311	0.603	0.278	0.311	0.554
C2-2	0.178	1.098	1.186	0.170	1.098	1.183
C2-3	0.115	1.210	**1.267**	0.110	1.210	**1.265**
B1-1	0.001	1.372	**1.372**	0.001	1.372	**1.372**
B1-2	0.046	1.049	1.072	0.043	1.049	1.070
B2-1	0.003	1.272	1.273	0.003	1.272	1.273
B2-2	0.096	1.117	1.165	0.083	1.117	1.158

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 1a, analysis results in Tables 3 and 4 compare ALR values obtained by DM and MDM when AISC interaction equation H1-1 equals 1.0. Analysis results in Table 3 were obtained using Direct Modeling Approach, whereas those in Table 4 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Column C2-3 (or beam B1-1) has the lowest ALR value at interaction equation value of 1.0, and thus this member is the most crucial member in determining the stability of the entire structural system. For this member, it is observed that

- ALR value by MDM is less than 1.0 by 21%,
- ALR value by MDM is the same as that of DM, and
- The difference between ALR values by the two methods is less than 4.5%.

Table 3: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 1a – Unsymmetrical Frame)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.899	0.113	1.000	0.850	0.165	1.250
C1-2	0.529	0.529	0.980	0.507	0.551	1.015
C1-3	0.536	0.508	1.700	0.496	0.555	1.705
C2-1	0.474	0.230	1.760	0.402	0.230	1.760
C2-2	0.150	0.924	0.845	0.144	0.924	0.845
C2-3	0.091	0.954	**0.790**	0.087	0.954	**0.790**
B1-1	0.001	0.996	**0.720**	0.001	0.996	**0.720**
B1-2	0.044	0.977	0.940	0.041	0.977	0.940
B2-1	0.002	0.994	0.780	0.002	0.994	0.780
B2-2	0.082	0.957	0.860	0.072	0.962	0.865

Table 4: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 1a – Unsymmetrical Frame)

Imperfection NL (0.002Yi) **Stiffness Adjustment** 0.8E and NL (0.001Y$_i$)
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.888	0.125	0.985	0.837	0.184	1.225
C1-2	0.516	0.539	0.955	0.495	0.563	0.990
C1-3	0.649	0.391	2.020	0.602	0.429	2.030
C2-1	0.528	0.529	1.610	0.482	0.581	1.730
C2-2	0.149	0.921	0.840	0.144	0.926	0.845
C2-3	0.091	0.954	**0.790**	0.087	0.954	**0.790**
B1-1	0.001	0.999	**0.725**	0.001	0.999	**0.725**
B1-2	0.043	0.973	0.930	0.040	0.979	0.935
B2-1	0.002	0.999	0.785	0.002	0.999	0.785
B2-2	0.083	0.958	0.860	0.072	0.958	0.860

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess stability of structural system 1a, since its H1-1 value for the controlling member (column C2-1 or beam B1-1) is greater than 1.0, and its ALR value for the controlling member is less than 1.0.

MDM tends to be a more accurate method than DM for this structural system, since the members controlling the design have AISC interaction equation H1-1 values closer to 1.0 or the same as that of the DM .

However, it should be kept in mind that MDM tends to be a less conservative method than DM, because it tends to yield lower H1-1 values or higher ALR values than DM for all other members.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 1a, because the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 1b – Unsymmetrical Frame

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 1b, Tables 5 and 6 compare the AISC interaction equation H-1 values at an applied load ratio of 1.0 obtained by Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. Table 5 analysis results were obtained using Direct Modeling Approach, whereas Table 6 results were obtained using the equivalent Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach, and lead to the following conclusions about DM and MDM.

DM and MDM do not result in the same member controlling the strength of the design. According to DM, column C2-3 has the largest H1-1 value at an applied load ratio of 1.0, but according to MDM, column C1-2 (or beam B2-1) has the largest H1-1 value at an applied load ratio of 1.0. However, for both controlling members by DM and MDM, it is observed that

- H1-1 value by MDM is lower than 1.0 by only 1.7%,
- H1-1 value by MDM is closer to 1.0 or the same as that of DM,
- H1-1 value by MDM is less than or the same as that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is not less than 4.5%.

Table 5: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 1b – Unsymmetrical Frame)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.051	0.455	0.480	0.037	0.455	0.473
C1-2	0.587	0.621	1.139	0.431	0.621	**0.983**
C1-3	0.282	0.043	0.321	0.207	0.043	0.246
C2-1	0.773	0.103	0.864	0.098	0.103	0.152
C2-2	0.551	0.297	0.815	0.313	0.297	0.577
C2-3	0.626	0.690	**1.240**	0.250	0.690	0.864
B1-1	0.005	0.934	0.937	0.005	0.934	0.937
B1-2	0.001	0.719	0.719	0.001	0.719	0.719
B2-1	0.000	1.021	**1.021**	0.000	1.021	**1.021**
B2-2	0.003	0.690	0.691	0.002	0.690	0.691

Table 6: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach
(Structural System 1b – Unsymmetrical Frame)

Imperfection NL (0.002Yi) **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.051	0.455	0.480	0.037	0.455	0.473
C1-2	0.587	0.621	1.139	0.431	0.621	**0.983**
C1-3	0.282	0.043	0.321	0.207	0.043	0.246
C2-1	0.772	0.103	0.864	0.098	0.103	0.152
C2-2	0.551	0.297	0.815	0.313	0.297	0.577
C2-3	0.626	0.690	**1.240**	0.250	0.690	0.864
B1-1	0.005	0.934	0.937	0.005	0.934	0.937
B1-2	0.001	0.719	0.719	0.001	0.719	0.719
B2-1	0.000	1.021	**1.021**	0.000	1.021	**1.021**
B2-2	0.003	0.690	0.691	0.002	0.690	0.691

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 1b, analysis results in Tables 7 and 8 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 7 were obtained using Direct Modeling Approach, whereas those in Table 8 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

DM and MDM do not result in the same controlling member. According to DM, column C2-3 has the lowest ALR value at interaction equation value of 1.0 but according to MDM, column C1-2 (or beam B2-1) has the lowest ALR value at interaction equation value of 1.0. However, for both controlling members by DM and MDM, it is observed that

- ALR value by MDM is greater than 1.0 by only 0.5 %.
- ALR value by MDM is closer to 1.0 or the same as that of DM,
- ALR value by MDM is greater than or the same as that of DM, and

- The difference between ALR values by the two methods is not less than 4.5%.

Table 7: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 1b – Unsymmetrical Frame)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.067	0.938	1.125	0.050	0.973	1.130
C1-2	0.547	0.500	0.930	0.433	0.632	**1.005**
C1-3	0.323	0.734	1.165	0.239	0.843	1.175
C2-1	0.883	0.128	1.135	0.209	0.126	1.354
C2-2	0.674	0.366	1.230	0.398	0.676	1.335
C2-3	0.507	0.552	**0.810**	0.285	0.800	1.135
B1-1	0.008	0.996	1.095	0.008	0.996	1.095
B1-2	0.002	0.987	1.240	0.002	0.987	1.240
B2-1	0.000	0.995	**0.975**	0.000	0.995	**0.975**
B2-2	0.005	0.994	1.350	0.004	0.994	1.350

Table 8: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 1b – Unsymmetrical Frame)

Imperfection NL (0.002Yi) **Stiffness Adjustment** 0.8E and No NL

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.067	0.938	1.125	0.050	0.973	1.130
C1-2	0.547	0.500	0.930	0.433	0.632	**1.005**
C1-3	0.323	0.734	1.165	0.239	0.843	1.175
C2-1	0.882	0.128	1.135	0.314	0.746	1.505
C2-2	0.674	0.366	1.230	0.398	0.676	1.335
C2-3	0.507	0.552	**0.810**	0.285	0.801	1.135
B1-1	0.008	0.996	1.095	0.008	0.996	1.095
B1-2	0.001	0.988	1.240	0.001	0.988	1.240
B2-1	0.000	0.995	**0.975**	0.000	0.995	**0.975**
B2-2	0.005	0.994	1.350	0.004	0.994	1.350

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess the stability of structural system 1b, since its H1-1 value for the controlling member (column C1-2 or beam B2-1) is less than 1.0 by only 1.7%, and its ALR value for the controlling member is greater than 1.0 by only 0.5%.

MDM is a more accurate method than DM for structural system 1b, since its controlling member has a H1-1 value closer to 1.0 or the same as that of DM, and it have an ALR value closer to 1.0 or the same as that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to yield lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 1b, since the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 2 – Industrial Frame

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 2, Tables 9 and 10 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 9 analysis results were obtained using Direct Modeling Approach, whereas Table 10 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when

ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Column C1-1 has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is greater than 1.0 by 12%,
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is not less than 4.5%.

Table 9: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 2 – Industrial Frame)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$
second-order elastic; P-C;increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.458	0.827	**1.193**	0.382	0.827	**1.117**
C1-2	0.473	0.588	0.996	0.395	0.588	0.918
B1-1	0.003	0.913	0.915	0.003	0.913	0.914

Table 10: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 2 – Industrial Frame)

Imperfection NL 0.002Yi **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.458	0.828	**1.193**	0.382	0.828	**1.118**
C1-2	0.473	0.588	0.996	0.395	0.588	0.918
B1-1	0.001	0.913	0.913	0.001	0.913	0.913

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 2, analysis results in Tables 11 and 12 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 11 were obtained using Direct Modeling Approach, whereas those in Table 12 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Column C1-1 has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is less than 1.0 by 3%,
- ALR value by MDM is closer to 1.0 than that of DM,
- ALR value by MDM is greater than that of DM, and
- The difference between ALR values by the two methods is not less than 4.5%.

Table 11: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 2 – Industrial Frame)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.435	0.631	**0.945**	0.372	0.707	**0.970**
C1-2	0.473	0.589	1.000	0.399	0.637	1.010
B1-1	0.001	0.994	1.045	0.003	0.983	1.040

Table 12: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 2 – Industrial Frame)

Imperfection NL 0.002Yi **Stiffness Adjustment** 0.8E and No NL

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.435	0.631	**0.945**	0.370	0.691	**0.965**
C1-2	0.473	0.589	1.000	0.402	0.664	1.015
B1-1	0.001	0.994	1.045	0.001	0.994	1.045

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess stability of structural system 2, since its H1-1 value for the controlling member (column C1-1) is greater than 1.0, and its ALR value for the controlling member is less than 1.0.

MDM can be a more accurate method than DM for structural system 2, since its controlling member has a H1-1 value closer to 1.0 than that of DM, and has an ALR value closer to 1.0 than that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 2, since the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 3– Grain Storage Bin

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 3, Tables 13 and 14 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 13 analysis results were obtained using Direct Modeling Approach, whereas Table 14 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Column C1-2 has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is less than 1.0 by only 1.6% (Direct Modeling Approach), or H1-1 value by MDM greater than 1.0 by 3.2% (Notional Load Approach),
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is less than 4.5%.

Table 13: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 3 – Grain Storage Bin)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.558	0.431	0.941	0.528	0.431	0.911
C1-2	0.630	0.437	**1.018**	0.596	0.437	**0.984**
C2-1	0.543	0.431	0.926	0.497	0.431	0.880
C2-2	0.586	0.437	0.974	0.537	0.437	0.925

Table 14: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 3 – Grain Storage Bin)

Imperfection NL 0.002Y_i **Stiffness Adjustment** 0.8E and NL (0.001Y_i)
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.555	0.482	0.983	0.525	0.482	0.953
C1-2	0.633	0.487	**1.066**	0.599	0.487	**1.032**
C2-1	0.541	0.482	0.969	0.495	0.482	0.924
C2-2	0.587	0.487	1.020	0.538	0.487	0.971

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 3, analysis results in Tables 15 and 16 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 15 were obtained using Direct Modeling Approach, whereas those in Table 16 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Column C1-1 has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is greater than 1.0 by only 0.5% (Direct Modeling Approach), or ALR value by MDM is less than 1.0 by 2% (Notional Load Approach),
- ALR value by MDM is closer to 1.0 than that of DM,
- ALR value by MDM is greater than that of DM, and
- The difference between ALR values by the two methods is less than 4.5%.

Table 15: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 3 – Grain Storage Bin)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$

second-order elastic; P-C;increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.573	0.477	1.030	0.549	0.503	1.045
C1-2	0.623	0.423	**0.990**	0.599	0.444	**1.005**
C2-1	0.561	0.486	1.035	0.525	0.532	1.060
C2-2	0.592	0.451	1.010	0.556	0.491	1.035

Table 16: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 3 – Grain Storage Bin)

Imperfection NL 0.002Y$_i$ **Stiffness Adjustment** 0.8E and NL (0.001Y$_i$)

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.557	0.490	1.005	0.534	0.515	1.020
C1-2	0.610	0.436	**0.965**	0.586	0.457	**0.980**
C2-1	0.548	0.507	1.015	0.512	0.543	1.035
C2-2	0.578	0.464	0.985	0.544	0.503	1.010

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess the stability of structural system 3, since its H1-1 value for the controlling member (column C1-2) is less than 1.0 by only 1.6% (Direct Modeling Approach), and its ALR value for the controlling member is greater than 1.0 by only 0.5% (Direct Modeling Approach).

MDM is a more accurate method than DM for structural system 3, since its controlling member has a H1-1 value closer to 1.0 than that of DM, and it has an ALR value closer to 1.0 than that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 3, since the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 4 – Multi-story Frame

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 4, Tables 17 and 18 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 17 analysis results were obtained using Direct Modeling Approach, whereas Table 18 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Column C1-2 has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is 23% greater than 1.0,
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is less than 4.5%.

Table 17: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 4 – Multi-story Frame)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and τ_b
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.395	0.353	0.709	0.366	0.353	0.680
C1-2	0.789	0.544	**1.273**	0.748	0.544	**1.232**
C1-3	0.534	0.564	1.036	0.495	0.564	0.996
C2-1	0.337	0.025	0.359	0.312	0.025	0.334
C2-2	0.646	0.438	1.036	0.612	0.438	1.002
C2-3	0.432	0.649	1.009	0.400	0.649	0.977
C3-1	0.272	0.058	0.324	0.252	0.058	0.304
C3-2	0.571	0.376	0.905	0.536	0.376	0.870
C3-3	0.330	0.598	0.862	0.306	0.598	0.838
C4-1	0.201	0.268	0.440	0.186	0.268	0.361
C4-2	0.419	0.271	0.660	0.393	0.271	0.633
C4-3	0.231	0.658	0.817	0.214	0.658	0.800
C5-1	0.228	0.284	0.480	0.196	0.284	0.382
C5-2	0.369	0.320	0.654	0.336	0.320	0.621
C5-3	0.249	0.809	0.969	0.215	0.809	0.934
C6-1	0.091	0.690	0.736	0.078	0.690	0.729
C6-2	0.145	0.102	0.174	0.132	0.102	0.168
C6-3	0.095	0.871	0.919	0.082	0.871	0.912

Table 18: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 4 – Multi-story Frame)

Imperfection No NL **Stiffness Adjustment** 0.8E and NL (0.001Y_i)
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.397	0.338	0.698	0.368	0.338	0.668
C1-2	0.787	0.532	**1.260**	0.746	0.532	**1.219**
C1-3	0.532	0.549	1.020	0.493	0.549	0.981
C2-1	0.338	0.018	0.354	0.313	0.018	0.329
C2-2	0.644	0.425	1.023	0.610	0.425	0.988
C2-3	0.431	0.640	0.999	0.399	0.640	0.968
C3-1	0.273	0.065	0.331	0.253	0.065	0.311
C3-2	0.571	0.365	0.896	0.536	0.365	0.860
C3-3	0.330	0.590	0.854	0.305	0.590	0.830
C4-1	0.201	0.273	0.445	0.187	0.273	0.367
C4-2	0.419	0.263	0.653	0.393	0.263	0.627
C4-3	0.231	0.653	0.811	0.214	0.653	0.794
C5-1	0.228	0.291	0.487	0.196	0.291	0.389
C5-2	0.369	0.311	0.645	0.336	0.311	0.612
C5-3	0.249	0.801	0.961	0.215	0.801	0.926
C6-1	0.091	0.693	0.738	0.078	0.693	0.732
C6-2	0.145	0.098	0.171	0.132	0.098	0.164
C6-3	0.095	0.868	0.915	0.082	0.868	0.909

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 4, analysis results in Tables 19 and 20 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 19 were obtained using Direct Modeling Approach, whereas those in Table 20 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Column C1-2 has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is less than 1.0 by 18%,
- ALR value by MDM is closer to 1.0 than that of DM,
- ALR value by MDM is greater than that of DM, and
- The difference between ALR values by the two methods is less than 4.5%.

Table 19: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 4 – Multi-story Frame)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_b
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.523	0.535	1.335	0.499	0.566	1.375
C1-2	0.624	0.421	**0.790**	0.613	0.439	**0.820**
C1-3	0.515	0.542	0.965	0.497	0.568	1.005
C2-1	0.502	0.008	1.490	0.465	0.008	1.490
C2-2	0.624	0.420	0.965	0.612	0.438	1.000
C2-3	0.428	0.642	0.990	0.411	0.667	1.025
C3-1	0.406	0.070	1.490	0.376	0.070	1.490
C3-2	0.625	0.419	1.095	0.610	0.439	1.140
C3-3	0.379	0.695	1.145	0.362	0.719	1.180
C4-1	0.300	0.400	1.490	0.278	0.400	1.490
C4-2	0.612	0.433	1.470	0.581	0.441	1.490
C4-3	0.281	0.807	1.210	0.267	0.829	1.240
C5-1	0.341	0.415	1.490	0.293	0.415	1.490
C5-2	0.542	0.512	1.475	0.498	0.519	1.490
C5-3	0.257	0.835	1.030	0.230	0.869	1.070
C6-1	0.123	0.935	1.350	0.108	0.950	1.370
C6-2	0.216	0.160	1.490	0.196	0.160	1.490
C6-3	0.104	0.946	1.085	0.090	0.955	1.095

Table 20: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 4 – Multi-story Frame)

Imperfection No NL **Stiffness Adjustment** 0.8E and NL ($0.001Y_i$)

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.547	0.506	1.395	0.527	0.534	1.455
C1-2	0.628	0.412	**0.795**	0.621	0.432	**0.830**
C1-3	0.522	0.537	0.980	0.503	0.561	1.020
C2-1	0.753	0.276	2.325	0.724	0.313	2.425
C2-2	0.630	0.413	0.975	0.619	0.430	1.010
C2-3	0.431	0.640	1.000	0.413	0.664	1.035
C3-1	0.758	0.271	2.925	0.721	0.314	3.020
C3-2	0.631	0.411	1.105	0.619	0.434	1.155
C3-3	0.383	0.694	1.160	0.366	0.717	1.195
C4-1	0.527	0.531	2.690	0.528	0.531	2.930
C4-2	0.622	0.425	1.485	0.605	0.445	1.540
C4-3	0.282	0.805	1.220	0.268	0.826	1.250
C5-1	0.553	0.502	2.460	0.551	0.505	2.860
C5-2	0.550	0.504	1.490	0.526	0.536	1.565
C5-3	0.258	0.830	1.035	0.232	0.868	1.080
C6-1	0.123	0.935	1.355	0.108	0.948	1.375
C6-2	0.536	0.520	3.675	0.508	0.556	3.820
C6-3	0.104	0.947	1.090	0.090	0.955	1.100

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess the stability of structural system 4, since its H1-1 value for the controlling member (column C1-2) is greater than 1.0, and its ALR value for the controlling member is less than 1.0.

MDM is a more accurate method than DM for structural system 4, since its controlling member has a H1-1 value closer to 1.0 than that of DM, and it has an ALR value closer to 1.0 than that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 4, since the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 5 – Gabled Frame

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 5, Tables 21 and 22 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 21 analysis results were obtained using Direct Modeling Approach, whereas Table 22 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Column C1-2 has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is greater than 1.0 by 59%,
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and

- The difference between AISC interaction equation H1-1 values by the two methods is less than 4.5%.

Table 21: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 5 – Gabled Frame)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_b
second-order elastic; P-C; increment0.1

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.058	0.152	0.180	0.045	0.152	0.174
C1-2	0.080	1.562	**1.602**	0.062	1.562	**1.593**
B1-1	0.010	0.434	0.439	0.007	0.434	0.437
B1-2	0.024	0.488	0.500	0.018	0.488	0.497

Table 22: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 5 – Gabled Frame)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL
second-order elastic; work control; increment 0.1

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.058	0.159	0.188	0.045	0.159	0.182
C1-2	0.080	1.554	**1.594**	0.062	1.554	**1.585**
B1-1	0.010	0.433	0.438	0.007	0.433	0.437
B1-2	0.024	0.486	0.498	0.018	0.486	0.495

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 5, analysis results in Tables 23 and 24 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 23 were obtained using Direct Modeling Approach, whereas those in Table 24 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Column C1-2 has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is less than 1.0 by 36%,
- ALR value by MDM is closer to 1.0 or the same as that of DM,
- ALR value by MDM is greater than or the same as that of DM, and
- The difference between ALR values by the two methods is less than 4.5%.

Table 23: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 5 – Gabled Frame)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_b
second-order elastic; P-C; increment 0.01

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.151	0.920	2.960	0.119	0.935	2.970
C1-2	0.051	0.966	**0.640**	0.040	0.966	**0.640**
B1-1	0.019	0.991	2.110	0.014	0.991	2.110
B1-2	0.043	0.976	1.820	0.032	0.983	1.830

Table 24: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 5 – Gabled Frame)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.01

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.153	0.914	2.980	0.120	0.929	2.990
C1-2	0.051	0.961	**0.640**	0.040	0.977	**0.650**
B1-1	0.019	0.989	2.110	0.014	0.989	2.110
B1-2	0.044	0.977	1.830	0.032	0.977	1.830

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess the stability of structural system 5, since its H1-1 value for the controlling member (column C1-2) is greater than 1.0, and its ALR value for the controlling member is less than 1.0.

MDM is a more accurate method than DM for structural system 5, since its controlling member has a H1-1 value closer to 1.0 than that of DM, and it has an ALR value closer to 1.0 or the same as that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM can be equivalent for assessing the stability of structural system 5, since the results by DM and MDM for all members always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 6 – Two-bay Frame with Irregular Geometry

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 6, Tables 25 and 26 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 25 analysis results were obtained using Direct Modeling Approach, whereas Table 26 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when

ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Column C1-2 has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is greater than 1.0 by 10%,
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is less than 4.5%.

Table 25: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 6 – Two-bay Frame with Irregular Geometry)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and τ_b
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.072	0.891	0.927	0.064	0.891	0.923
C1-2	0.222	0.997	**1.108**	0.197	0.997	**1.096**
C1-3	0.126	0.633	0.696	0.096	0.633	0.681
C2-1	0.062	0.099	0.130	0.055	0.099	0.127
C2-2a	0.124	0.291	0.353	0.120	0.291	0.351
C2-2b	0.101	0.434	0.485	0.098	0.434	0.484
C3-1	0.021	0.019	0.029	0.019	0.019	0.028
C3-2	0.047	0.269	0.292	0.042	0.269	0.289
C3-3	0.034	0.279	0.296	0.026	0.279	0.292
B1-1	0.008	0.952	0.956	0.007	0.952	0.955
B2-1	0.012	0.520	0.526	0.010	0.520	0.525
B2-2	0.001	0.589	0.589	0.001	0.589	0.589
B3-1	0.047	0.633	0.656	0.038	0.633	0.652
B3-2	0.019	0.509	0.518	0.015	0.509	0.517

Table 26: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 6 – Two-bay Frame with Irregular Geometry)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.074	0.872	0.909	0.065	0.872	0.905
C1-2	0.221	0.981	**1.093**	0.197	0.981	**1.079**
C1-3	0.126	0.624	0.687	0.096	0.624	0.672
C2-1	0.062	0.096	0.127	0.055	0.096	0.123
C2-2a	0.124	0.290	0.352	0.120	0.290	0.350
C2-2b	0.101	0.431	0.482	0.098	0.431	0.480
C3-1	0.021	0.020	0.031	0.019	0.020	0.030
C3-2	0.047	0.267	0.291	0.042	0.267	0.288
C3-3	0.034	0.278	0.295	0.026	0.278	0.291
B1-1	0.007	0.940	0.943	0.007	0.940	0.943
B2-1	0.012	0.516	0.522	0.010	0.516	0.522
B2-2	0.001	0.583	0.583	0.001	0.583	0.583
B3-1	0.047	0.631	0.654	0.038	0.631	0.649
B3-2	0.019	0.507	0.516	0.015	0.507	0.514

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 6, analysis results in Tables 27 and 28 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 27 were obtained using Direct Modeling Approach, whereas those in Table 28 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Column C1-2 has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is less than 1.0 by 7%,
- ALR value by MDM is closer to 1.0 than that of DM,

- ALR value by MDM is greater than that of DM, and

- The difference between ALR values by the two methods is less than 4.5%.

Table 27: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 6 – Two-bay Frame with Irregular Geometry)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and τ_b

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.076	0.958	1.055	0.067	0.971	1.065
C1-2	0.203	0.892	**0.915**	0.183	0.910	**0.930**
C1-3	0.171	0.912	1.335	0.133	0.935	1.360
C2-1	0.191	0.899	3.380	0.170	0.916	3.395
C2-2a	0.253	0.836	2.135	0.247	0.848	2.150
C2-2b	0.198	0.898	1.945	0.194	0.904	1.955
C3-1	0.127	0.809	4.690	0.113	0.809	4.690
C3-2	0.150	0.925	3.325	0.134	0.933	3.345
C3-3	0.107	0.945	3.045	0.083	0.961	3.080
B1-1	0.008	0.992	1.035	0.007	0.998	1.040
B2-1	0.023	0.987	1.825	0.020	0.993	1.835
B2-2	0.003	0.995	1.555	0.003	0.999	1.560
B3-1	0.071	0.962	1.505	0.057	0.975	1.525
B3-2	0.034	0.982	1.890	0.027	0.988	1.900

Table 28: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 6 – Two-bay Frame with Irregular Geometry)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.078	0.957	1.070	0.069	0.970	1.080
C1-2	0.204	0.890	**0.925**	0.185	0.908	**0.940**
C1-3	0.172	0.912	1.350	0.134	0.935	1.375
C2-1	0.193	0.898	3.385	0.171	0.915	3.400
C2-2a	0.256	0.837	2.160	0.250	0.845	2.170
C2-2b	0.200	0.900	1.960	0.195	0.905	1.970
C3-1	0.127	0.897	4.695	0.113	0.820	4.700
C3-2	0.150	0.923	3.340	0.134	0.933	3.365
C3-3	0.107	0.945	3.060	0.083	0.959	3.090
B1-1	0.008	0.990	1.045	0.007	0.996	1.050
B2-1	0.023	0.986	1.835	0.020	0.992	1.845
B2-2	0.003	0.996	1.570	0.003	1.000	1.575
B3-1	0.071	0.962	1.510	0.057	0.975	1.530
B3-2	0.034	0.981	1.895	0.027	0.989	1.910

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess the stability of structural system 6, since its H1-1 value for the controlling member (column C1-2) is greater than 1.0, and its ALR value for the controlling member is less than 1.0.

MDM is a more accurate method than DM for structural system 6, since its controlling member has a H1-1 value closer to 1.0 than that of DM, and it has an ALR value closer to 1.0 than that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM can be equivalent for assessing the stability of structural system 6, since the results by DM and MDM for all members always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 7a – Two-bay Frame with Unequal Heights

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 7a, Tables 29 and 30 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 29 analysis results were obtained using Direct Modeling Approach, whereas Table 30 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Beam B1-2 has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is greater than 1.0 by 27%,
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is less than 4.5%.

Table 29: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 7a – Two-bay Frame with Unequal Heights)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.221	0.631	0.782	0.158	0.631	0.710
C1-2a	0.229	0.539	0.709	0.205	0.539	0.684
C1-2b	0.115	0.476	0.533	0.109	0.476	0.530
C1-3	0.112	0.204	0.259	0.099	0.204	0.253
B1-1	0.006	1.062	1.065	0.005	1.062	1.065
B1-2	0.018	1.260	**1.269**	0.016	1.260	**1.268**

Table 30: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 7a – Two-bay Frame with Unequal Heights)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.221	0.618	0.770	0.158	0.618	0.697
C1-2a	0.229	0.518	0.689	0.204	0.518	0.665
C1-2b	0.115	0.484	0.542	0.110	0.484	0.539
C1-3	0.113	0.231	0.287	0.100	0.231	0.280
B1-1	0.006	1.061	1.065	0.006	1.061	1.064
B1-2	0.018	1.233	**1.242**	0.016	1.233	**1.241**

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 7a, analysis results in Tables 31 and 32 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 11 were obtained using Direct Modeling Approach, whereas those in Table 12 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Beam B1-2 has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is less than 1.0 by 20%,
- ALR value by MDM is closer to 1.0 or the same as that of DM,
- ALR value by MDM is greater than or the same as that of DM, and
- The difference between ALR values by the two methods is less than 4.5%.

Table 31: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 7a – Two-bay Frame with Unequal Heights)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_b
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.281	0.807	1.265	0.218	0.883	1.375
C1-2a	0.306	0.776	1.325	0.281	0.810	1.365
C1-2b	0.219	0.878	1.935	0.211	0.888	1.960
C1-3	0.210	0.883	2.355	0.186	0.898	2.360
B1-1	0.006	0.992	0.935	0.005	0.998	0.940
B1-2	0.015	0.992	**0.805**	0.014	0.992	**0.805**

Table 32: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 7a – Two-bay Frame with Unequal Heights)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.285	0.802	1.285	0.222	0.880	1.400
C1-2a	0.313	0.771	1.360	0.288	0.803	1.400
C1-2b	0.220	0.878	1.930	0.211	0.888	1.955
C1-3	0.216	0.871	2.405	0.191	0.903	2.415
B1-1	0.006	0.997	0.940	0.005	1.002	0.945
B1-2	0.016	0.991	**0.820**	0.014	0.998	**0.825**

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess the stability of structural system 7a, since its H1-1 value for the controlling member (beam B1-2) is greater than 1.0, and its ALR value for the controlling member is less than 1.0.

MDM is a more accurate method than DM for structural system 7a, since its controlling member has a H1-1 value closer to 1.0 or the same as that of DM, and it has an ALR value closer to 1.0 or the same as that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 7a, since the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 7b – Two-bay Frame with Unequal Heights

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 7b, Tables 33 and 34 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 33 analysis results were obtained using Direct Modeling Approach, whereas Table 34 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Column C1-1 (beam B1-1) has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is less than 1.0 by 6%,
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is less than 4.5%.

Table 33: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 7b – Two-bay Frame with Unequal Heights)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_b
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.249	0.855	**1.009**	0.177	0.855	**0.944**
C1-2a	0.244	0.124	0.354	0.217	0.124	0.327
C1-2b	0.130	0.355	0.419	0.123	0.355	0.416
C1-3	0.146	0.133	0.206	0.129	0.133	0.197
B1-1	0.016	1.097	**1.106**	0.015	1.097	**1.105**
B1-2	0.005	0.980	0.983	0.004	0.980	0.982

Table 34: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 7b – Two-bay Frame with Unequal Heights)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.248	0.841	**0.996**	0.176	0.841	**0.929**
C1-2a	0.244	0.128	0.358	0.218	0.128	0.331
C1-2b	0.130	0.366	0.431	0.123	0.366	0.428
C1-3	0.146	0.119	0.192	0.129	0.119	0.184
B1-1	0.016	1.097	**1.105**	0.015	1.097	**1.104**
B1-2	0.005	0.986	0.989	0.004	0.986	0.988

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 7b, analysis results in Tables 35 and 36 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 35 were obtained using Direct Modeling Approach, whereas those in Table 36 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Column C1-1 (beam B1-1) has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is greater than 1.0 by 6%,
- ALR value by MDM is closer to 1.0 than that of DM,
- ALR value by MDM is greater than that of DM, and
- The difference between ALR values by the two methods is less than 4.5%.

Table 35: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 7b – Two-bay Frame with Unequal Heights)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_b

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.246	0.847	**0.990**	0.187	0.908	**1.060**
C1-2a	0.615	0.431	2.545	0.574	0.481	2.670
C1-2b	0.498	0.564	4.150	0.473	0.581	4.155
C1-3	0.397	0.677	2.725	0.361	0.721	2.805
B1-1	0.015	0.992	**0.905**	0.014	0.998	**0.910**
B1-2	0.005	0.995	1.015	0.004	1.000	1.020

Table 36: Comparison 2 (ALR at H1-1=1.0) Using Notional Load Approach (Structural System 7b – Two-bay Frame with Unequal Heights)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.248	0.841	**1.000**	0.190	0.906	**1.075**
C1-2a	0.630	0.415	2.600	0.591	0.461	2.740
C1-2b	0.533	0.515	4.430	0.508	0.557	4.450
C1-3	0.410	0.662	2.805	0.374	0.708	2.890
B1-1	0.015	0.992	**0.905**	0.014	0.997	**0.910**
B1-2	0.005	0.996	1.010	0.004	1.001	1.015

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is not adequate to assess the stability of structural system 7b, since its H1-1 value for the controlling member (column C1-1) is less than 1.0 by 6%, and its ALR value for the controlling member is greater than 1.0 by 6%.

MDM is a less accurate method than DM for structural system 7b, since its controlling member has a H1-1 value less closer to 1.0 than that of DM, and it has an ALR value less closer to 1.0 than that of DM.

It should also be kept in mind that MDM is a less conservative design than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 7b, since the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 7c – Two-bay Frame with Unequal Heights

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 7c, Tables 37 and 38 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 37 analysis results were obtained using Direct Modeling Approach, whereas Table 38 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when

ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Column C1-2a has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is less than 1.0 by only 1.2%,
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is not less than 4.5%.

Table 37: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 7c – Two-bay Frame with Unequal Heights)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.202	0.000	0.202	0.142	0.000	0.071
C1-2a	0.256	0.898	**1.054**	0.180	0.898	**0.988**
C1-2b	0.146	0.898	0.971	0.102	0.898	0.949
C1-3	0.303	0.000	0.303	0.237	0.000	0.237
B1-1	0.020	0.419	0.430	0.018	0.419	0.429
B1-2	0.015	0.398	0.406	0.014	0.398	0.405
BRACE	0.131	0.000	0.066	0.131	0.000	0.066

DM: KL = 20' for C1-2 and C1-2b

Table 38: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 7c – Two-bay Frame with Unequal Heights)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.201	0.000	0.201	0.141	0.000	0.071
C1-2a	0.256	0.892	**1.049**	0.180	0.892	**0.981**
C1-2b	0.146	0.892	0.965	0.102	0.892	0.943
C1-3	0.303	0.000	0.303	0.237	0.000	0.237
B1-1	0.020	0.419	0.430	0.018	0.419	0.429
B1-2	0.015	0.398	0.406	0.014	0.398	0.405
BRACE	0.129	0.000	0.065	0.129	0.000	0.065

DM: KL = 20' for C1-2 and C1-2b

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 7c, analysis results in Tables 39 and 40 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 39 were obtained using Direct Modeling Approach, whereas those in Table 40 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Column C1-2a has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is greater than 1.0 by only 1%,
- ALR value by MDM is closer to 1.0 than that of DM,
- ALR value by MDM is greater than that of DM, and
- The difference between ALR values by the two methods is not less than 4.5%.

Table 39: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 7c – Two-bay Frame with Unequal Heights)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.813	0.000	3.800	0.570	0.000	3.800
C1-2a	0.243	0.847	**0.950**	0.181	0.908	**1.010**
C1-2b	0.150	0.924	1.025	0.107	0.945	1.045
C1-3	0.999	0.000	3.280	0.906	0.000	3.800
B1-1	0.050	0.974	2.305	0.045	0.976	2.310
B1-2	0.041	0.979	2.440	0.037	0.981	2.445
BRACE	0.616	0.000	3.800	0.616	0.000	3.800

DM: KL = 20' for C1-2 and C1-2b

Table 40: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 7c – Two-bay Frame with Unequal Heights)

Imperfection No NL **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.999	0.000	4.540	0.997	0.000	5.790
C1-2a	0.245	0.846	**0.955**	0.182	0.907	**1.015**
C1-2b	0.150	0.923	1.030	0.107	0.943	1.050
C1-3	0.999	0.000	3.280	0.999	0.000	4.175
B1-1	0.050	0.974	2.305	0.045	0.976	2.310
B1-2	0.041	0.979	2.440	0.036	0.981	2.445
BRACE	0.998	0.000	4.975	0.998	0.000	4.975

DM: KL = 20' for C1-2 and C1-2b

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess the stability of structural system 7c, since its H1-1 value for the controlling member (column C1-2a) is less than 1.0 by only 1.2%, and its ALR value for the controlling member is greater than 1.0 by only 1%.

MDM is a more accurate method than DM for structural system 7c, since its controlling member has a H1-1 value closer to 1.0 than that of DM, and it has an ALR value closer to 1.0 than that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 7c, since the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 7d – Two-bay Frame with Unequal Heights

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 7d, Tables 41 and 42 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 41 analysis results were obtained using Direct Modeling Approach, whereas Table 42 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Column C1-2a has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is less than 1.0 by 11 %,
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is not less than 4.5%.

Table 41: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 7d – Two-bay Frame with Unequal Heights)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.352	0.000	0.352	0.186	0.000	0.093
C1-2a	0.571	0.661	**1.158**	0.301	0.661	**0.889**
C1-2b	0.326	0.661	0.914	0.172	0.661	0.747
C1-3	0.755	0.000	0.755	0.386	0.000	0.386
B1-1	0.008	0.590	0.594	0.007	0.590	0.594
B1-2	0.006	0.614	0.617	0.006	0.614	0.617
Bracing	0.097	0.000	0.049	0.097	0.000	0.049

DM: KL = 20' for C1-2 and C1-2b

Table 42: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 7d – Two-bay Frame with Unequal Heights)

Imperfection NL 0.002Yi **Stiffness Adjustment** 0.8E and No NL
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.352	0.000	0.352	0.186	0.000	0.093
C1-2a	0.571	0.661	**1.158**	0.301	0.661	**0.889**
C1-2b	0.326	0.661	0.914	0.172	0.661	0.747
C1-3	0.755	0.000	0.755	0.386	0.000	0.386
B1-1	0.008	0.590	0.594	0.007	0.590	0.594
B1-2	0.006	0.614	0.617	0.006	0.614	0.617
Bracing	0.097	0.000	0.049	0.097	0.000	0.049

DM: KL = 20' for C1-2 and C1-2b

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 7d, analysis results in Tables 43 and 44 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. Analysis results in Table 43 were obtained using Direct Modeling Approach, whereas those in Table 44 were obtained using Notional Load Approach. However, these results both lead to the same conclusions.

Column C1-2a has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is greater than 1.0 by 8%,
- ALR value by MDM is closer to 1.0 than that of DM,
- ALR value by MDM is greater than that of DM, and
- The difference between ALR values by the two methods is not less than 4.5%.

Table 43: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 7d – Two-bay Frame with Unequal Heights)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.514	0.000	1.440	0.271	0.000	1.440
C1-2a	0.511	0.547	**0.895**	0.325	0.762	**1.080**
C1-2b	0.346	0.736	1.060	0.201	0.895	1.170
C1-3	0.997	0.000	1.320	0.556	0.000	1.440
B1-1	0.012	0.638	1.440	0.011	0.638	1.440
B1-2	0.010	0.664	1.440	0.009	0.664	1.440
Bracing	0.167	0.000	1.440	0.167	0.000	1.440

DM: KL = 20' for C1-2 and C1-2b

Table 44: Comparison 2(ALR at H1-1=1.0) Using Notional Load Approach (Structural System 7d – Two-bay Frame with Unequal Heights)

Imperfection NL 0.002Yi **Stiffness Adjustment** 0.8E and No NL

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.514	0.000	1.440	0.271	0.000	1.440
C1-2a	0.511	0.547	**0.895**	0.324	0.756	**1.075**
C1-2b	0.345	0.736	1.060	0.201	0.895	1.170
C1-3	0.997	0.000	1.320	0.556	0.000	1.440
B1-1	0.012	0.638	1.440	0.012	0.638	1.440
B1-2	0.010	0.664	1.440	0.010	0.664	1.440
Bracing	0.167	0.000	1.440	0.167	0.000	1.440

DM: KL = 20' for C1-2 and C1-2b

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is not adequate to assess the stability of structural system 7d, since its H1-1 value for the controlling member (column C1-

2a) is less than 1.0 by 11 %, and its ALR value for the controlling member is greater than 1.0 by 8%

MDM is a more accurate method than DM for structural system 7d, since its controlling member (column C1-2a) has a H1-1 value closer to 1.0 than that of DM, and it has an ALR value closer to 1.0 than that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 7d, since the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Structural System 8 – Vierendeel Truss

Comparison 1: Comparing H1-1 when ALR = 1.0

For structural system 8, Tables 45 and 46 compare the AISC interaction equation H1-1 values at an applied load ratio of 1.0 obtained by the Direct Analysis Method (DM) and Modified Direct Analysis Method (MDM) procedure. However, Table 45 analysis results were obtained using Direct Modeling Approach, whereas Table 46 analysis results were obtained using Notional Load Approach. These tables show that Direct Modeling Approach and Notional Load Approach lead to the same conclusions in comparing H1-1 values by DM and MDM when ALR =1.0. This confirms the equivalency of Direct Modeling Approach and Notional Load Approach. Both of these approaches lead to the following conclusions about DM and MDM.

Top chord 3 has the largest H1-1 value at an applied load ratio of 1.0. For this controlling member, it is observed that

- H1-1 value by MDM is greater than 1.0 by 23%,
- H1-1 value by MDM is closer to 1.0 than that of DM,
- H1-1 value by MDM is less than that of DM, and
- The difference between AISC interaction equation H1-1 values by the two methods is not less than 4.5%.

Table 45: Comparison 1(H1-1 at ALR =1.0) Using Direct Modeling Approach (Structural System 8 – Vierendeel Truss)

Imperfection	Direct Modeling	**Stiffness Adjustment**	0.8E and tau_b
	second-order elastic; P-C; increment 0.01		

1	Eq. H1-1 at an Applied Load Ratio =1.00							
	DM: K = 1				MDM: $P_n = P_y$			
Member	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	Eq. H1-1	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	Eq. H1-1
TC-1	0.278	0.414	0.361	0.967	0.072	0.414	0.361	0.811
TC-2	0.653	0.372	0.770	1.668	0.169	0.372	0.770	1.226
TC-3	0.858	0.198	0.932	**1.862**	0.222	0.198	0.932	**1.226**
BC-1	0.074	0.576	0.083	0.696	0.074	0.576	0.083	0.696
BC-2	0.169	0.409	0.046	0.540	0.169	0.409	0.046	0.540
BC-3	0.222	0.205	0.009	0.411	0.222	0.205	0.009	0.411
W-1	0.108	0.629	0.034	0.717	0.104	0.629	0.034	0.715
W-2	0.042	0.810	0.036	0.866	0.040	0.791	0.031	0.842
W-3	0.029	0.434	0.024	0.473	0.028	0.434	0.022	0.470
W-4	0.029	0.000	0.025	0.040	0.028	0.000	0.015	0.030

DM: KL = 48' for TC-1, TC-2, and TC-3; KL = 8' for W-1,W-2,W-3 and W-4

Table 46: Comparison 1(H1-1 at ALR =1.0) Using Notional Load Approach (Structural System 8 – Vierendeel Truss)

Imperfection	NL 0.002Y_i	**Stiffness Adjustment**	0.8E and No NL
	second-order elastic; P-C; increment 0.01		

1	Eq. H1-1 at an Applied Load Ratio =1.00							
	DM: K = 1				MDM: $P_n = P_y$			
Member	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	Eq. H1-1	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	Eq. H1-1
TC-1	0.278	0.410	0.397	0.995	0.072	0.410	0.397	0.843
TC-2	0.653	0.369	0.832	1.720	0.169	0.369	0.832	1.285
TC-3	0.857	0.196	1.001	**1.921**	0.221	0.196	1.001	**1.286**
BC-1	0.074	0.578	0.078	0.693	0.074	0.578	0.078	0.693
BC-2	0.169	0.409	0.039	0.533	0.169	0.409	0.039	0.533
BC-3	0.221	0.204	0.010	0.412	0.221	0.204	0.010	0.412
W-1	0.108	0.630	0.034	0.718	0.104	0.630	0.034	0.716
W-2	0.042	0.809	0.036	0.866	0.040	0.789	0.036	0.846
W-3	0.029	0.433	0.025	0.473	0.028	0.433	0.024	0.472
W-4	0.029	0.000	0.027	0.041	0.028	0.000	0.017	0.031

DM: KL = 48' for TC-1, TC-2, and TC-3; KL = 8' for W-1,W-2,W-3 and W-4

Comparison 2: Comparing ALR when H1-1 = 1.0

For structural system 8, analysis results in Table 47 compare ALR values obtained by DM and MDM when the interaction equation H1-1 equals unity. The analysis results were obtained using Direct Modeling Approach and lead to the following conclusions.

Top chord 3 has the lowest ALR value at interaction equation value of 1.0. For this controlling member, it is observed that

- ALR value by MDM is less than 1.0 by 3%,
- ALR value by MDM is closer to 1.0 than that of DM,
- ALR value by MDM is greater than that of DM, and
- The difference between ALR values by the two methods is not less than 4.5%.

Table 47: Comparison 2(ALR at H1-1=1.0) Using Direct Modeling Approach (Structural System 8 – Vierendeel Truss)

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_b$

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00							
	DM: K = 1				MDM: $P_n = P_y$			
Member	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	ALR	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	ALR
TC-1	0.280	0.403	0.398	1.010	0.076	0.279	0.680	1.085
TC-2	0.546	0.329	0.173	0.830	0.163	0.372	0.532	0.965
TC-3	0.691	0.168	0.176	**0.800**	0.216	0.197	0.678	**0.970**
BC-1	0.297	0.664	0.110	1.040	0.081	0.814	0.137	1.095
BC-2	0.628	0.384	0.033	0.955	0.215	0.656	0.226	1.320
BC-3	0.814	0.198	0.009	0.945	0.296	0.351	0.434	1.485
W-1	0.111	0.868	0.076	1.165	0.107	0.868	0.076	1.165
W-2	0.068	0.891	0.074	1.100	0.065	0.891	0.074	1.100
W-3	0.004	0.225	0.536	2.515	0.004	0.225	0.536	2.515
W-4	0.191	0.000	0.277	2.515	0.184	0.000	0.277	2.515

DM: KL = 48' for TC-1, TC-2, and TC-3; KL = 8' for W-1,W-2,W-3 and W-4

Conclusions

The observations from Comparisons 1 and 2 suggest that MDM is adequate to assess the stability of structural system 8, since its H1-1 value for the controlling member (top chord 3) is greater than 1.0, and its ALR value for the controlling member is less than 1.0.

MDM is a more accurate method than DM for structural system 8, since its controlling member has a H1-1 value closer to 1.0 than that of DM, and it has an ALR value closer to 1.0 than that of DM.

However, it should be kept in mind that MDM is a less conservative method than DM, since it tends to result in lower H1-1 values or higher ALR values than DM.

Moreover, it should be noted that DM and MDM are not equivalent for assessing the stability of structural system 8, since the results by DM and MDM for all members do not always match within 4.5%.

On a side note, this case study confirms the equivalence between Direct Modeling Approach and Notional Load Approach, since these approaches lead to similar results.

Column Study

As mentioned earlier in Section 2.1.3, different from all other case studies, in this column study, whether DM and MDM is a more accurate method will be determined based on comparing the axial strengths of the column (P_u/P_y) obtained by these two methods against the advanced inelastic analysis results (Appx 1).

Major Axis Bending

As can be seen in Table 48 and Figure 17,

For $L/r \geq 100$,

- P_u/P_y value by MDM has negative percent difference from that of Appendix 1, and
- P_u/P_y value by MDM has greater negative percent differences from Appendix 1 than that by DM.

Table 48: Comparison of Major Axis Strength of Column Obtained by Different Analysis Methods (Appx1, DM and MDM) and Their Percent Differences

	Major Axis Strength (P_u/P_y)			Percent Difference (%)	
L/r_x	Appx 1	DM	MDM (tau_B)	DM vs Appx 1	MDM (tau_B) vs Appx 1
0	0.9	0.9	0.9	0.000	0.000
10	0.890	0.893	0.891	0.395	0.158
20	0.874	0.874	0.882	0.060	0.965
30	0.848	0.843	0.87	-0.601	2.597
40	0.813	0.801	0.852	-1.478	4.841
50	0.770	0.750	0.821	-2.705	6.565
60	0.717	0.692	0.769	-3.559	7.228
70	0.656	0.629	0.694	-4.079	5.857
80	0.583	0.564	0.603	-3.341	3.373
90	0.506	0.498	0.506	-1.652	0.037
100	0.435	0.433	0.422	-0.348	-3.031
110	0.373	0.372	0.354	-0.257	-4.942
120	0.321	0.314	0.301	-2.120	-6.195
130	0.278	0.267	0.258	-3.744	-7.083
140	0.242	0.231	0.224	-4.895	-7.633
150	0.213	0.201	0.196	-5.732	-8.091
160	0.189	0.176	0.173	-6.428	-8.369
170	0.168	0.156	0.153	-6.971	-8.863
180	0.151	0.139	0.137	-7.422	-8.893
190	0.136	0.125	0.123	-7.793	-9.248
200	0.123	0.113	0.111	-8.129	-9.333

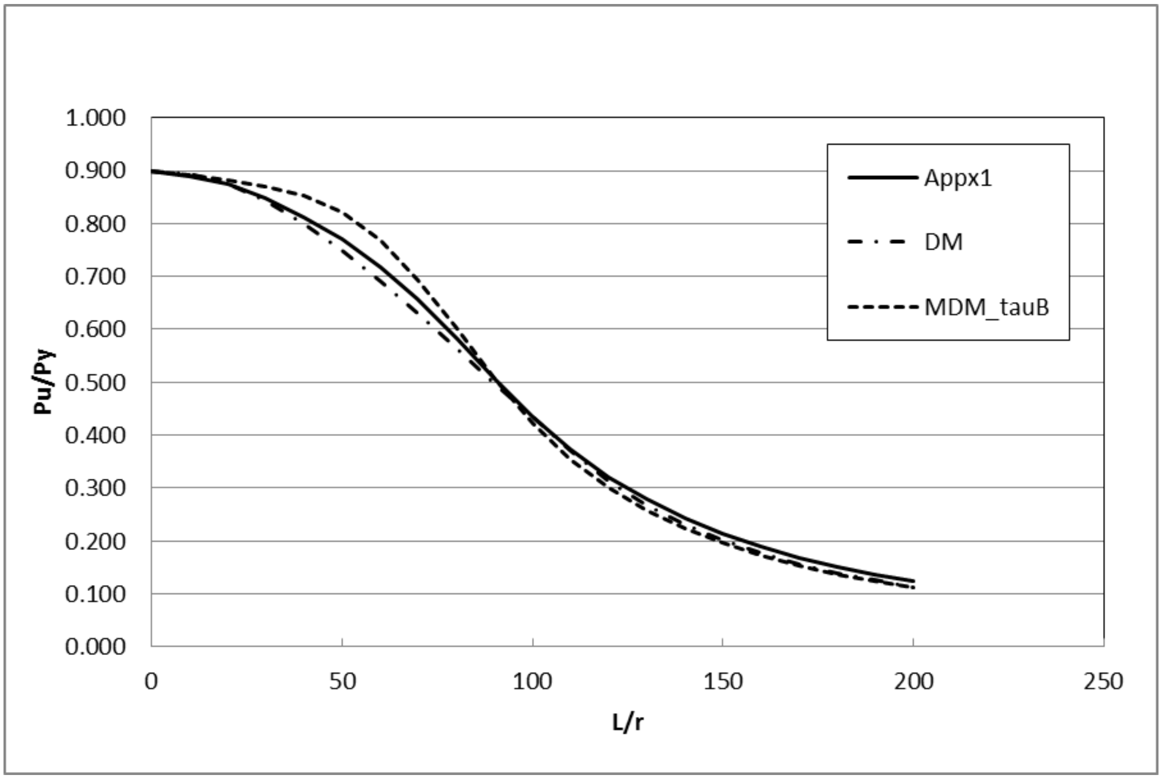


Figure 17: Comparison of Major Axis Strength of Column Obtained by Different Analysis Methods

Minor Axis Bending

As can be seen in Table 49 and Figure 18,

For L/r ≥ 120,

- P_u/P_y value by MDM has negative percent difference from that of Appendix 1, and
- P_u/P_y value by MDM has greater negative percent differences from Appendix 1 than that by DM.

Table 49: Comparison of Minor Axis Strength of Column Obtained by Different Analysis Methods (Appx1, DM and MDM) and Their Percent Differences

	Minor Axis Strength (P_u/P_y)			Percent Difference (%)	
L/r_y	Appx 1	DM	MDM (tau_B)	DM vs Appx 1	MDM (tau_B) vs Appx 1
0	0.9	0.9	0.9	0.000	0.000
10	0.892	0.893	0.889	0.126	-0.320
20	0.872	0.874	0.878	0.194	0.617
30	0.838	0.843	0.863	0.510	2.961
40	0.776	0.801	0.842	3.197	8.549
50	0.706	0.75	0.809	6.169	14.561
60	0.646	0.692	0.755	7.015	16.812
70	0.585	0.629	0.68	7.432	16.160
80	0.521	0.564	0.59	8.285	13.378
90	0.457	0.498	0.496	9.000	8.718
100	0.397	0.433	0.415	9.133	4.543
110	0.344	0.372	0.349	7.926	1.497
120	0.299	0.314	0.297	4.982	-0.486
130	0.261	0.267	0.256	2.495	-1.957
140	0.229	0.231	0.222	0.709	-3.008
150	0.202	0.201	0.194	-0.688	-3.868
160	0.18	0.176	0.171	-1.820	-4.638
170	0.161	0.156	0.152	-2.743	-5.303
180	0.145	0.139	0.136	-3.522	-5.703
190	0.131	0.125	0.123	-4.155	-6.026
200	0.119	0.113	0.111	-4.717	-6.361

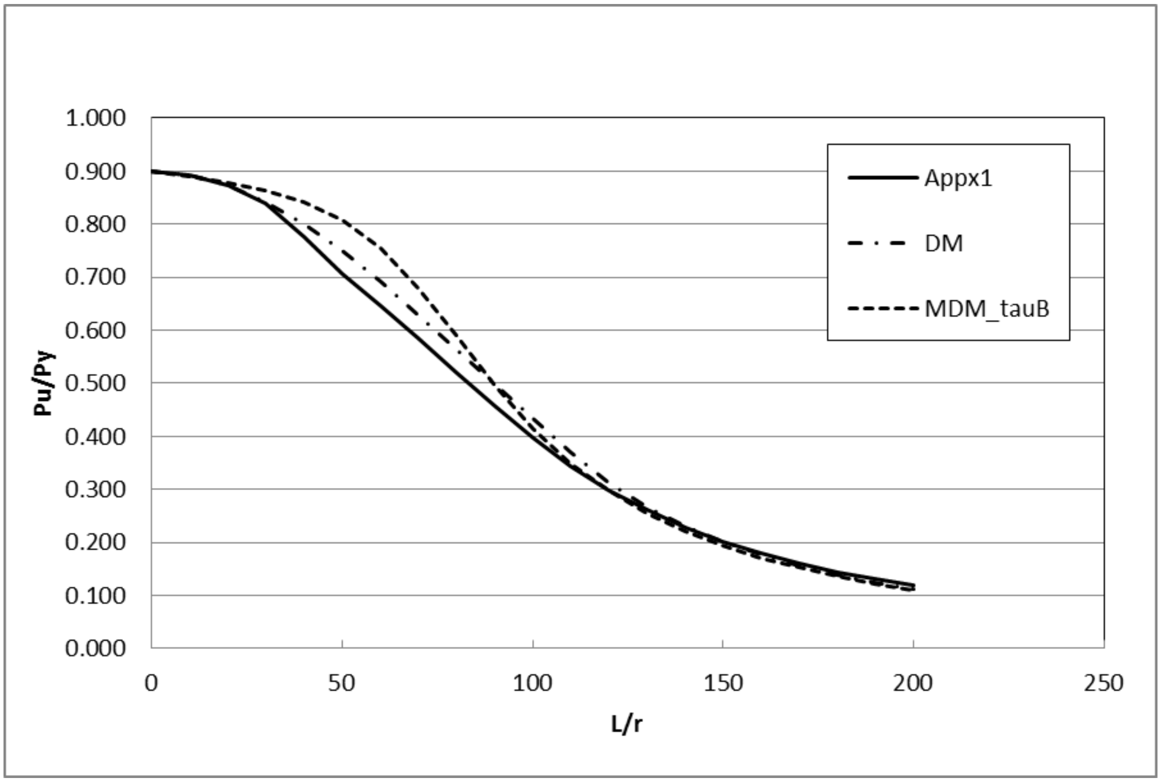


Figure 18: Comparison of Minor Axis Strength of Column Obtained by Different Analysis Methods

Conclusions

The observations suggest that, in cases where system is of low to no redundancy,

For major axis column bending with L/r ≥ 100 and for minor axis bending with L/r ≥ 120,

MDM is adequate to assess the stability of the system, since its P_u/P_y value is less than that by Appendix 1(negative percent difference). The negative percent difference means that MDM indicates that system can resist less applied load than predicted by Appendix 1.

When comparing to DM, MDM is a less accurate method than DM, since P_u/P_y value by MDM has greater percent difference from Appendix 1 than that by DM.

However, MDM is a more conservative method than DM, since P_u/P_y value by MDM has greater negative percent difference from Appendix 1 than that by DM.

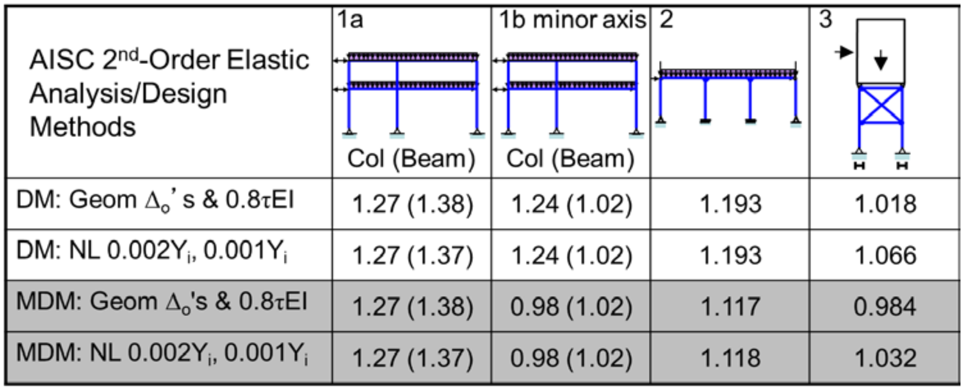
CHAPTER4: SUMMARY OF RESULTS

Based on the conclusions for each case study presented in Chapter 3, the following overall conclusions can be made:

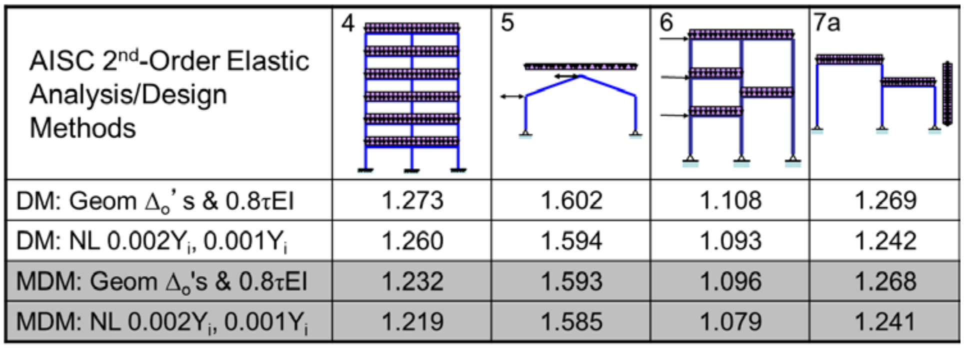
- The Modified Direct Analysis Method (MDM) method is adequate to assess the stability of structural steel systems with a few exceptions.
    - MDM analyses consistently result in predicting conservative to acceptable member strength limits, with AISC interaction equation H1-1 values greater than 1.0 or ALR values less than 1.0 (Tables 51 and 52).
    - In a few cases, including systems 7b and 7d, MDM may not be adequate to assess the stability of structural systems. MDM indicates non-conservative results, with interaction equation H1-1 values less than 1.0 by more than 5% and ALR values greater than 1.0 by more than 5%.
    - For structural systems with little or no redundancy, for example the column study presented in this thesis, both the DM and MDM appear inadequate to assess the stability of structural systems for cases in which slenderness ratio, $L/r < 100$ for major axis bending, and $L/r < 120$ for minor axis bending. This is because the predicted strengths (P_u/P_y values) by both DM and MDM are greater than those by Appendix 1 for these cases (positive percent difference).
- In general, MDM is a more accurate method than DM for assessing the stability of structural steel systems with a few exceptions.
    - MDM analyses result in member strength limit states defined by interaction equation H1-1 or ALR values closer to 1.0 than those obtained by DM analyses (Tables 50 and 51).

  - One of these exceptions is that the MDM method is a less accurate than DM for assessing the stability of structural system 7b. MDM indicate failure of column C1-1 with H1-1 and ALR values further away from 1.0 than those obtained by DM.
  - For structural systems with little or no redundancy (for example, the column study in this thesis), MDM is a less accurate method than DM for cases in which $L/r \geq 100$ for major axis orientation and $L/r \geq 120$ for minor axis orientation. This is because P_u/P_y values by MDM have greater percent difference from Appendix 1 than those by DM.
- In general, the MDM method is a less conservative design procedure than DM for assessing the stability of structural steel systems with a few exceptions.
  - MDM tends to consistently result in lower, but often acceptable, H1-1 values or higher ALR values than DM (Tables 50 and 51).
  - Interestingly, for structural systems with little or no redundancy (again, the column study in this thesis), MDM is a little more conservative than DM for cases for slender columns in which $L/r \geq 100$ for major axis bending and $L/r \geq 120$ for minor axis bending. This is because P_u/P_y values by MDM have greater negative percent difference from Appendix 1 than those by DM.
- DM and MDM differ in assessing the stability of structural systems.
  - All case studies, except the gabled frame (Structural System 5) and the two-bay frame with irregular geometry (Structural System 6), Tables 50 and 51 show that DM and MDM do not always result in the same H1-1 or ALR values for all members.

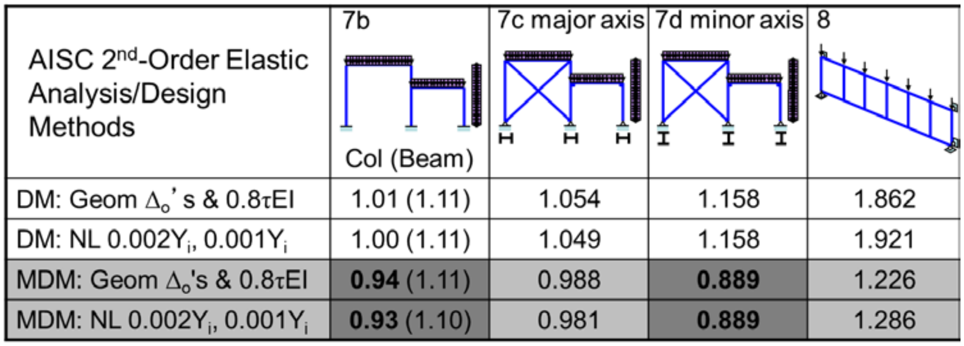
Table 50: Summary Table of Comparisons of DM and MDM for All Case Study Steel Frames
(Comparison 1: H1-1 when ALR=1.0)



AISC 2nd-Order Elastic Analysis/Design Methods	1a Col (Beam)	1b minor axis Col (Beam)	2	3
DM: Geom Δ_o' s & 0.8τEI	1.27 (1.38)	1.24 (1.02)	1.193	1.018
DM: NL 0.002Y_i, 0.001Y_i	1.27 (1.37)	1.24 (1.02)	1.193	1.066
MDM: Geom Δ_o's & 0.8τEI	1.27 (1.38)	0.98 (1.02)	1.117	0.984
MDM: NL 0.002Y_i, 0.001Y_i	1.27 (1.37)	0.98 (1.02)	1.118	1.032

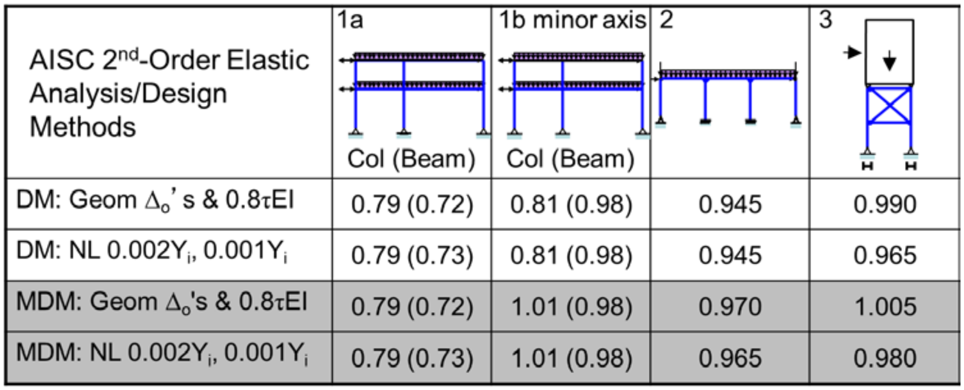


AISC 2nd-Order Elastic Analysis/Design Methods	4	5	6	7a
DM: Geom Δ_o' s & 0.8τEI	1.273	1.602	1.108	1.269
DM: NL 0.002Y_i, 0.001Y_i	1.260	1.594	1.093	1.242
MDM: Geom Δ_o's & 0.8τEI	1.232	1.593	1.096	1.268
MDM: NL 0.002Y_i, 0.001Y_i	1.219	1.585	1.079	1.241

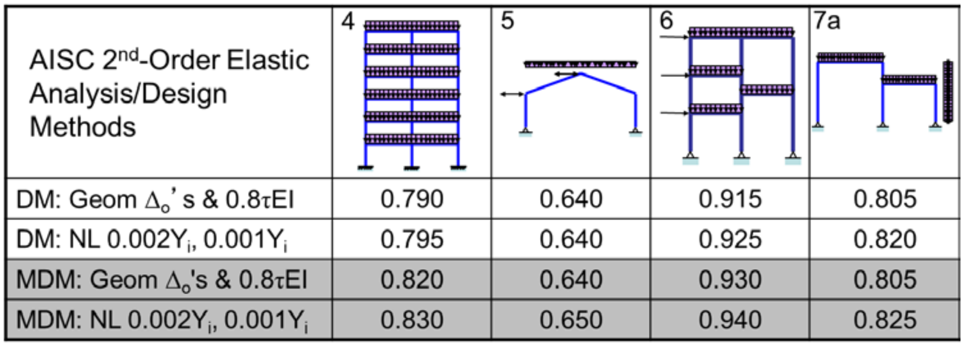


AISC 2nd-Order Elastic Analysis/Design Methods	7b Col (Beam)	7c major axis	7d minor axis	8
DM: Geom Δ_o' s & 0.8τEI	1.01 (1.11)	1.054	1.158	1.862
DM: NL 0.002Y_i, 0.001Y_i	1.00 (1.11)	1.049	1.158	1.921
MDM: Geom Δ_o's & 0.8τEI	**0.94** (1.11)	0.988	**0.889**	1.226
MDM: NL 0.002Y_i, 0.001Y_i	**0.93** (1.10)	0.981	**0.889**	1.286

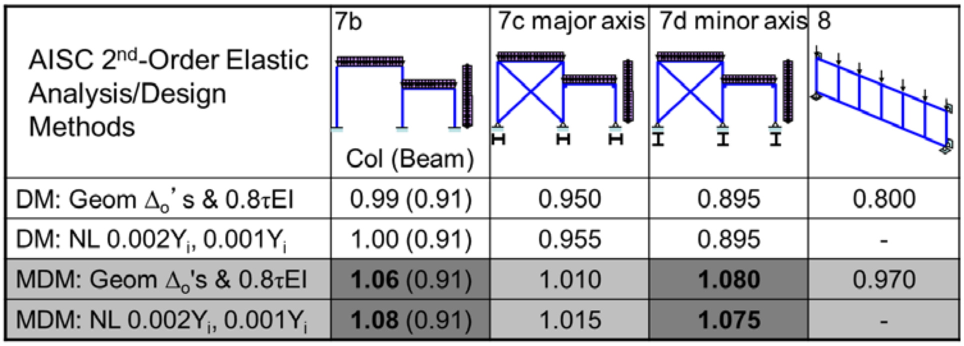
Table 51: Summary Table of Comparisons of DM and MDM for All Case Study Steel Frames
(Comparison 2: ALR when H1-1=1.0)



AISC 2^{nd}-Order Elastic Analysis/Design Methods	1a Col (Beam)	1b minor axis Col (Beam)	2	3
DM: Geom Δ_o' s & $0.8\tau EI$	0.79 (0.72)	0.81 (0.98)	0.945	0.990
DM: NL $0.002Y_i$, $0.001Y_i$	0.79 (0.73)	0.81 (0.98)	0.945	0.965
MDM: Geom Δ_o's & $0.8\tau EI$	0.79 (0.72)	1.01 (0.98)	0.970	1.005
MDM: NL $0.002Y_i$, $0.001Y_i$	0.79 (0.73)	1.01 (0.98)	0.965	0.980



AISC 2^{nd}-Order Elastic Analysis/Design Methods	4	5	6	7a
DM: Geom Δ_o' s & $0.8\tau EI$	0.790	0.640	0.915	0.805
DM: NL $0.002Y_i$, $0.001Y_i$	0.795	0.640	0.925	0.820
MDM: Geom Δ_o's & $0.8\tau EI$	0.820	0.640	0.930	0.805
MDM: NL $0.002Y_i$, $0.001Y_i$	0.830	0.650	0.940	0.825



AISC 2^{nd}-Order Elastic Analysis/Design Methods	7b Col (Beam)	7c major axis	7d minor axis	8
DM: Geom Δ_o' s & $0.8\tau EI$	0.99 (0.91)	0.950	0.895	0.800
DM: NL $0.002Y_i$, $0.001Y_i$	1.00 (0.91)	0.955	0.895	-
MDM: Geom Δ_o's & $0.8\tau EI$	**1.06** (0.91)	1.010	**1.080**	0.970
MDM: NL $0.002Y_i$, $0.001Y_i$	**1.08** (0.91)	1.015	**1.075**	-

CHAPTER 5: SUMMARY AND CONCLUSIONS

5.1. Summary

This thesis investigates a new stability assessment procedure for use in the design of structural steel systems, namely the Modified Direct Analysis Method (MDM) method. This new method proposes that by employing a rigorous second-order elastic analysis that accounts for the destabilizing effects of imperfections and inelasticity, structural steel systems can be adequately designed with only the need to check the cross section strength of members.

A structural system is considered stable when the load effects acting on each of its members are less than or equal to their strength to resist them. In structural systems that are modeled to include initial imperfections, both axial and bending load effects tend to be present in each member (that is, all members become beam-column), and thus it becomes necessary to understand how the interaction between these two load effects and their corresponding strengths impact the stability of the member. The interaction between axial and bending moment effects on a member follows the concept that one load effect (say, axial force) will reduce the member's ability to resist the other load effect (say, bending). The AISC interaction equations used to represent this concept were derived, following the process of determining axial strength in the presence of a given bending moment, or determining bending moment strength in the presence of a given axial load. A structural member subjected to both axial load and bending moment is considered stable if its load effects and corresponding strengths satisfy the AISC interaction equation.

AISC recognizes two existing methods in evaluating structural stability by means of interaction equations, including the effective length method (ELM) and the direct analysis method (DM). ELM makes use of effective length factor (K) for each structural member in determining frame and member stability. The process of finding K for every single structural member can be laborious, time-consuming, and can involve inaccuracies. DM takes resovles this problem by assuming unit effective length factors for every member, and thus eliminating the need to calculate K values. This assumption is made possible by utilizing a second-order elastic analysis that accounts for inelasticity and member imperfections in the modeling. The method proposed in this research, Modified Direct Analysis Method (MDM), is intended to further simplify DM by assuming the analysis will detect member and frame instabilities, and thereby resulting in the need to checking only the cross section strength of members to assess their stability.

To study the feasibility of MDM, this thesis utilized a set of 12 benchmark structural steel systems, and a column study. DM and MDM were compared in two ways to determine which method is a more accurate method for accessing stability. The first comparison was made based on accessing the stability of the systems at the given applied loads. For this comparison, the AISC interaction equation values at the applied load ratio of 1.0 (H1-1 when ALR =1.0) were calculated for all members. Since the given applied loads were calibrated failure loads defined by advanced inelastic analyses, the DM and MDM methods were determined adequate for assessing the stability of the systems if the memthod resulted in interaction equation values of 1.0 or greater. Moreover, the method that resulted in the AISC interaction equation values closer to 1.0 was considered a more accurate procedure. The second comparison was made based on accessing the stability of the systems at the corresponding failure loads by each method. For this comparison, the applied load ratios at which the failure of the system occurs were achieved (ALR when H1-1 =1.0). A method was considered adequate to assess the stability of the systems if it resulted in applied load ratios of 1.0 or smaller. Similarly, the method that resulted in applied load ratios closer to 1.0 for failure was considered a more accurate method.

5.2. Conclusions

Based on the stability analysis results of the case studies presented in this thesis, it is observed that MDM is adequate to assess the stability of structural systems, with perhaps a few exceptions. For all structural systems, except Structural Systems 7b and 7d, MDM analyses provided conservative results for predicting strength limits with AISC interaction equation H1-1 values of 1.0 or greater and ALR values of 1.0 or less. For structural system 7d, in which the columns are oriented for minor axis bending, it is observed that MDM provides non-conservative results, with an H1-1 value of less than 1.0 by 11% and ALR value of greater than 1.0 by 8%. Moreover, for structural systems with little or no redundancy (for example, the column study in this thesis), MDM appears inadequate to assess the stability for cases in which slenderness ratio of $L/r < 100$ for major axis bending and $L/r < 120$ for minor axis bending, because the MDM indicates that the system can resist more applied loads than predicted by the advanced analysis procedure of Appendix 1.

Secondly, MDM appears to be a more accurate method for assessing stability of structural steel systems with a few exceptions. MDM consistently provided AISC interaction equation H1-1 values and ALR values closer to 1.0 than DM for all case studies investigated except Structural System 7b and the column study. It appears that for structural systems with little or no redundancy, specifically the column study in this thesis, MDM is a less accurate method compared to DM to assessing the stability for cases in which the column slenderness $L/r \geq 100$ for major axis bending and $L/r \geq 120$ for minor axis bending. For these cases, the differences between strengths predicted by MDM and Appendix 1 are greater than the differences between strengths predicted by DM and Appendix 1.

Thirdly, it is observed that MDM provides less conservative (but still acceptable) results compared to DM with a few exceptions. In general, MDM tends to provide lower AISC interaction equation H1-1 values or higher ALR values compared to DM for vast majority of the structural systems investigated in this study. For structural systems with little or no redundancy (again, the column study in this thesis), MDM is a little more conservative method compared to DM to assess the stability for cases in which $L/r \geq 100$ for major axis bending and $L/r \geq 120$ for minor axis bending. For these cases, MDM indicated that the column would resist less applied loads than predicted by both Appendix 1 and DM.

It is also noteworthy that DM and MDM are not identical in assessing the stability of structural systems, because they do not always provide results within an acceptable tolerance of say 4.5%. Moreover, and as a side study, the research performed as part of this thesis confirms the equivalency of the two approaches for modeling the destabilizing effects of initial imperfections and material inelasticity – Direct Modeling Approach and Notional Load Approach.

Overall, and noting the few exception described above, the results of case studies investigated in this research confirm the thesis statement; employing a rigorous second-order elastic analysis that accounts for the destabilizing effects of imperfections and inelasticity, the stability of structural steel systems can be adequately assessed with only the need to check the cross section strength of members. In other words, the stability of structural steel systems can be adequately assessed using the new proposed stability assessment method, Direct Analysis of Member Imperfections, MDM.

Considering that both DM and MDM are adequate to assess the stability of structural systems, a list of trade-offs between DM and MDM are now provided. Firstly, simplifying DM into MDM will be particularly useful for cases in which it is not clear how to define member slenderness L/r when the laterally unbraced length L is not apparent, such as arches and the compression chord of an unbraced truss. Secondly, MDM appears to be a more accurate method than DM for assessing the stability of structural systems. However, the trade-offs for these advantages will be that MDM would require more modeling and computational time if member imperfections and P-δ effects are included in each model by subdividing members into many more elements. Moreover, MDM tends to sacrifice its conservativeness to achieve more accuracy in assessing structural system stability.

5.3. Recommendations for Further Research

It is recommended that further studies on structural systems with beam-columns subject to minor axis flexure (such as structural system 7d) should be performed to validate the adequacy of employing Modified Direct Analysis Method (MDM) to assess their stability. When studying these structural systems, it is suggested that the modulus of elasticity E be reduced to a smaller value than 0.8E such as 0.7E or 0.75E. This reduction may result in increased moment load effects M_u and consequently result in increased AISC interaction equation H1-1 values. This may cause MDM to always predict lower strength limits than those predicted by the use advanced inelastic analysis (AISC Appendix 1).

In addition, all the structural systems in this thesis except structural System 8 (Vierendeel Truss) were assumed to be fully braced out of plane. Further study on the adequacy of MDM should be performed for cases in which the members are no longer fully braced out of plane. For these cases, it will be interesting to observe whether the controlling moment strength of each member (M_n) will still be the member cross-section plastic yielding strength (M_p) Note that in Structural System 8, the loadings and member sections were defined so that $M_n = M_p$. It will be of particular interest to examine the validity of MDM when M_n is no longer M_p.

Moreover, it is recommended that further study on the adequacy of MDM should be performed on structural members with shapes other than wide-flange sections such as hollow rectangular HSS and channels.

Furthermore, this thesis only investigated structural systems comprised of members with compact cross sections. It will be interesting to further study whether MDM is adequate to assess the stability of structural systems that include non-compact and non-slender elements.

APPENDIX A: MASTAN2 ANALYSIS MODELS

Structural System 1a

Direct Modeling Approach

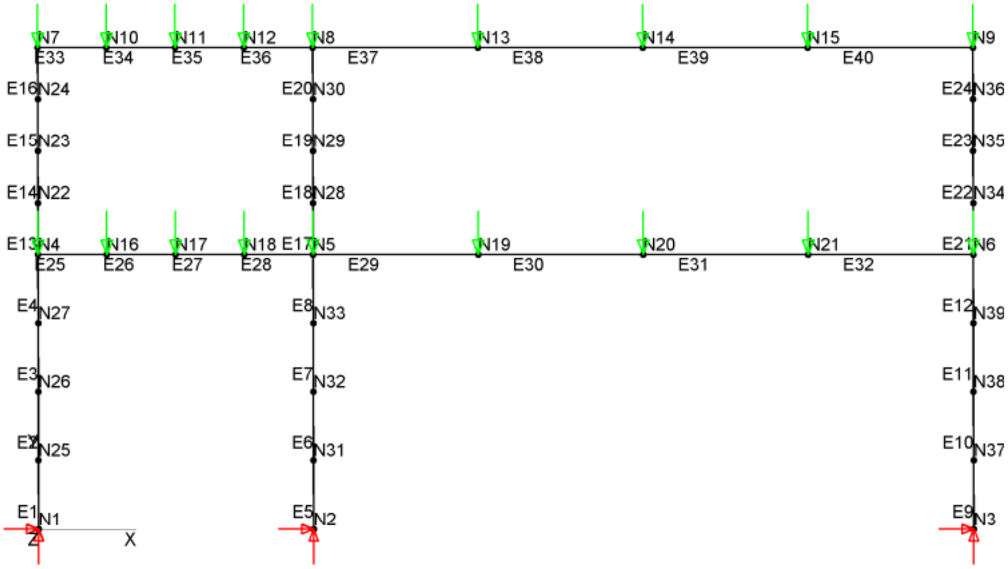


Figure 1: MASTAN2 Analysis Model

Notional Load Approach

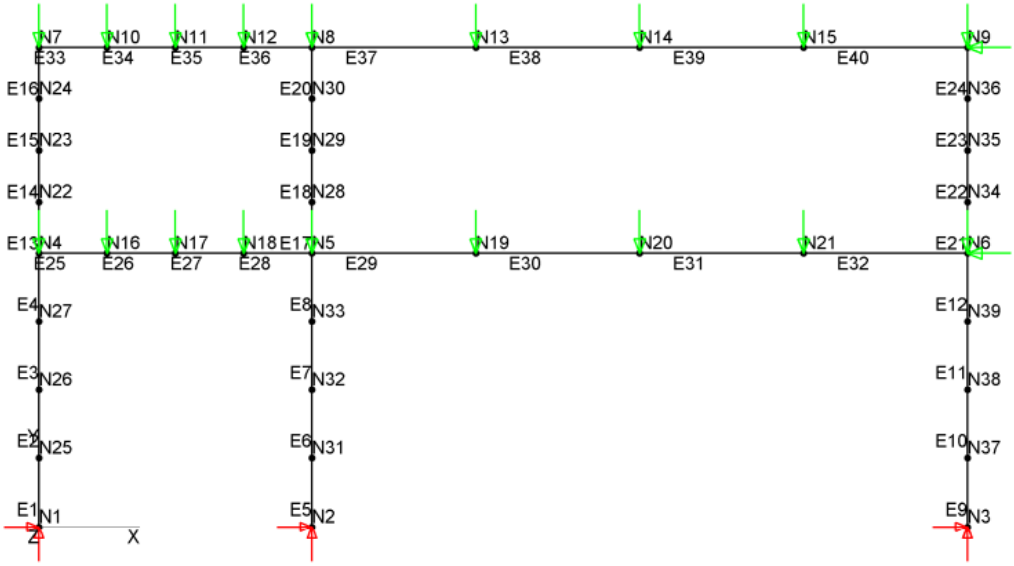


Figure 2: MASTAN2 Analysis Model

Structural System 1b

Direct Modeling Approach

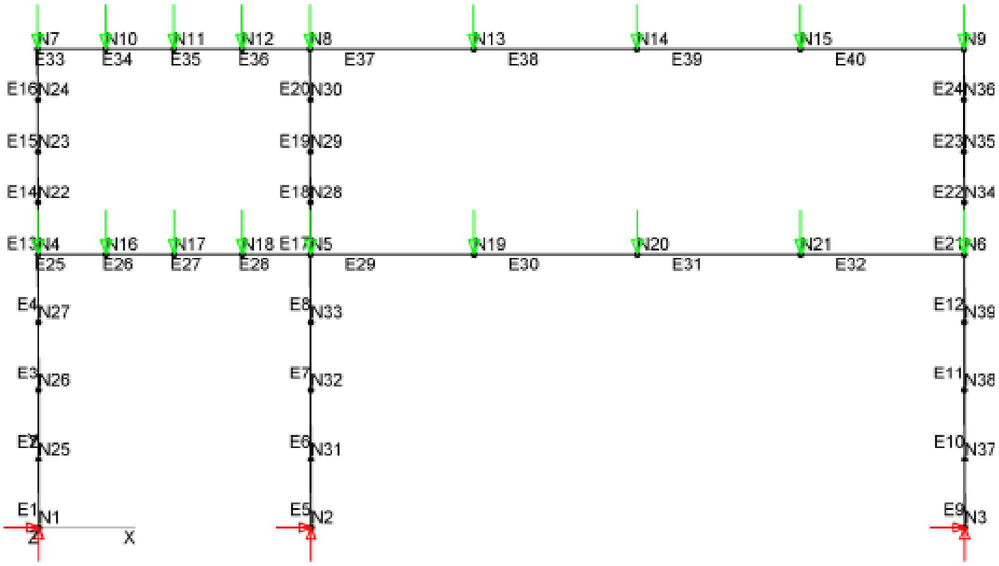


Figure 3: MASTAN2 Analysis Model

Notional Load Approach

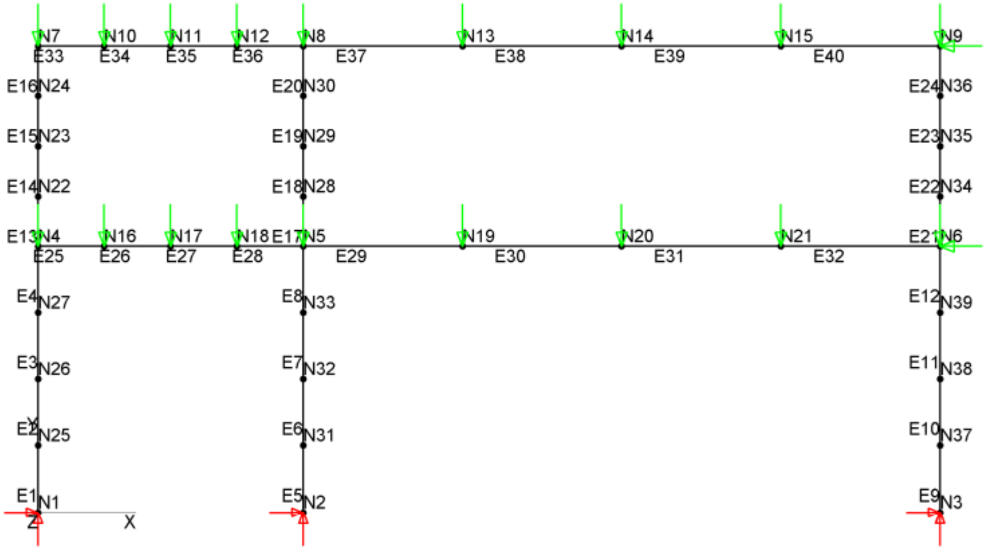


Figure 4: MASTAN2 Analysis Model

Structural System 2

Direct Modeling Approach

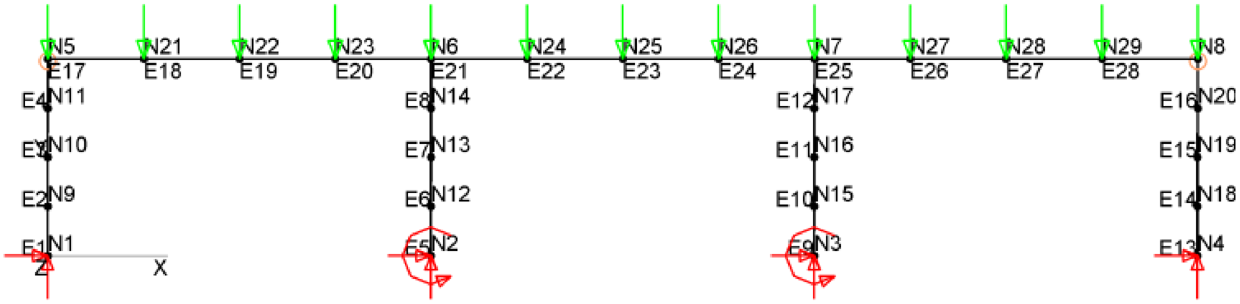


Figure 5: MASTAN2 Analysis Model

Notional Load Approach

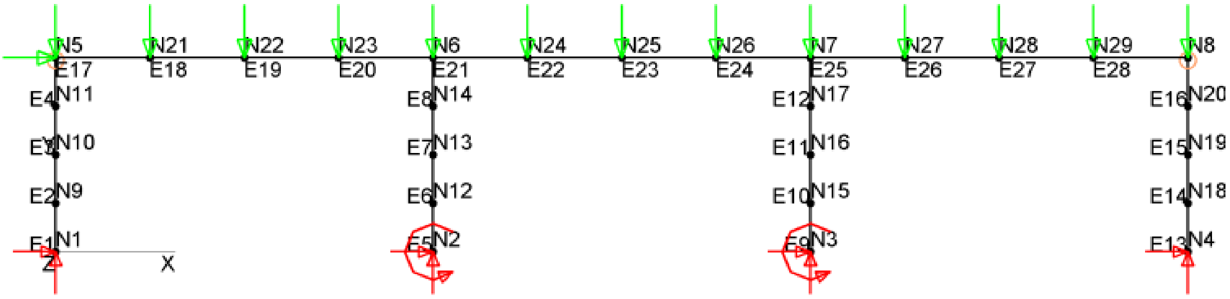


Figure 6: MASTAN2 Analysis Model

Structural System 3

Direct Modeling Approach

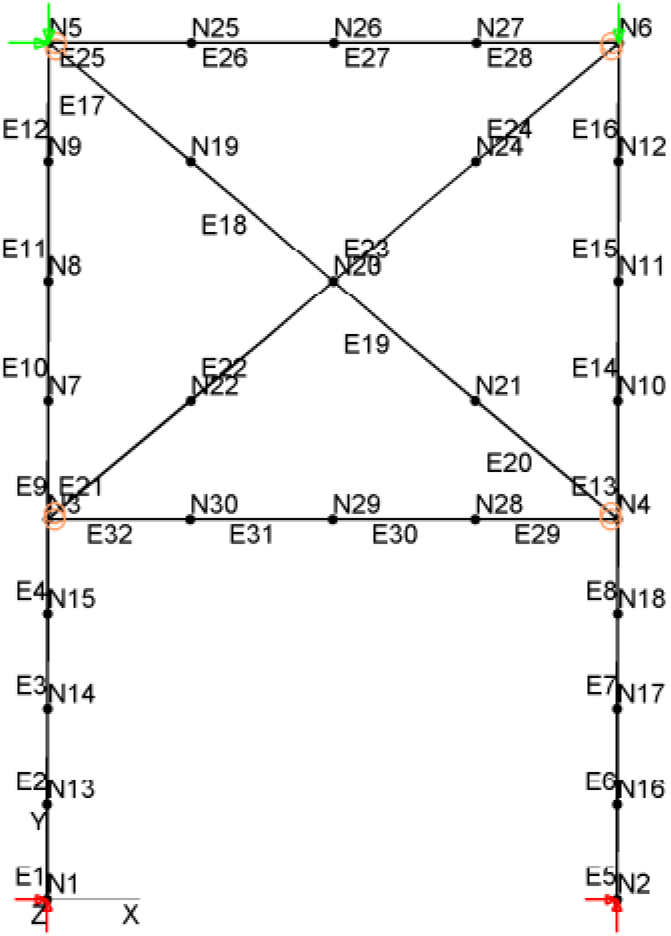


Figure 7: MASTAN2 Analysis Model

Notional Load Approach

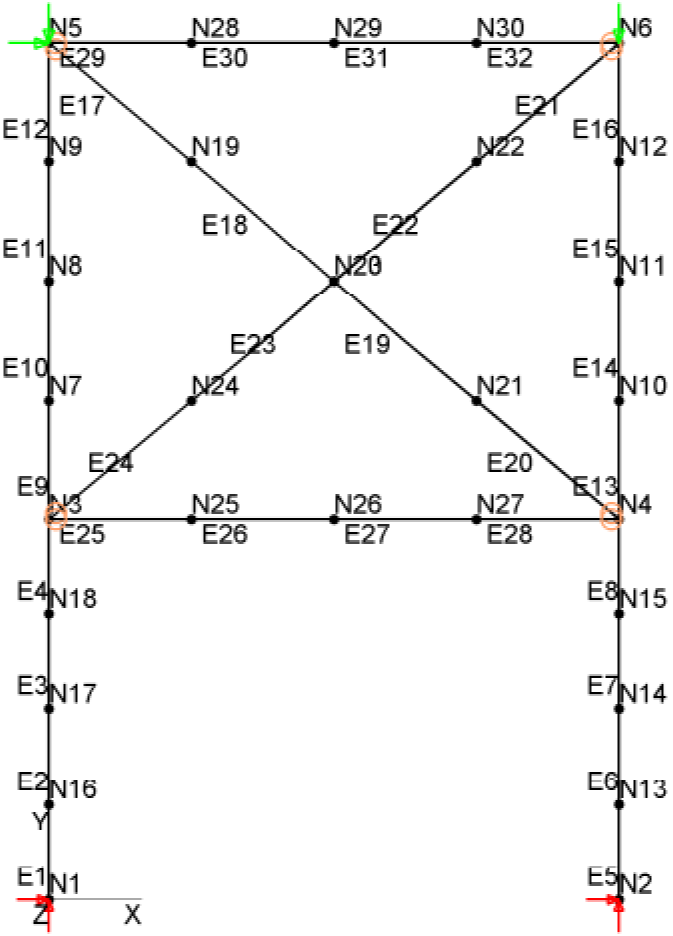


Figure 8: MASTAN2 Analysis Model

Structural System 4

Direct Modeling Approach

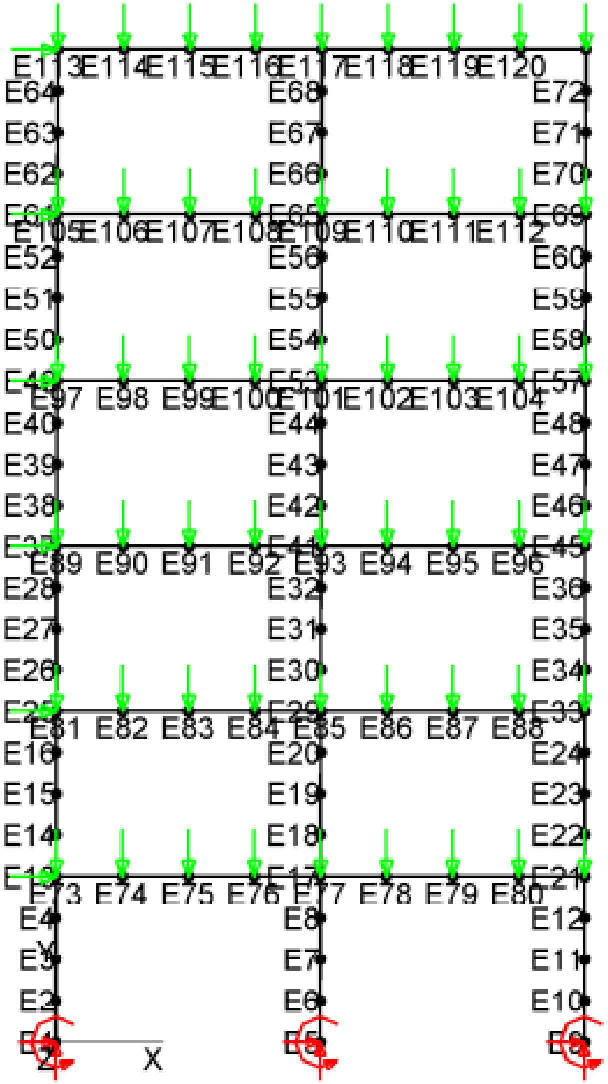


Figure 9: MASTAN2 Analysis Model

Notional Load Approach

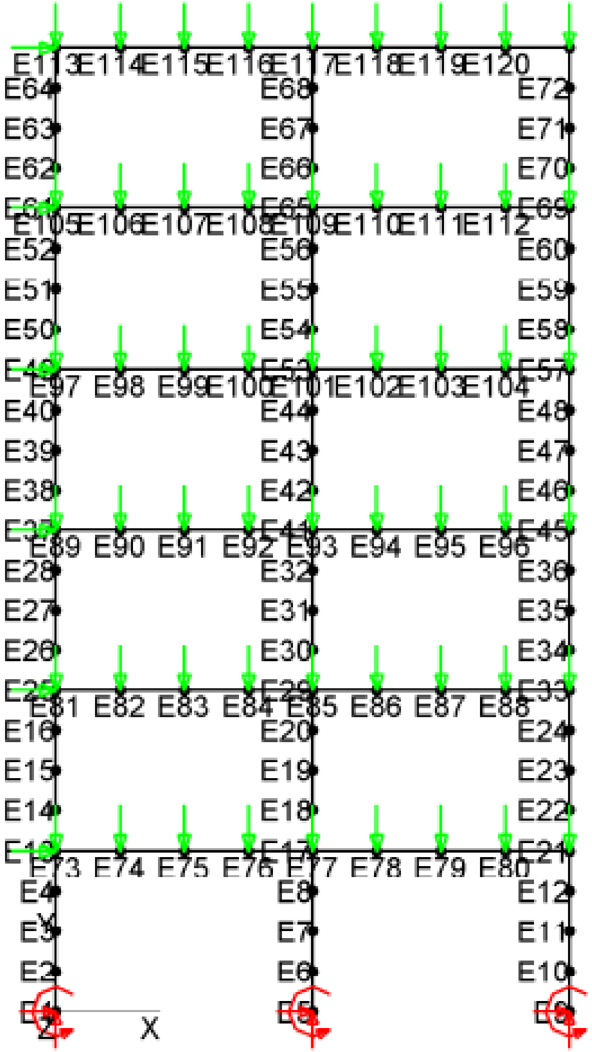


Figure 10: MASTAN2 Analysis Model

Structural System 5

Direct Modeling Approach

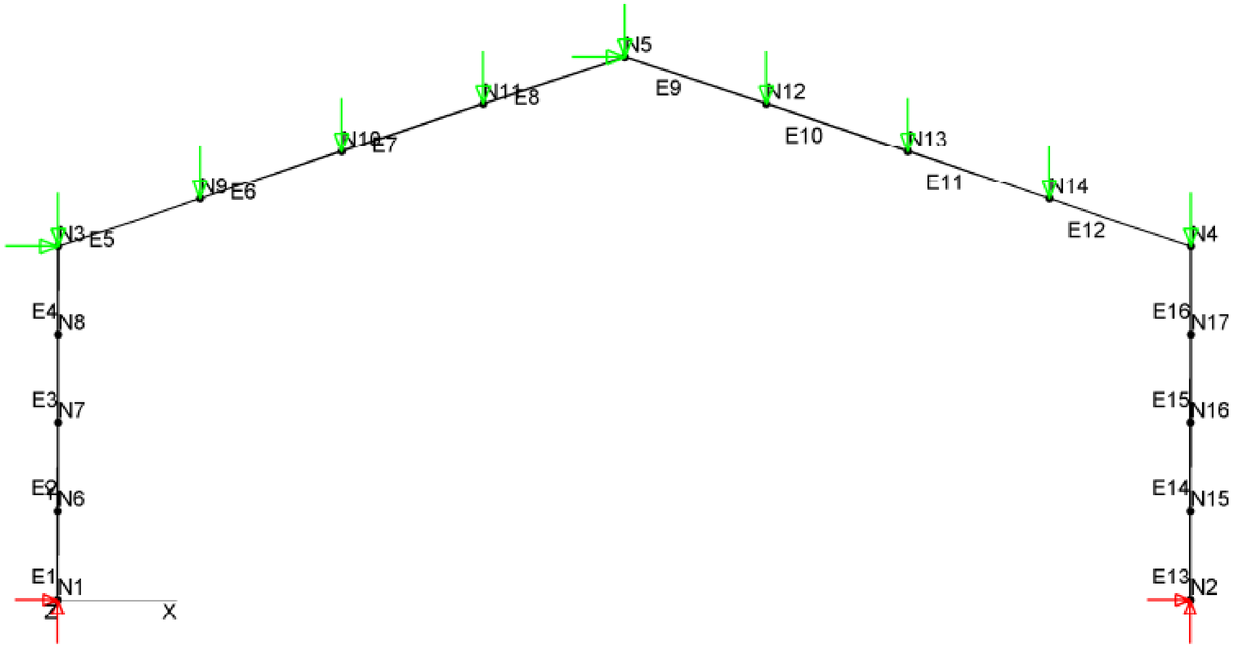


Figure 11: MASTAN2 Analysis Model

Notional Load Approach

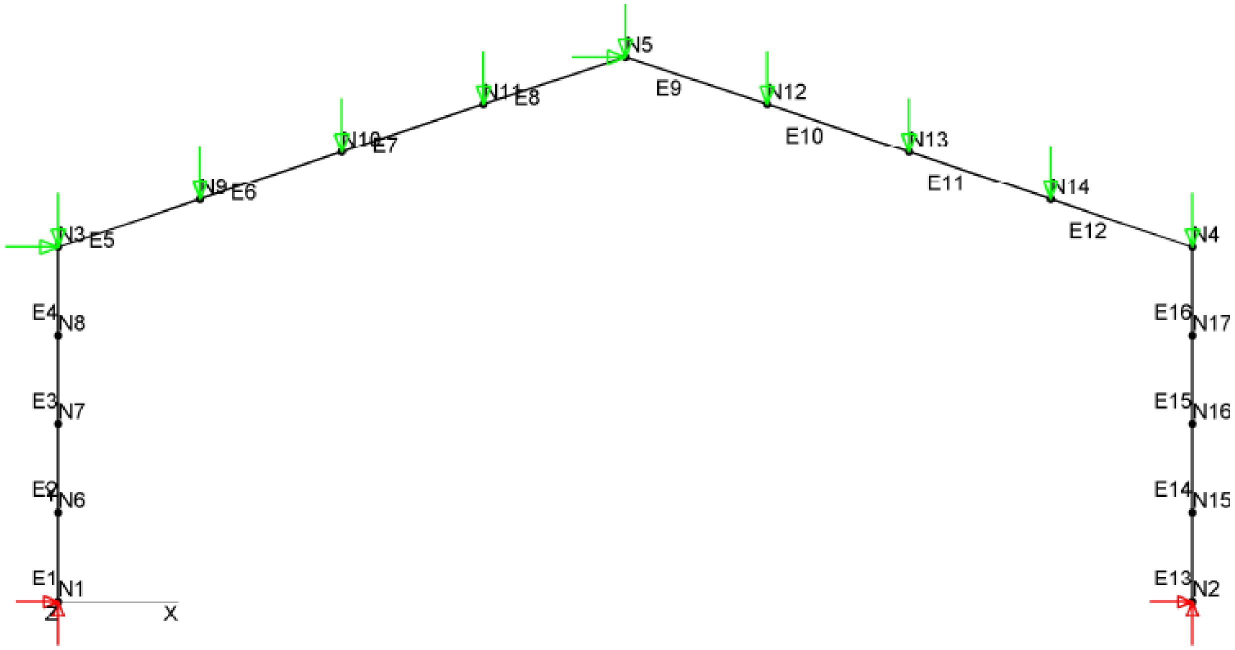


Figure 12: MASTAN2 Analysis Model

Structural System 6

Direct Modeling Approach

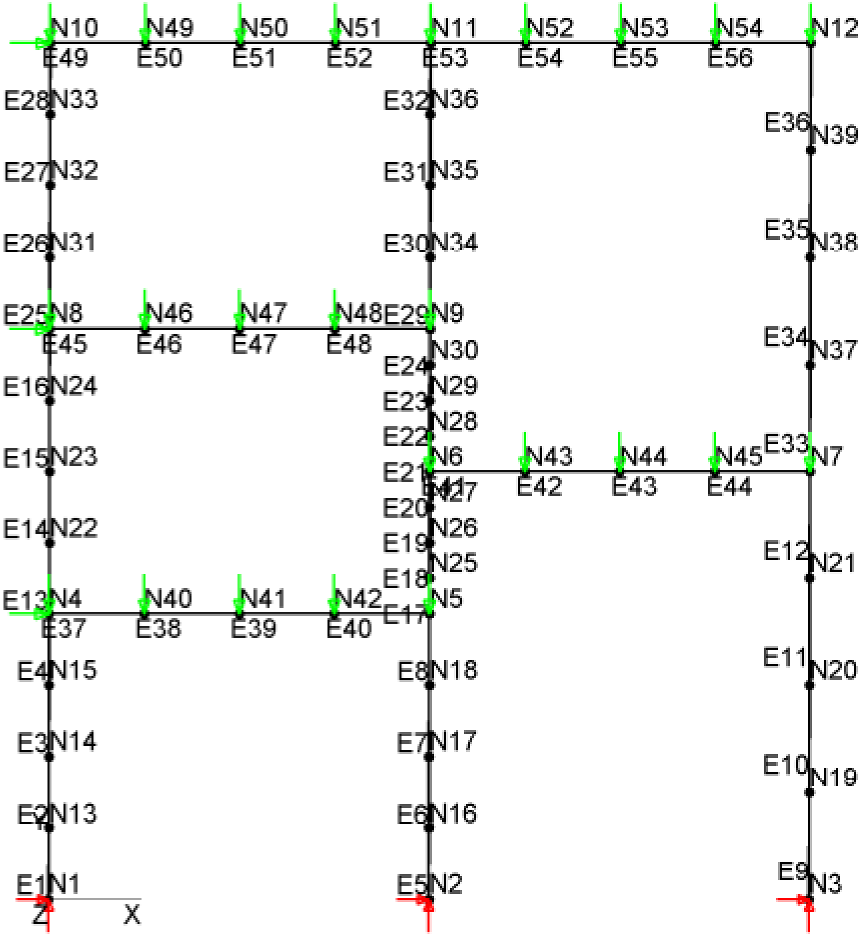


Figure 13: MASTAN2 Analysis Model

Notional Load Approach

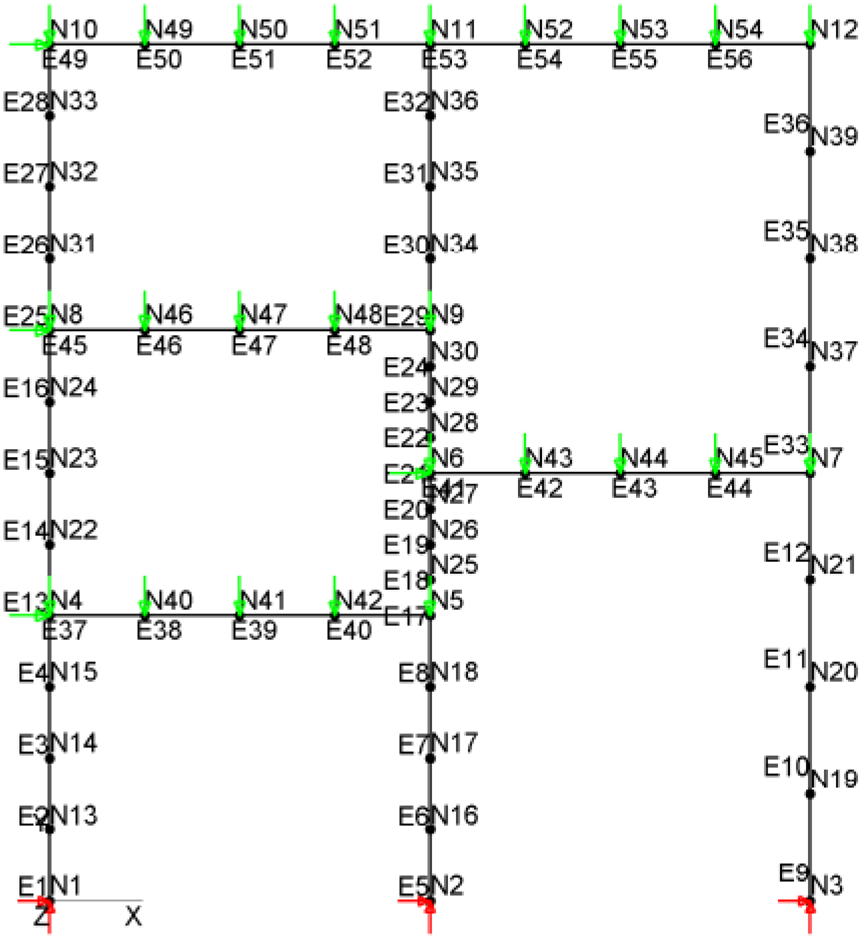


Figure 14: MASTAN2 Analysis Model

Structural System 7a

Direct Modeling Approach

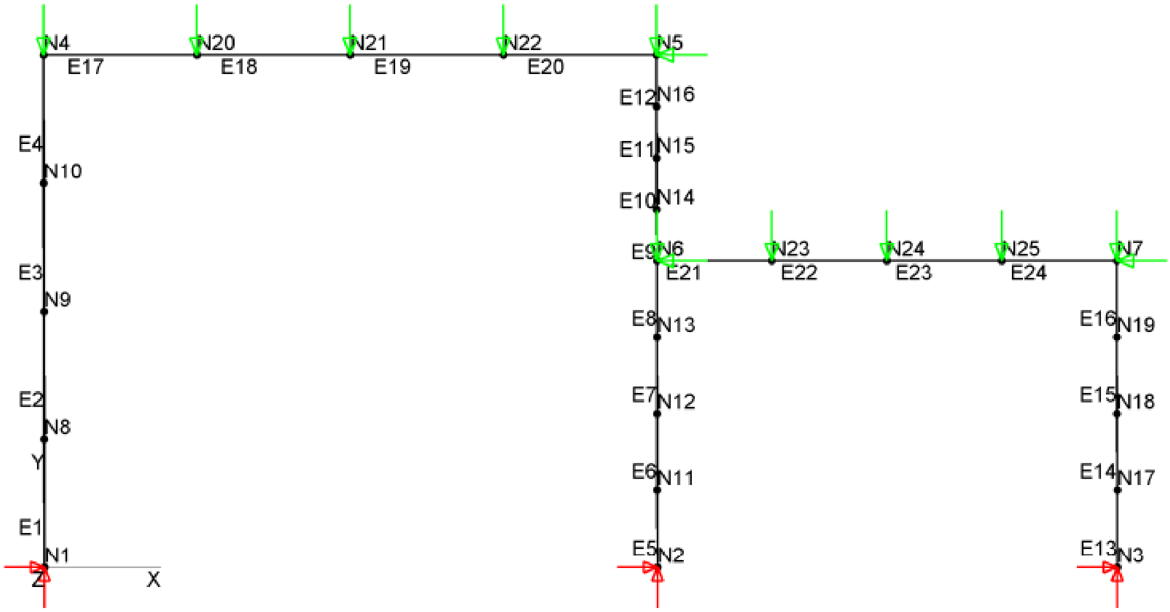


Figure 15: MASTAN2 Analysis Model

Notional Load Approach

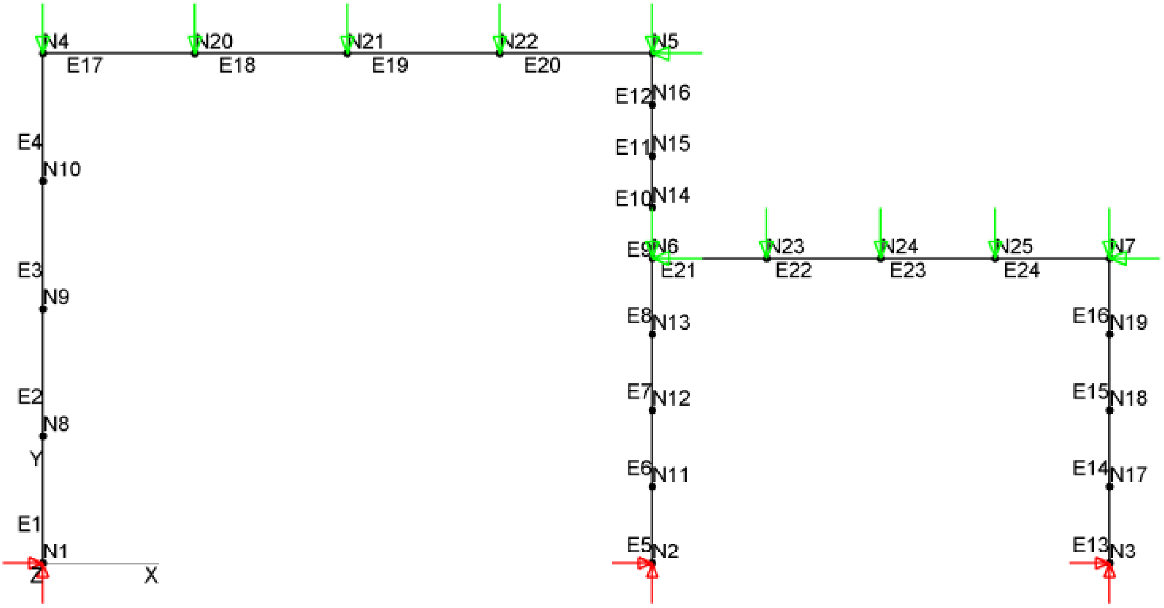


Figure 16: MASTAN2 Analysis Model

Structural System 7b

Direct Modeling Approach

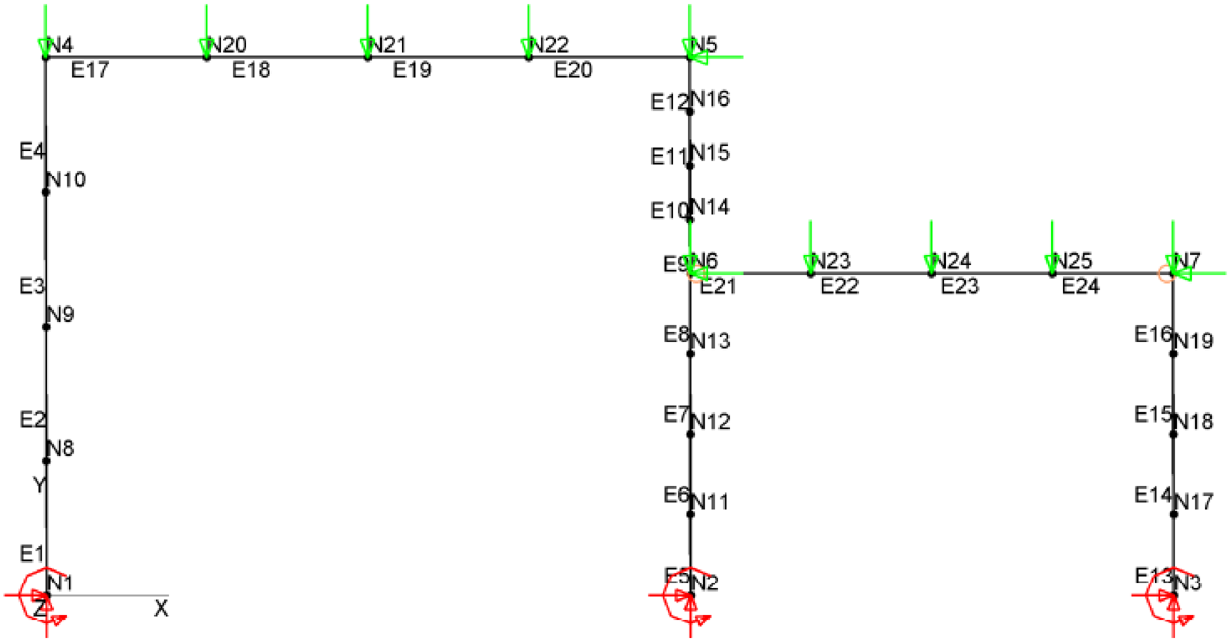


Figure 17: MASTAN2 Analysis Model

Notional Load Approach

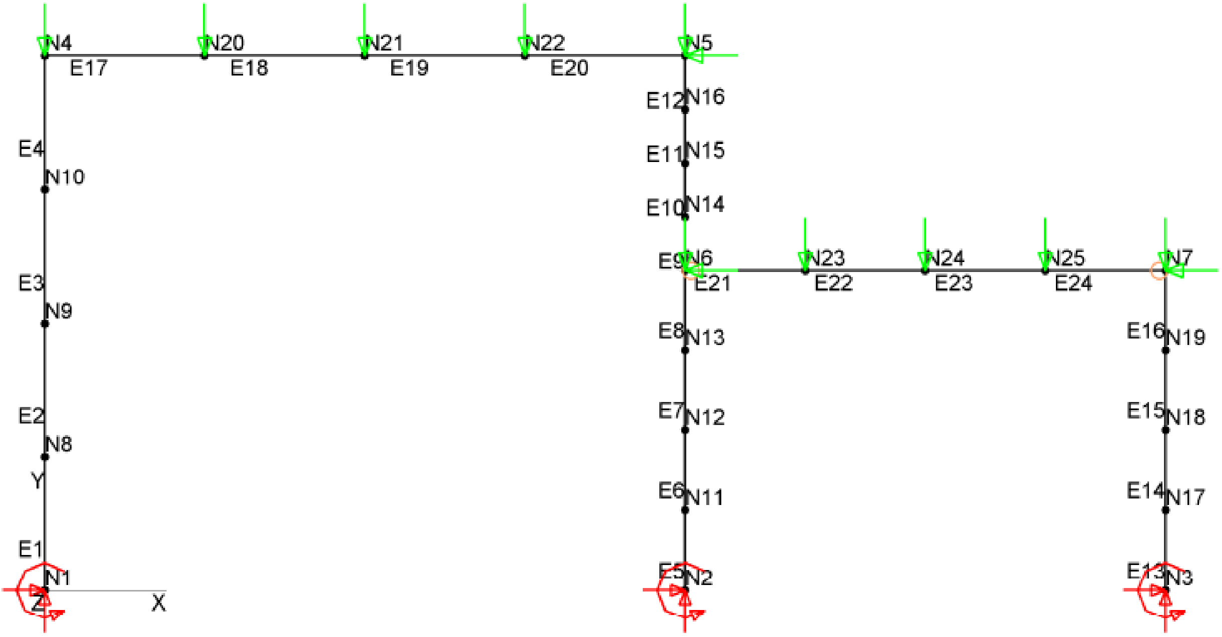


Figure 18: MASTAN2 Analysis Model

Structural System 7c

Direct Modeling Approach

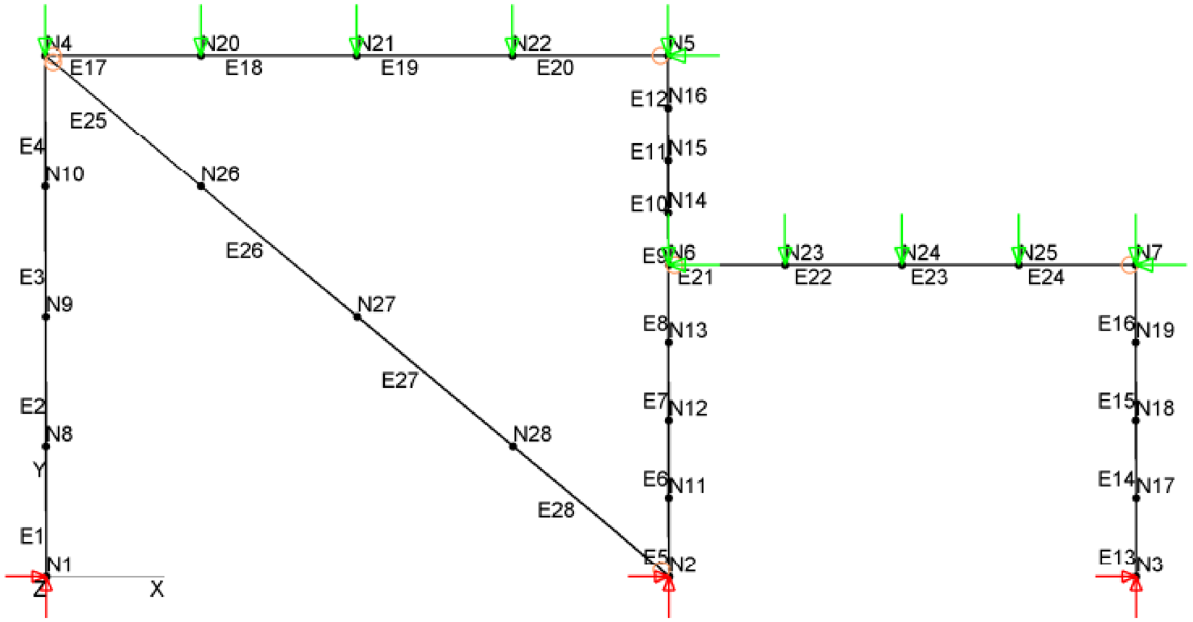


Figure 19: MASTAN2 Analysis Model

Notional Load Approach

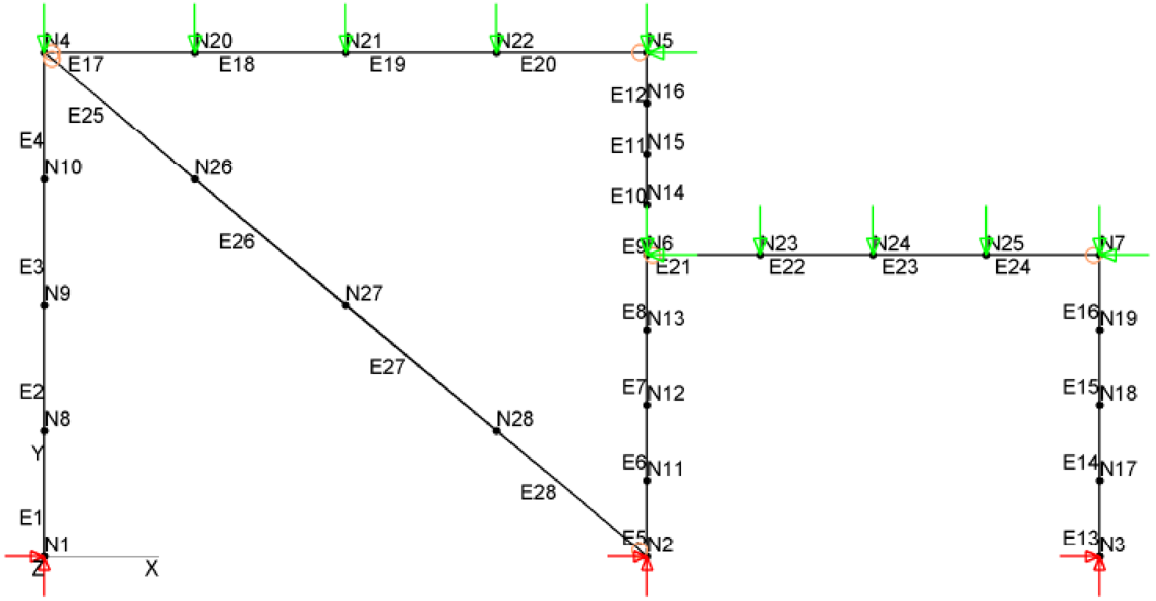


Figure 20: MASTAN2 Analysis Model

Structural System 7d

Direct Modeling Approach

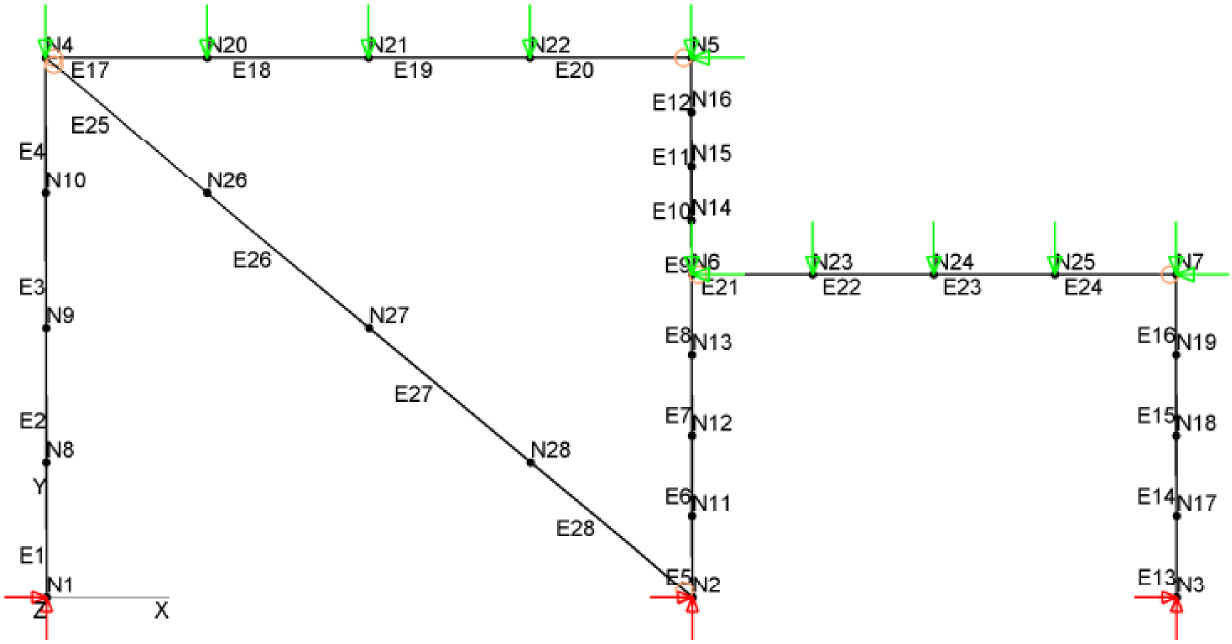


Figure 21: MASTAN2 Analysis Model

Notional Load Approach

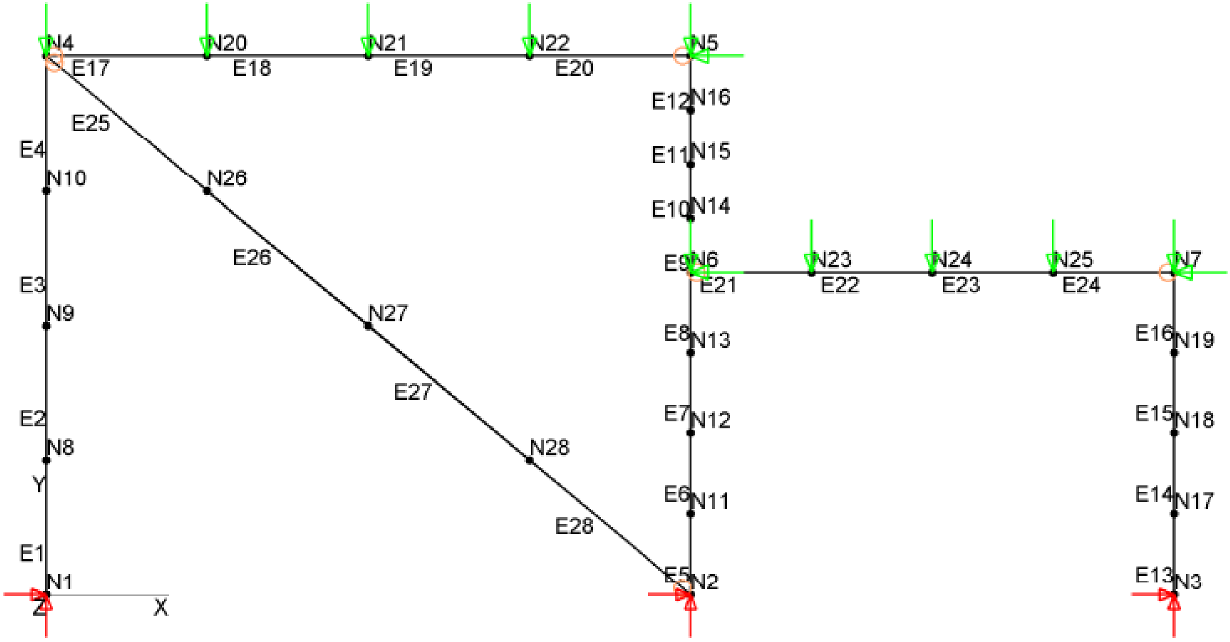


Figure 22: MASTAN2 Analysis Model

Structural System 8

Direct Modeling Approach

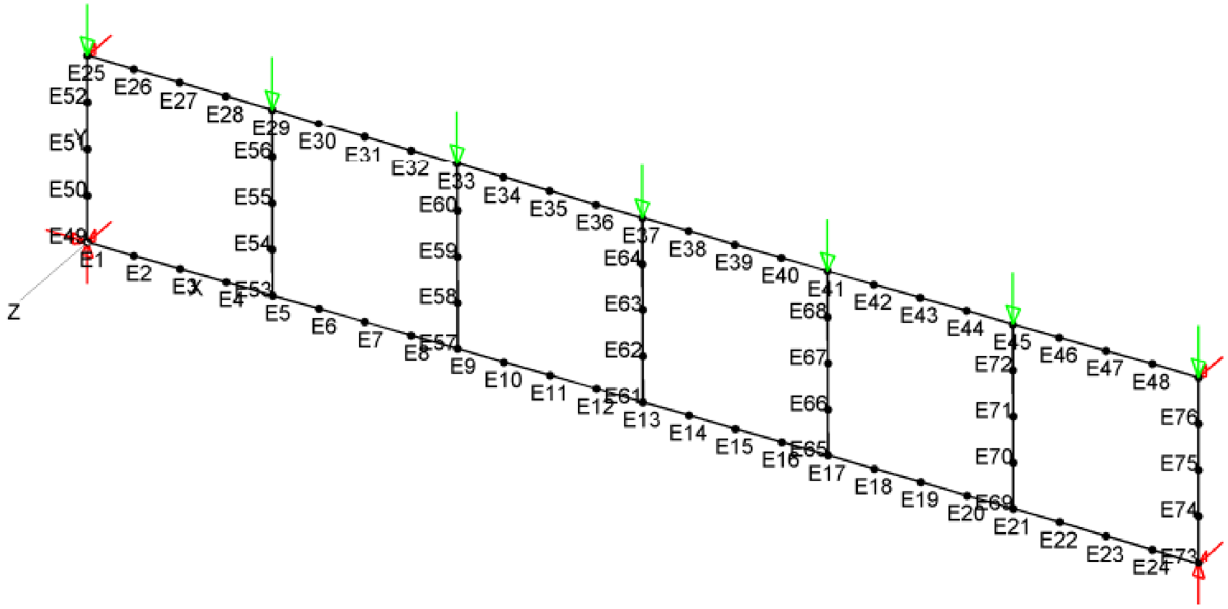


Figure 23: MASTAN2 Analysis Model

Notional Load Approach

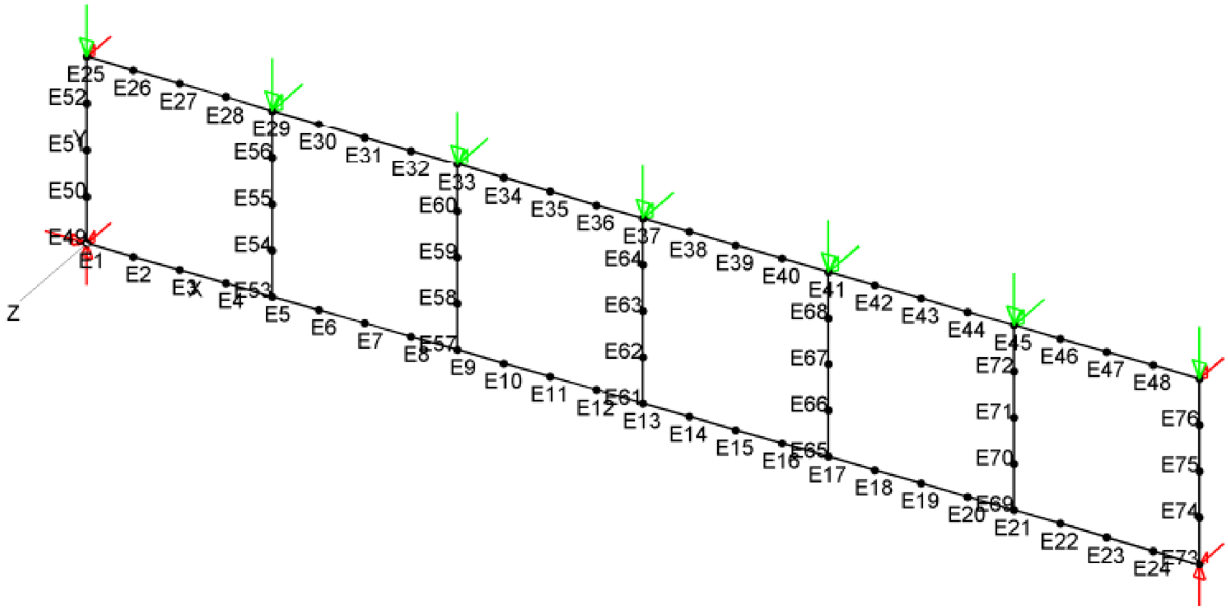


Figure 24: MASTAN2 Analysis Model

Column Study

Major/Minor Axis Orientation

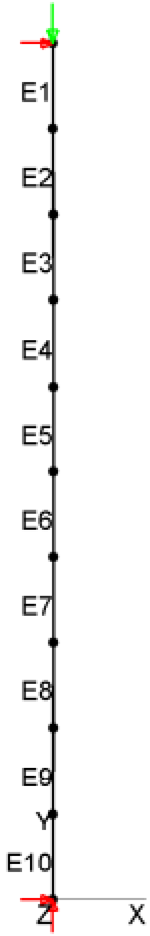


Figure 25: MASTAN2 Analysis Model

APPENDIX B: ANALYSIS RESULTS USING DIRECT MODELING APPROACH WITH τ_{AISC}

For all case study structural systems, additional stability analyses were conducted using Direct Modeling Approach with τ_{AISC} (Et_AISC option in MASTAN2 second-order inelastic analysis) instead of τ_b (Et option in MASTAN2 second-order inelastic analysis) (Tables 1-26 and Figures 26-27 below).

These analysis results also lead to similar conclusions as in regards to evaluating DM and MDM in assessing the stability of the structural systems.

Structural System 1a

Table 1: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.1

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.898	0.109	0.995	0.679	0.109	0.775
C1-2	0.540	0.542	1.022	0.500	0.542	0.982
C1-3	0.333	0.368	0.659	0.308	0.368	0.634
C2-1	0.327	0.303	0.596	0.277	0.303	0.546
C2-2	0.177	1.095	1.184	0.170	1.095	1.180
C2-3	0.116	1.209	**1.267**	0.111	1.209	**1.265**
B1-1	0.002	1.380	**1.381**	0.002	1.380	**1.381**
B1-2	0.047	1.042	1.065	0.044	1.042	1.063
B2-1	0.002	1.276	1.277	0.002	1.276	1.277
B2-2	0.096	1.115	1.163	0.083	1.115	1.156

Table 2: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.898	0.109	1.000	0.848	0.168	1.250
C1-2	0.529	0.529	0.980	0.507	0.552	1.015
C1-3	0.510	0.522	1.615	0.472	0.570	1.620
C2-1	0.471	0.103	1.680	0.399	0.103	1.680
C2-2	0.150	0.924	0.845	0.144	0.924	0.845
C2-3	0.091	0.954	**0.790**	0.087	0.954	**0.790**
B1-1	0.001	0.997	**0.720**	0.001	0.997	**0.720**
B1-2	0.044	0.977	0.940	0.041	0.977	0.940
B2-1	0.002	0.994	0.780	0.002	0.994	0.780
B2-2	0.082	0.957	0.860	0.072	0.962	0.865

Structural System 1b

Table 3: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.051	0.455	0.480	0.037	0.455	0.473
C1-2	0.587	0.621	1.139	0.431	0.621	**0.983**
C1-3	0.282	0.043	0.321	0.207	0.043	0.246
C2-1	0.773	0.103	0.864	0.098	0.103	0.152
C2-2	0.551	0.297	0.815	0.313	0.297	0.577
C2-3	0.626	0.690	**1.240**	0.250	0.690	0.864
B1-1	0.005	0.934	0.937	0.005	0.934	0.937
B1-2	0.001	0.719	0.719	0.001	0.719	0.719
B2-1	0.000	1.021	**1.021**	0.000	1.021	**1.021**
B2-2	0.003	0.690	0.691	0.002	0.690	0.691

Table 4: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.067	0.947	1.125	0.049	0.947	1.125
C1-2	0.547	0.500	0.930	0.433	0.632	**1.005**
C1-3	0.322	0.712	1.160	0.238	0.823	1.170
C2-1	0.883	0.128	1.135	0.209	0.128	1.349
C2-2	0.671	0.367	1.225	0.396	0.675	1.330
C2-3	0.507	0.552	**0.810**	0.285	0.801	1.135
B1-1	0.008	0.996	1.095	0.008	0.995	1.095
B1-2	0.002	0.987	1.240	0.002	0.989	1.235
B2-1	0.000	0.995	**0.975**	0.000	0.995	**0.975**
B2-2	0.004	0.994	1.350	0.003	0.795	1.349

Structural System 2

Table 5: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.458	0.827	**1.193**	0.382	0.827	**1.117**
C1-2	0.473	0.588	0.996	0.395	0.588	0.918
B1-1	0.003	0.913	0.915	0.003	0.913	0.914

Table 6: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_{AISC}
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.435	0.631	**0.945**	0.372	0.707	**0.970**
C1-2	0.473	0.589	1.000	0.399	0.637	1.010
B1-1	0.001	0.994	1.045	0.003	0.983	1.040

Structural System 3

Table 7: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_{AISC}
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.558	0.444	0.953	0.528	0.444	0.923
C1-2	0.630	0.445	**1.026**	0.596	0.445	**0.992**
C2-1	0.542	0.444	0.937	0.497	0.444	0.892
C2-2	0.586	0.445	0.981	0.537	0.445	0.932

Table 8: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_{AISC}
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.568	0.481	1.020	0.544	0.512	1.035
C1-2	0.620	0.422	**0.985**	0.596	0.445	**1.000**
C2-1	0.555	0.491	1.025	0.518	0.534	1.045
C2-2	0.589	0.453	1.005	0.554	0.498	1.030

Structural System 4

Table 9: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.395	0.360	0.716	0.366	0.360	0.686
C1-2	0.789	0.535	**1.264**	0.747	0.535	**1.223**
C1-3	0.535	0.572	1.043	0.495	0.572	1.004
C2-1	0.337	0.022	0.357	0.312	0.022	0.332
C2-2	0.646	0.439	1.036	0.612	0.439	1.002
C2-3	0.433	0.650	1.010	0.401	0.650	0.978
C3-1	0.272	0.059	0.325	0.252	0.059	0.305
C3-2	0.571	0.375	0.905	0.535	0.375	0.869
C3-3	0.331	0.600	0.864	0.306	0.600	0.840
C4-1	0.201	0.270	0.441	0.186	0.270	0.363
C4-2	0.419	0.271	0.659	0.392	0.271	0.633
C4-3	0.232	0.660	0.818	0.214	0.660	0.801
C5-1	0.228	0.285	0.481	0.196	0.285	0.384
C5-2	0.369	0.320	0.654	0.336	0.320	0.621
C5-3	0.250	0.811	0.970	0.215	0.811	0.935
C6-1	0.091	0.692	0.737	0.078	0.692	0.731
C6-2	0.145	0.102	0.174	0.132	0.102	0.168
C6-3	0.095	0.873	0.920	0.082	0.873	0.914

Table 10: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.513	0.546	1.305	0.488	0.576	1.340
C1-2	0.624	0.419	**0.790**	0.613	0.436	**0.820**
C1-3	0.513	0.545	0.960	0.495	0.572	1.000
C2-1	0.505	0.004	1.490	0.467	0.004	1.490
C2-2	0.623	0.421	0.965	0.612	0.439	1.000
C2-3	0.428	0.643	0.990	0.411	0.668	1.025
C3-1	0.408	0.075	1.490	0.378	0.075	1.490
C3-2	0.625	0.418	1.095	0.612	0.441	1.145
C3-3	0.378	0.695	1.140	0.361	0.720	1.175
C4-1	0.302	0.412	1.490	0.279	0.412	1.490
C4-2	0.610	0.438	1.470	0.579	0.442	1.490
C4-3	0.280	0.807	1.205	0.266	0.829	1.235
C5-1	0.342	0.428	1.490	0.295	0.428	1.490
C5-2	0.542	0.515	1.480	0.496	0.519	1.490
C5-3	0.256	0.832	1.025	0.230	0.871	1.070
C6-1	0.122	0.936	1.340	0.107	0.948	1.355
C6-2	0.215	0.160	1.490	0.196	0.160	1.490
C6-3	0.103	0.945	1.080	0.090	0.958	1.095

Structural System 5

Table 11: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.151	0.920	2.960	0.119	0.935	2.970
C1-2	0.051	0.966	**0.640**	0.040	0.966	**0.640**
B1-1	0.019	0.991	2.110	0.014	0.991	2.110
B1-2	0.043	0.976	1.820	0.032	0.983	1.830

Table 12: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.151	0.920	2.960	0.119	0.935	2.970
C1-2	0.051	0.966	**0.640**	0.040	0.966	**0.640**
B1-1	0.019	0.991	2.110	0.014	0.991	2.110
B1-2	0.043	0.976	1.820	0.032	0.983	1.830

Structural System 6

Table 13: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.072	0.891	0.927	0.064	0.891	0.923
C1-2	0.222	0.997	**1.108**	0.197	0.997	**1.096**
C1-3	0.126	0.633	0.696	0.096	0.633	0.681
C2-1	0.062	0.099	0.130	0.055	0.099	0.127
C2-2a	0.124	0.291	0.353	0.120	0.291	0.351
C2-2b	0.101	0.434	0.485	0.098	0.434	0.484
C3-1	0.021	0.019	0.029	0.019	0.019	0.028
C3-2	0.047	0.269	0.292	0.042	0.269	0.289
C3-3	0.034	0.279	0.296	0.026	0.279	0.292
B1-1	0.008	0.952	0.956	0.007	0.952	0.955
B2-1	0.012	0.520	0.526	0.010	0.520	0.525
B2-2	0.001	0.589	0.589	0.001	0.589	0.589
B3-1	0.047	0.633	0.656	0.038	0.633	0.652
B3-2	0.019	0.509	0.518	0.015	0.509	0.517

Table 14: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.067	0.965	1.060	0.067	0.965	1.060
C1-2	0.182	0.904	**0.925**	0.182	0.904	**0.925**
C1-3	0.132	0.930	1.355	0.132	0.930	1.355
C2-1	0.167	0.912	3.320	0.162	0.912	3.180
C2-2a	0.247	0.844	2.145	0.247	0.844	2.145
C2-2b	0.193	0.901	1.950	0.193	0.901	1.950
C3-1	0.064	0.052	3.415	0.063	0.063	3.335
C3-2	0.133	0.933	3.335	0.130	0.933	3.305
C3-3	0.083	0.957	3.070	0.082	0.958	3.050
B1-1	0.007	0.992	1.035	0.007	0.992	1.035
B2-1	0.020	0.990	1.830	0.020	0.990	1.830
B2-2	0.003	0.995	1.555	0.003	0.995	1.555
B3-1	0.057	0.971	1.520	0.057	0.971	1.520
B3-2	0.027	0.985	1.895	0.027	0.985	1.895

Structural System 7a

Table 15: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.221	0.631	0.782	0.158	0.631	0.710
C1-2a	0.229	0.539	0.709	0.205	0.539	0.684
C1-2b	0.115	0.476	0.533	0.109	0.476	0.530
C1-3	0.112	0.204	0.259	0.099	0.204	0.253
B1-1	0.006	1.062	1.065	0.005	1.062	1.065
B1-2	0.018	1.260	**1.269**	0.016	1.260	**1.268**

Table 16: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.281	0.807	1.265	0.218	0.883	1.375
C1-2a	0.306	0.776	1.325	0.281	0.810	1.365
C1-2b	0.219	0.878	1.935	0.211	0.888	1.960
C1-3	0.210	0.877	2.350	0.186	0.908	2.360
B1-1	0.006	0.992	0.935	0.005	0.998	0.940
B1-2	0.015	0.992	**0.805**	0.014	0.992	**0.805**

Structural System 7b

Table 17: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.249	0.855	**1.009**	0.177	0.855	**0.944**
C1-2a	0.244	0.124	0.354	0.217	0.124	0.327
C1-2b	0.130	0.355	0.419	0.123	0.355	0.416
C1-3	0.146	0.133	0.206	0.129	0.133	0.197
B1-1	0.016	1.097	**1.106**	0.015	1.097	**1.105**
B1-2	0.005	0.980	0.983	0.004	0.980	0.982

Table 18: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.246	0.847	**0.990**	0.187	0.908	**1.060**
C1-2a	0.616	0.431	2.550	0.575	0.480	2.675
C1-2b	0.472	0.583	3.935	0.449	0.625	3.945
C1-3	0.395	0.678	2.715	0.360	0.724	2.795
B1-1	0.015	0.992	**0.905**	0.014	0.998	**0.910**
B1-2	0.005	0.995	1.015	0.004	1.000	1.020

Structural System 7c

Table 19: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.202	0.000	0.202	0.142	0.000	0.071
C1-2a	0.256	0.898	**1.054**	0.180	0.898	**0.988**
C1-2b	0.146	0.898	0.971	0.102	0.898	0.949
C1-3	0.303	0.000	0.303	0.237	0.000	0.237
B1-1	0.020	0.419	0.430	0.018	0.419	0.429
B1-2	0.015	0.398	0.406	0.014	0.398	0.405
BRACE	0.131	0.000	0.066	0.131	0.000	0.066

DM: KL = 20' for C1-2 and C1-2b

Table 20: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.757	0.000	3.560	0.531	0.000	3.560
C1-2a	0.243	0.847	**0.950**	0.181	0.908	**1.010**
C1-2b	0.150	0.924	1.025	0.107	0.945	1.045
C1-3	0.999	0.000	3.280	0.848	0.000	3.560
B1-1	0.050	0.974	2.305	0.045	0.976	2.310
B1-2	0.041	0.979	2.440	0.037	0.981	2.445
BRACE	0.564	0.000	3.560	0.564	0.000	3.560

DM: KL = 20' for C1-2 and C1-2b

Structural System 7d

Table 21: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1	$P_u/\phi P_n$	$M_u/\phi M_n$	Eq. H1-1
C1-1	0.352	0.000	0.352	0.186	0.000	0.093
C1-2a	0.571	0.661	**1.158**	0.301	0.661	**0.889**
C1-2b	0.326	0.661	0.914	0.172	0.661	0.747
C1-3	0.755	0.000	0.755	0.386	0.000	0.386
B1-1	0.008	0.590	0.594	0.007	0.590	0.594
B1-2	0.006	0.614	0.617	0.006	0.614	0.617
Bracing	0.097	0.000	0.049	0.097	0.000	0.049

DM: KL = 20' for C1-2 and C1-2b

Table 22: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_{AISC}
second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00					
	DM: K = 1			MDM: $P_n = P_y$		
Member	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR	$P_u/\phi P_n$	$M_u/\phi M_n$	ALR
C1-1	0.491	0.000	1.380	0.259	0.000	1.380
C1-2a	0.511	0.547	**0.895**	0.325	0.762	**1.080**
C1-2b	0.346	0.736	1.060	0.201	0.895	1.170
C1-3	0.997	0.000	1.320	0.533	0.000	1.380
B1-1	0.011	0.611	1.380	0.011	0.611	1.380
B1-2	0.009	0.636	1.380	0.009	0.636	1.380
Bracing	0.155	0.000	1.380	0.155	0.000	1.380

DM: KL = 20' for C1-2 and C1-2b

Structural System 8

Table 23: Comparison 1 (H1-1 at ALR =1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau_{AISC}
second-order elastic; P-C; increment 0.01

1	Eq. H1-1 at an Applied Load Ratio =1.00							
	DM: K = 1				MDM: $P_n = P_y$			
Member	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	Eq. H1-1	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	Eq. H1-1
TC-1	0.278	0.414	0.361	0.967	0.072	0.414	0.361	0.811
TC-2	0.653	0.372	0.770	1.668	0.169	0.372	0.770	1.226
TC-3	0.858	0.198	0.932	**1.862**	0.222	0.198	0.932	**1.226**
BC-1	0.074	0.576	0.083	0.696	0.074	0.576	0.083	0.696
BC-2	0.169	0.409	0.046	0.540	0.169	0.409	0.046	0.540
BC-3	0.222	0.205	0.009	0.411	0.222	0.205	0.009	0.411
W-1	0.108	0.629	0.034	0.717	0.104	0.629	0.034	0.715
W-2	0.042	0.810	0.036	0.866	0.040	0.791	0.031	0.842
W-3	0.029	0.434	0.024	0.473	0.028	0.434	0.022	0.470
W-4	0.029	0.000	0.025	0.040	0.028	0.000	0.015	0.030

DM: KL = 48' for TC-1, TC-2, and TC-3; KL = 8' for W-1,W-2,W-3 and W-4

Table 24: Comparison 2 (ALR at H1-1=1.0) Using Direct Modeling Approach with τ_{AISC}

Imperfection Direct Modeling **Stiffness Adjustment** 0.8E and tau$_{AISC}$

second-order elastic; P-C; increment 0.005

1	Applied Load Ratio when Eq. H1-1 = 1.00							
	DM: K = 1				MDM: $P_n = P_y$			
Member	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	ALR	$P_u/\phi P_n$	$M_{ux}/\phi M_{nx}$	$M_{uy}/\phi M_{ny}$	ALR
TC-1	0.280	0.403	0.398	1.010	0.076	0.279	0.680	1.085
TC-2	0.546	0.329	0.173	0.830	0.163	0.372	0.532	0.965
TC-3	0.691	0.168	0.176	**0.800**	0.216	0.197	0.678	**0.970**
BC-1	0.297	0.664	0.110	1.040	0.081	0.814	0.137	1.095
BC-2	0.628	0.384	0.033	0.955	0.215	0.656	0.226	1.320
BC-3	0.814	0.198	0.009	0.945	0.296	0.351	0.434	1.485
W-1	0.111	0.868	0.076	1.165	0.107	0.868	0.076	1.165
W-2	0.068	0.891	0.074	1.100	0.065	0.891	0.074	1.100
W-3	0.004	0.225	0.536	2.515	0.004	0.225	0.536	2.515
W-4	0.191	0.000	0.277	2.515	0.184	0.000	0.277	2.515

DM: KL = 48' for TC-1, TC-2, and TC-3; KL = 8' for W-1,W-2,W-3 and W-4

Column Study

Major Axis Orientation

Table 25: Comparison of Major Axis Strength of Column Obtained by Different Analysis Methods (Appx1, DM and MDM (τ_{AISC})) and Their Percent Differences

	Major Axis Strength (P_u/P_y)			Percent Difference (%)	
L/r_x	Appx 1	DM	MDM (tau_AISC)	DM vs Appx 1	MDM (tau_AISC) vs Appx 1
0	0.900	0.9	0.9	0.000	0.000
10	0.890	0.893	0.891	0.395	0.158
20	0.874	0.874	0.881	0.060	0.912
30	0.848	0.843	0.868	-0.601	2.431
40	0.813	0.801	0.846	-1.478	4.150
50	0.770	0.750	0.803	-2.705	4.195
60	0.717	0.692	0.736	-3.559	2.657
70	0.656	0.629	0.659	-4.079	0.429
80	0.583	0.564	0.577	-3.341	-1.044
90	0.506	0.498	0.496	-1.652	-1.906
100	0.435	0.433	0.421	-0.348	-3.139
110	0.373	0.372	0.354	-0.257	-4.942
120	0.321	0.314	0.301	-2.120	-6.195
130	0.278	0.267	0.258	-3.744	-7.083
140	0.242	0.231	0.224	-4.895	-7.633
150	0.213	0.201	0.196	-5.732	-8.091
160	0.189	0.176	0.173	-6.428	-8.369
170	0.168	0.156	0.153	-6.971	-8.863
180	0.151	0.139	0.137	-7.422	-8.893
190	0.136	0.125	0.123	-7.793	-9.248
200	0.123	0.113	0.111	-8.129	-9.333

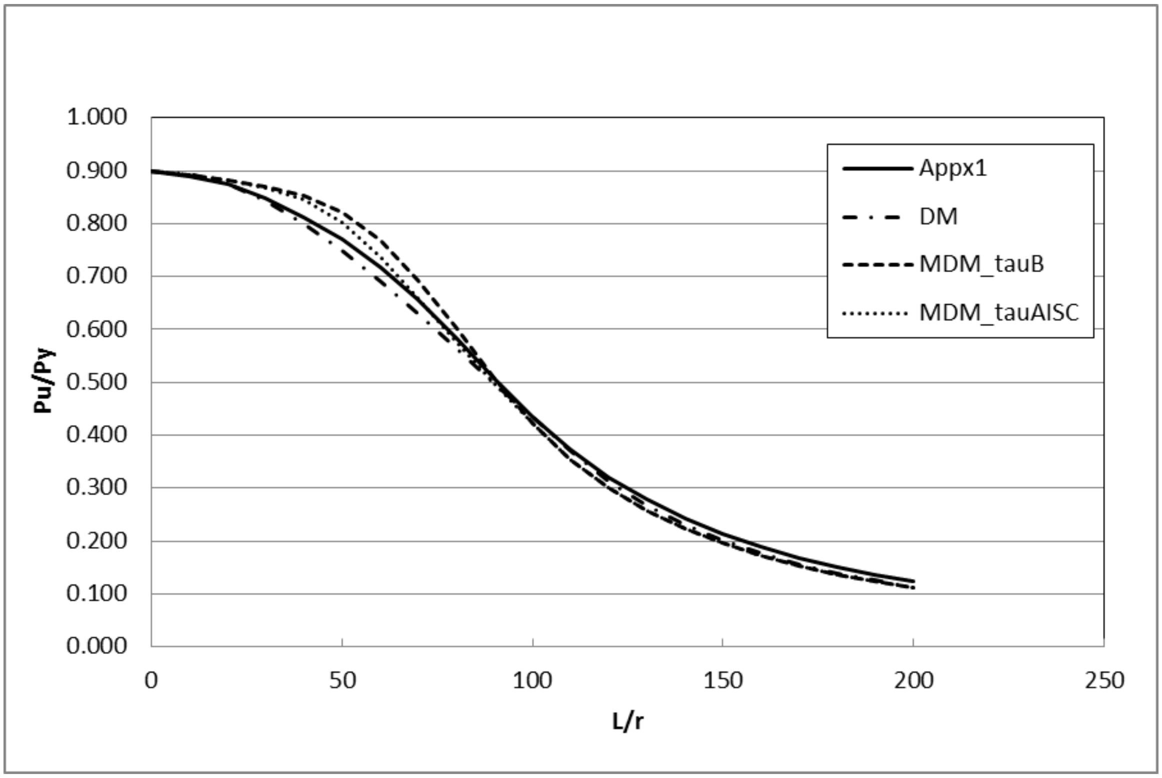


Figure 26: Comparison of Major Axis Strength of Column Obtained by Different Analysis Methods

Minor Axis Orientation

Table 26: Comparison of Minor Axis Strength of Column Obtained by Different Analysis Methods (Appx1, DM and MDM (τ_{AISC})) and Their Percent Differences

	Minor Axis Strength (P_u/P_y)			Percent Difference (%)	
L/r_y	Appx 1	DM	MDM (tau_AISC)	DM vs Appx 1	MDM (tau_AISC) vs Appx 1
0	0.9	0.9	0.9	0.000	0.000
10	0.892	0.893	0.889	0.126	-0.320
20	0.872	0.874	0.878	0.194	0.617
30	0.838	0.843	0.862	0.510	2.793
40	0.776	0.801	0.837	3.197	7.885
50	0.706	0.75	0.793	6.169	12.371
60	0.646	0.692	0.728	7.015	12.609
70	0.585	0.629	0.651	7.432	11.120
80	0.521	0.564	0.567	8.285	8.879
90	0.457	0.498	0.489	9.000	7.077
100	0.397	0.433	0.415	9.133	4.425
110	0.344	0.372	0.349	7.926	1.497
120	0.299	0.314	0.297	4.982	-0.486
130	0.261	0.267	0.256	2.495	-1.957
140	0.229	0.231	0.222	0.709	-3.008
150	0.202	0.201	0.194	-0.688	-3.868
160	0.18	0.176	0.171	-1.820	-4.638
170	0.161	0.156	0.152	-2.743	-5.303
180	0.145	0.139	0.136	-3.522	-5.703
190	0.131	0.125	0.123	-4.155	-6.026
200	0.119	0.113	0.111	-4.717	-6.361

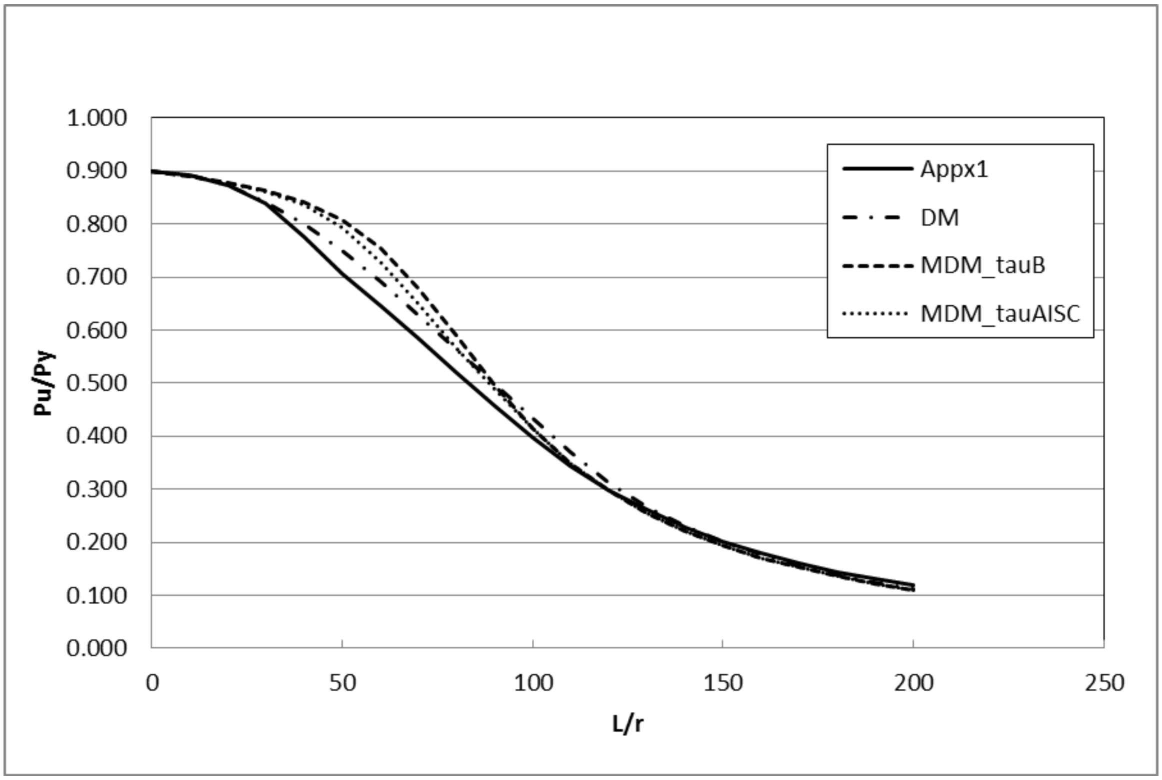


Figure 27: Comparison of Minor Axis Strength of Column Obtained by Different Analysis Methods

REFERENCES

Clarke, M., & Zablotskii, S. (1995). *NIFA: Non linear inelastic frame analysis*. University of Sydney, Australia:

Geschwindner, L. (2012). *Unified design of steel structures*. Hoboken, NJ: Wiley.

Martinez-Garcia, J. (2002). *Benchmark studies to evaluate new provisions for frame stability using second-order analysis.* (Master of Science in Civil and Environmental Engineering, Bucknell University, Lewisburg, PA).

Steel Construction Manual (2011). [Chicago, Ill.]: American Institute of Steel Construction.

Ziemian, R. (2010). *Guide to stability design criteria for metal structures*. Hoboken, N.J.: John Wiley & Sons.

Ziemian, R., & McGuire, W. (2013). *www.mastan2.com*