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Effective Length K-Factors For Flexural Buckling Strengths Of Web Members In Open Web Steel Joists

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EFFECTIVE LENGTH K-FACTORS FOR FLEXURAL BUCKLING STRENGTHS OF WEB MEMBERS IN OPEN WEB STEEL JOISTS

By

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I, Sugyu Lee, do grant permission for my thesis to be copied.

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Most of all, my sincere thanks goes to Dr. Ronald D. Ziemian for his teaching, constant guidance, and kind care throughout my two years at Bucknell University. His encouragement and faith in me provided the confidence and motivation I needed to complete the Master of Science degree program. It is a particular pleasure for me to have worked with him on this research related to the stability of steel joists.

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ABSTRACT

Open web steel joists are designed in the United States following the governing specification published by the Steel Joist Institute. For compression members in joists, this specification employs an effective length factor, or *K*-factor, in confirming their adequacy. In most cases, these *K*-factors have been conservatively assumed equal to 1.0 for compression web members, regardless of the fact that intuition and limited experimental work indicate that smaller values could be justified. Given that smaller *K*-factors could result in more economical designs without a loss in safety, the research presented in this thesis aims to suggest procedures for obtaining more rational values. Three different methods for computing in-plane and out-of-plane *K*-factors are investigated, including (1) a hand calculation method based on the use of alignment charts, (2) computational critical load (eigenvalue) analyses using uniformly distributed loads, and (3) computational analyses using a compressive strain approach. The latter method is novel and allows for computing the individual buckling load of a specific member within a system, such as a joist. Four different joist configurations are investigated, including an 18K3, 28K10, and two variations of a 32LH06. Based on these methods and the very limited number of joists studied, it appears promising that in-plane and out-of-plane *K*-factors of 0.75 and 0.85, respectively, could be used in computing the flexural buckling strength of web members in routine steel joist design. Recommendations for future work, which include systematically investigating a wider range of joist configurations and connection restraint, are provided.

Chapter 1: Introduction

In recent years, steel joist systems have become increasingly more popular in buildings and industrial complexes, in which they are most often used for supporting roof loads. Steel joists serve as an alternative to rolled steel beams because of economic benefits. The load capacity to self-weight ratio of a steel joist can be much higher than that of other structural steel member types.

Since 1928, the Steel Joist Institute (SJI 2010) is the governing agency that defines the specification for designing steel joists in the United States. The design provisions for steel joists and joist girders are presented in this specification. In general, an open web steel joist takes on the form of a truss that has been designed with top and bottom chords of back-to-back angles and a web formed from either segments of continuous bent bar and/or single angles. The SJI has continued to develop other types of joists, including pitched joists and joist girders. In general, joists are fabricated with welded-connections.

Given that the top and bottom chords of joists are continuous and the ends of the web members are almost always welded between these chords, the joist is not a traditional truss comprised of two-force members joined with pinned connections. Although this additional end-restraint does not result in any substantial reduction in the deflection or variations in force distribution, it does have the potential to significantly impact the buckling strength of web members subject to compression.

In general, the flexural buckling strength P_{cr} of a compression member can be represented by

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \tag{1.1}$$

where E is a measure of the material stiffness, I is the a moment of inertia representing the bending stiffness of the cross-section, L is the unbraced length of the member, and K is an effective length factor that accounts for the degree of rotational restraint provided at the member ends (Ziemian 2010). It is important to note that as the amount of this rotational restraint increases, the effective length K-factor decreases, and the flexural buckling strength or capacity increases.

Members in compression are often designed using a computational buckling analysis to determine their buckling strengths. These buckling strengths can then be used to back-calculate effective length *K*-factors according to

$$K = \frac{\pi}{L}\sqrt{\frac{EI}{P_{cr}}} \tag{1.2}$$

where P_{cr} is the compressive strength determined by the analysis.

A wide range of effective length *K*-factors is shown in Table C-A-7.1 (AISC 2011) and is dependent on the end conditions of a member, as illustrated in Fig.1.1. For a web member within a joist system, these conditions can be represented by the extremes of rigid (no-rotation) constraints at both ends (i.e. Case *a*) to the pinned-end (free to rotate) conditions of Case *d*. Furthermore, the presence of a lateral support can also result in a different range of *K*-factors. As shown in Fig.1.1 (cases *a*, *b*, and *d*) the theoretical *K*-values can range from 0.5 to 1.0 when it assumed that lateral support from other neighboring members prevents the end support from moving horizontally in-plane. In assessing out-of-plane behavior of steel joists, this lateral support will come from the joist’s top and bottom chords, which are often braced.

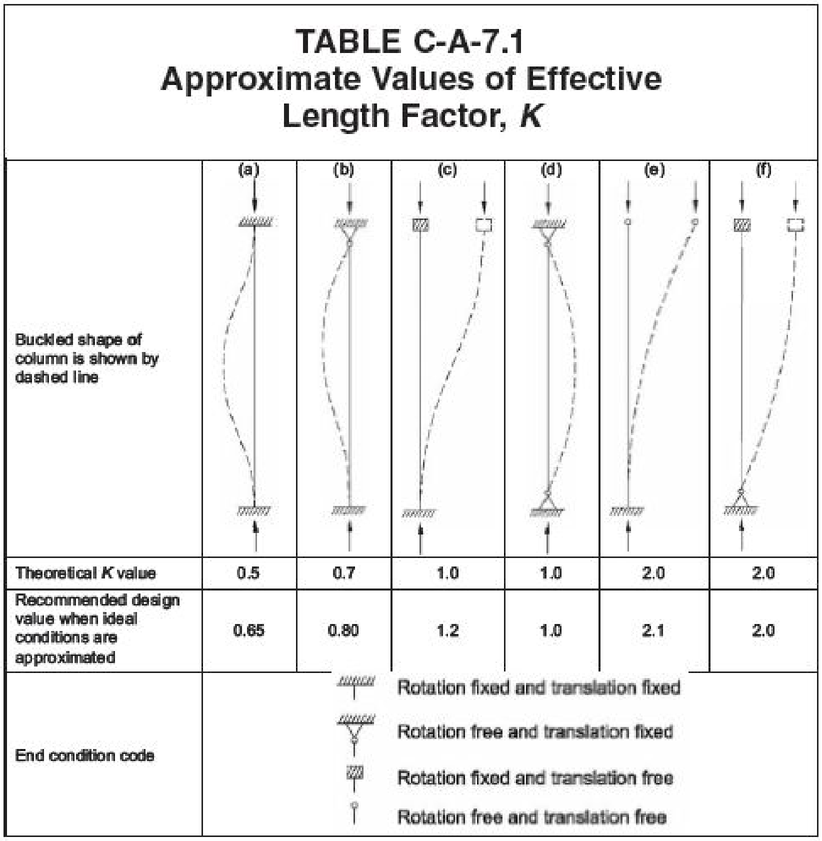


TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code	Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free					

Figure 1.1**.** Effective length factors for various end supports (AISC 2010).

In preparing provisions for the design of web members in K-series joists, the SJI specification committee has conservatively assumed K = 1 for web members in compression (SJI 2010). In doing so, they do not account for any rotational restraint provided by the top and bottom chords and neighboring tension web members when computing the in-plane and out-of-plane buckling strengths of compression web members. As may be expected, some member companies of the SJI have in recent years been evaluating all of the conservative assumptions associated with the SJI specifications in an effort to continue to remain competitive in the construction industry. With regard to joist web members, a member company funded researchers Yost et al. (2004) to conduct an experimental study on the strength and stiffness of K-Series joists with crimped web members. As part of this study, these researchers found for the few joists that they investigated the assumption of K = 1.0 is conservative for crimped web members and extremely conservative for round bar members. Yost et al. suggested a more reasonable value of $K = 0.8$. The SJI is now interested in conducting more studies to provide further insight to this topic.

1.1 Objective

The purpose of this project is to conduct a parametric study that investigates the possibility of increasing the compressive strength of web members in open web steel joists by taking into account the rotational end restraint provided by neighboring members, which include the top chord, bottom chord, and neighboring tension members. Parameters that will be studied include any factors that significantly impact the relative stiffness of the web members in compression and their neighboring members; stiffness that is known to impact buckling strength. Although using a *K*-factor of 1.0 is convenient and safer in the routine design of joists, it can be at times very conservative with joists being over-designed. Thus, designers have a responsibility to consider accurate *K*-factors in terms of both economy and stability. The research described in this thesis explores three methods for computing effective length *K*-factors for webs in open web steel joists.

1.2 Research Method

In this thesis, and in consultation with the Steel Joist Institute, K-series and LH-series joist standard configurations were selected for study. Refined finite element analysis models were prepared for 18K3, 28K10, and 32LH06 joists. These joists were previously investigated in experimental studies by Emerson (2001) and Schwarz (2002), and hence, all member properties were readily available. On all joists, in-plane and out-of-plane *K*-factors are determined using three methods including: computational analyses of the K- and LH- series joists with both uniformly distributed loads and induced compressive strains (termed self-equilibrating induced-compression, SEIC), and a hand calculation method using the alignment charts appearing in the 14th edition of the AISC manual (AISC 2011). For the two computational analyses, the finite element analyses program MASTAN2, developed by Ziemian and McGuire (2010), is used.

MASTAN2 is an interactive graphics program that provides pre-processing, analysis, and post-processing capabilities. Pre- processing options include definition of structural geometry, support conditions, applied loads, and element properties. The analysis routines provide the user the opportunity to perform first- or second- order elastic or inelastic analyses of two- or three- dimensional frames and trusses subjected to static loads. Post- processing capabilities include the interpretation of structural behavior through deformation and force diagrams, printed output, and facilities for plotting response curves.

In many ways, MASTAN2 is similar to today's commercially available structural analysis software in functionality. The number of pre- and post- processing options, however, have been limited in order to minimize the amount of time needed for a user to become proficient at its use. The program's linear and nonlinear analysis routines are based on the theoretical and numerical formulations presented in the text Matrix Structural Analysis, 2nd Edition, by McGuire, Gallagher, and Ziemian (John Wiley & Sons, Inc. 2000). In this regard, the reader is strongly encouraged to use this software as a tool for demonstration, reviewing examples, solving problems, and perhaps performing analysis and design studies. Where MASTAN2 has been written in modular format, the reader is also provided the opportunity to develop and implement additional or alternative analysis routines directly within the program. Finally, it should be noted that MASTAN2 will execute on any computing platform where MATLAB is available. (Introduction, McGuire, Gallagher, and Ziemian, MATRIX STRUCTURAL ANALYSIS, Wiley, Inc. 2000)

The two computational methods employed are based on using two- and three-dimensional elastic critical load analyses (eigenvalue analyses) to compute the in-plane and out-of-plane buckling strengths of the web members of interest. Based on the buckling strength, the K-factor is back-calculated according to Eq. (1.2). The first computational method is based on buckling strengths computed for joists subject to a uniformly distributed load. This load is used to model the actual gravity load effects on joists. Unlike the buckling analysis with uniformly distributed loads, the novel SEIC method uses temperature loads to produce a compressive strain in only the member of interest. To avoid producing axial load in neighboring web members, this method requires the use of an additional parallel member of equal axial stiffness to "self-equilibrate" the axial force produced by the induced strain. In the studies completed as part of this research, artificial temperature loading of the parallel member is used to produce this self-straining effect. To avoid providing any additional bending stiffness and produce only axial force in the member of interest, both ends of the extra parallel member are pinned connected. As a consequence, equilibrium between the web member of interest and the artificial parallel member results in no axial force being distributed to the other neighboring members.

The third method used to determine the *K*-factors employs the alignment charts, which also considers the resisting stiffness at both end nodes of the web member. The alignment chart method is presented in the commentary to the AISC Specification (AISC 2011) and is a popular method for computing effective length factors for columns in buildings. In this thesis, the relative stiffness values used in the alignment chart (*G*-factors) are based on members framing into the ends of the web member of interest. For the purpose of this research, these stiffness values are derived using the direct stiffness method (McGuire et al. 2000) with a significant amount of MATLAB programming employed. The expressions obtained for these stiffness values are confirmed using MASTAN2 to analyze a portion of one of the joist configurations studied (Fig.1.2).

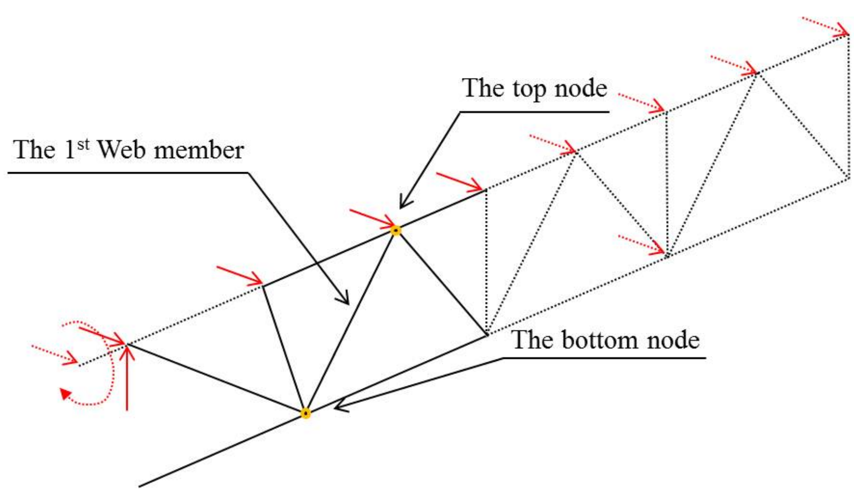


Figure 1.2. Portion of joist used to confirm hand method.

1.3 Thesis Overview

The three main objectives of this research are:

1) Use computational analyses (MASTAN2) to compute in-plane and out-of-plane buckling strengths of compressive web members, from which effective length *K*-factors may be back-calculated.
2) Develop a hand method for calculating these effective length *K*-factors based on the alignment chart method.
3) Make recommendations for in-plane and out-of-plane effective length *K*-factors for compression web members in steel joists.

This thesis is composed of five chapters.

Chapter 1 includes the introduction, the objectives, and the research method.

Chapter 2 provides background to the study, including the theory of the elastic buckling analysis and the effective length factor.

Chapter 3 describes the methods to compute *K*-factors for the joists. This chapter addresses the derivation process for the stiffness at both ends of the compressed web member, i.e. the two computational analyses with the joists. This chapter also includes a description of the two simulation analyses, including uniformly distributed loading and the SEIC method, which are used to determine *K*-factors from an elastic critical load (eigenvalue) analyses.

Chapter 4 compares the *K*-factors computed from the above methods for four joists configurations, including 18K3, 28K10, 32LH06_L1, and 32LH06_L2. Two- and three-dimensional analyses are used to provide in-plane and out-of-plane effective length factors.

Chapter 5 presents a summary of the work, conclusions, and recommendations for future research.

Chapter 2: Background

This chapter describes the theoretical background associated with using the flexural buckling strength to compute effective length factors. It also provides basic information about the effective length *K*-factor. The types of joists studied in this research, including K-series joists and LH-series joists, are described.

2.1 Flexural Buckling Strengths by Elastic Critical Load Analysis

The topic of elastic buckling of columns is well covered in the literature. The primary theory developed therein is often based on the Euler column. This column is assumed perfectly straight, prismatic, elastic material, and concentrically loaded, which implies that the axial load should be applied through the centroidal longitudinal axis of the member, thereby producing no bending or twisting (Ziemian 2010). In using Euler column theory these assumptions should be considered because actual columns rarely abide by them. The theory predicts that the column remains perfectly straight as the applied load increases, until it eventually bifurcates, or reaches the buckling load. At the instant of buckling, the column deforms into its buckled shape, which indicates the column has become unstable. Based on the deformed geometry along the length of the member, the governing differential equation for Euler buckling can be derived from an equilibrium formulation. Upon applying the boundary conditions of the mathematical problem, the resulting buckling load or critical load is

$$P_E = \frac{\pi^2 EI}{L^2} \tag{2.1}$$

where E is the modulus of elasticity of the material, I is the moment of inertia about the axis which buckling takes place, and L is the unbraced length of the column. The Euler load P_E is a reference value to which the strength of actual columns is often compared.

Equation (2.1) is derived under conditions of frictionless pinned-end supports. If different support conditions exist at the ends of the column, then it would seem reasonable that a coefficient related to the actual length should be included. This coefficient, or effective length K-factor, can also be derived from a differential equation similar to that used in determining the Euler buckling load. The critical buckling load that includes this coefficient can be expressed by

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \tag{2.2}$$

where KL is an effective length, which defines the portion of the deflected shape between points of zero curvature (inflection points). In other words, KL is the length of an equivalent pinned-ended column that is buckling at the same load as the end-restrained column (Ziemian 2010).

To compute P_{cr}, an elastic critical load analysis, as performed by the finite element method (basis for MASTAN2) reduces to solving the eigenvalue problem

$$\mathbf{det}\left|\left[\boldsymbol{K_e}\right] + \lambda\left[\boldsymbol{K_g}\right]\right| = \mathbf{0} \tag{2.3}$$

in which $[K_e]$ is the first-order elastic stiffness matrix for the system and $[K_g]$ is the system's corresponding geometric stiffness. Given that $[K_g]$ is a linear function of the element forces and moments resulting from the application of loads, temperature effects, and/or prescribed displacements, the computed eigenvalues are the ratios or load factors at which buckling occurs. Using these factors or ratios, the element forces and moments used to originally define $[K_g]$ can then be directly scaled to determine the buckling loads in members of interest. The eigenvectors coming from the solution of Eq. (2.4) represent the buckling modes or deformed shapes. More details for this type of analysis are provided in McGuire, et al. (2000).

The critical load plays a significant role in this thesis. By using the critical load of each compressive web member of interest, as provided by a computational analysis such as that available in MASTAN2, the effective length K-factor can be back-calculated as

$$K = \frac{\pi}{L}\sqrt{\frac{EI}{P_{cr}}} \tag{2.4}$$

2.2 The Effective Length Factor

For the design of a column restrained by hypothetical end conditions (fixed, pinned, or free), an effective length factor table is provided in the AISC manual (Fig.2.3).

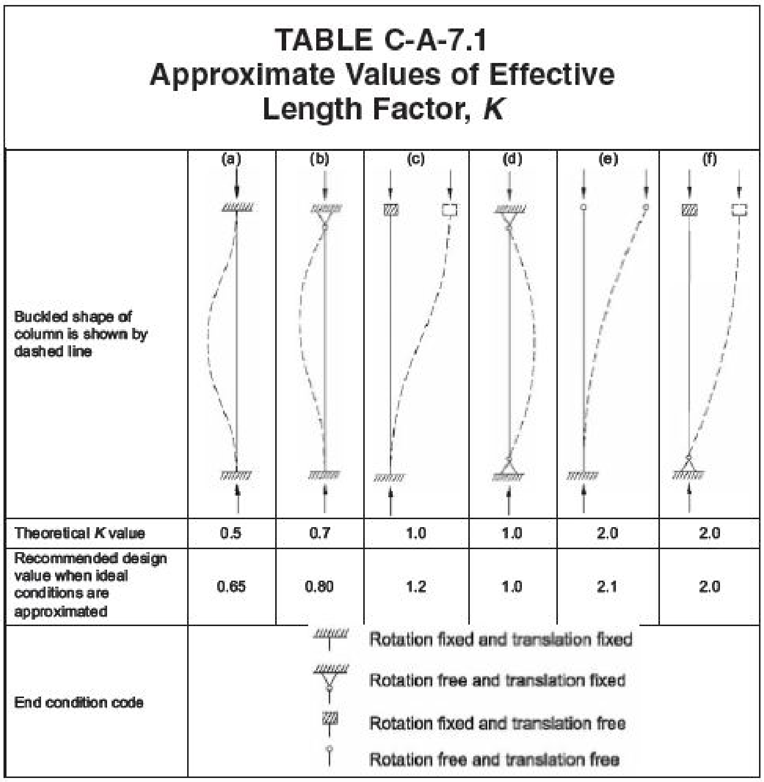


TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code	Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free					

Figure 2.3. The effective length factor table (AISC 2011).

The table in Fig.2.3 shows the theoretical K values and recommended design K values for the ideal condition depending on the end conditions. In the table, cases *(a)*, *(b)*, and *(d)* represent sidesway inhibited conditions. As shown in the table, the range of the K-factor appears between 0.5 and 1.0 for the theoretical K-values and between 0.65 and 1.0 for the recommended design K-values. For cases *(c)*, *(e)*, and *(f)*, which represent the sidesway uninhibited condition, the range of *K*-values is above 1.0 for both the theoretical and the recommended design *K*-values.

The effective length *K*-factor is defined as a proportion of the original unbraced length of the compression member. The *K*-factor can be defined as the distance between the inflection points where the curvatures are zero, as shown in Fig. 2.4.

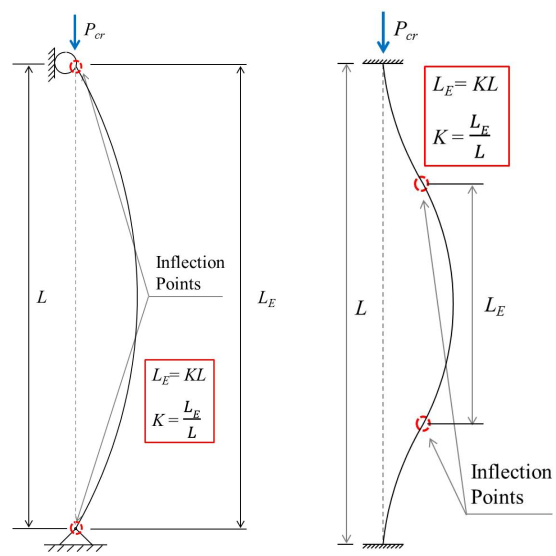


Figure 2.4. Column buckling with *K*-factors defined by inflection points.

With the advent of more advanced computational analysis, effective lengths of columns may be estimated using the critical axial loads (Nishino 1999). As seen from the critical load equation Eq. (2.2), it is important to note that the effective length is related to the buckling strength. As explained previously in regard to Eq. (2.2), the effective length *K*-factor is located in the denominator of the buckling strength expression, which means that the buckling strength and *K*-factor are inversely proportion to each other. This inverse relationship can be shown with simple example Eq. (2.5). When the *K*-factor decreases from 1.0 to 0.5, the flexural buckling strength increases by a factor of 4.

$$K = 1.0,\ P_E = \frac{\pi^2 EI}{L^2} \quad vs.\ K = 0.5,\ P_{cr} = 4.0\frac{\pi^2 EI}{L^2} = 4.0P_E \tag{2.5}$$

As shown above in this calculation, and as expected in general, the effective length can have a dramatic impact on the buckling strength of the member.

The Steel Joist Institute (SJI) specification committee has conservatively assumed $K = 1$ for the design of web members in K-series joists (SJI 2010). In that assumption, they are not accounting for any rotational restraint provided by the top and bottom chords and the neighboring tension web members in specifying the in-plane and out-of-plane buckling strength of web members. Using $K = 1$ as the effective length factor in joist design is more conservative, but with respect to economy, adopting 1.0 as the *K*-factor can be very inefficient. This is because a joist web member designed with $K= 1$ will mostly likely be able support more load than anticipated. With this in mind, this thesis expects the actual *K*-factor values for compression web members to be less than 1.0, especially when the rotational restraint provided by adjacent members is accounted for.

2.3 Alignment Charts

Alignment charts have been widely used in the design of compression members in structural frames since the time that the effective length method was first introduced in the AISC specification in 1961. Although researchers presented other methods for computing *K*-factors in frames with sidesway (Lu and Kavanagh 1962; Johnston 1976), they were typically not practical even though they did provide sufficient accuracy. The alignment chart method (Julian and Lawrence 1959), however, did provide an accurate and practical design procedure. The alignment chard method has since been adopted by many present day standards, including AISC (2010) and ACI-318 (2011).

In the alignment chart method, the effective length *K*-factor is expressed in terms of the relative joint bending stiffness ratio, *G*, at each end of the column for both sidesway-prevented and sidesway-permitted frames (Duan 1989). The sidesway inhibited and not inhibited equations, which are used for plotting the alignment charts in the AISC manual, and used for computing *K*-factors are

$$\frac{G_A G_B}{4}(\pi / K)^2 + \left(\frac{G_A + G_B}{2}\right)\left(1 - \frac{\pi / K}{\tan(\pi / K)}\right) + \frac{2\tan(\pi / 2K)}{(\pi / K)} - 1 = 0 \quad \text{(C-A-7.1, AISC)} \quad (2.6)$$

$$\frac{G_A G_B (\pi / K)^2 - 36}{6(G_A + G_B)} - \frac{(\pi / K)}{\tan(\pi / K)} = 0 \quad \text{(C-A-7.2, AISC)} \quad (2.7)$$

These two equations are derived with the following idealized assumptions, which should be considered carefully when using the alignment charts.

(1) Behavior is purely elastic.

(2) All members have constant cross section.

(3) All joints are rigid.

(4) For columns in frames with sidesway inhibited, rotations at opposite ends of the restraining beams are equal in magnitude and opposite in direction, producing single curvature bending.

(5) For columns in frames with sidesway uninhibited, rotations at opposite ends of the restraining beams are equal in magnitude and direction, producing reverse curvature bending.

(6) The stiffness parameter LP / EI of all columns is equal.

(7) Joint restraint is distributed to the column above and below the joint in proportion to EI/L for the two columns

(8) All columns buckle simultaneously.

(9) No significant axial compression force exists in the girders.

(AISC 2011)

As shown in Eqs. (2.6) and (2.7), the alignment chart can be separated into two types of cases depending on whether or not sidesway is prevented (e.g. braced frame) or permitted (e.g., rigid frame). As shown in Fig. 2.1, the system to the left is assumed restrained from horizontal movement. Under this condition, the *K*-factor for the column will range between 0.5 and 1.0, as shown in the alignment chart (Fig.2.2). The effective length factor, *K*, of the components of the braced frame is normally taken as 1.0 (AISC 2011). However, as the rotational stiffness at the far ends of the compressive column increase, the effective length factor of this system decreases to 0.5.

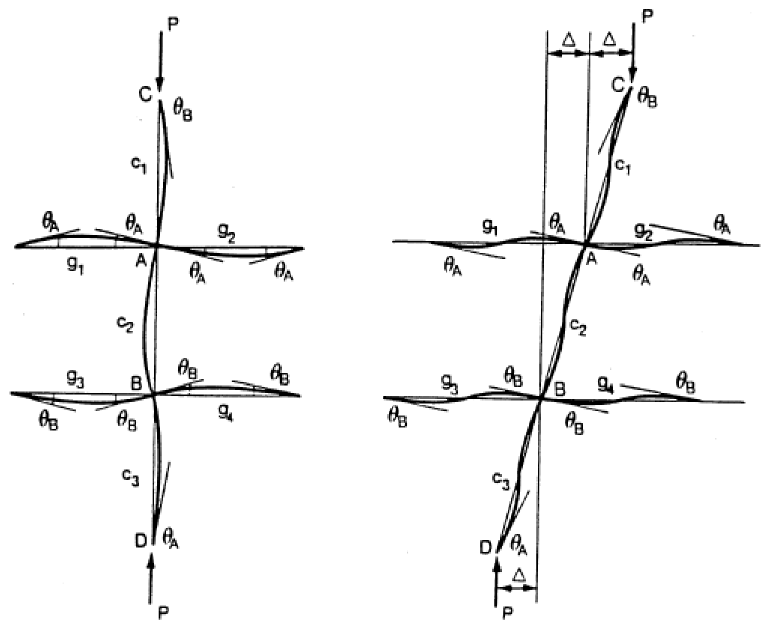


Figure 2.1. Subassemblage model for *K*-factors in braced (left) and unbraced (right) frames (AISC 2011)

The other case shown in Fig. 2.1 is for sidesway permitted, such as a column in an unbraced frame. Unlike the braced frame, the horizontal translation is permitted. Because of this permitted sidesway, when the top and bottom beams or girders (connected by the column of interest) are very flexible, the structure will become unstable, which implies a *K*-factor approaching infinity. If the supporting beams are rigid, then the *K*-factor would be 1.0. Therefore, the range of the *K*-factor is above 1.0 in an unbraced frame, as shown in Fig. 2.2.

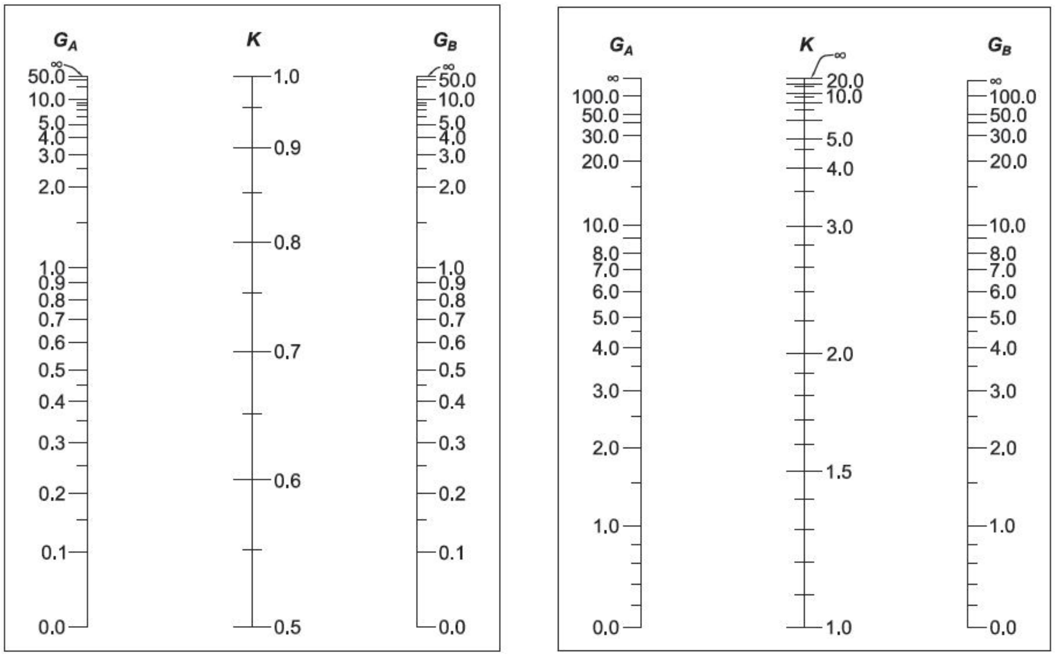


Figure 2.2. Alignment charts for sidesway inhibited (left) and uninhibited (right) (AISC 2011).

The two alignment charts are composed of 3 gradations. Both side gradations indicate the stiffness ratio, as shown in Eqs. (2.6) and (2.7). The *K*-factor gradation is placed in the center of the alignment chart. Depending on the top and bottom relative column-to-beam stiffnesses ($\boldsymbol{G_A}$ and $\boldsymbol{G_B}$), the *K*-factor is determined by drawing a straight line segment between these values. As shown in the two alignment charts, the subscripts A and B of the stiffness ratio *G*, indicate the ends of the compression member of interest. For example, $\boldsymbol{G_A}$ could represent the top joint of the compression member and $\boldsymbol{G_B}$ the bottom joint, or vice-versa because the charts are symmetrical. In the alignment charts, $\boldsymbol{G_A}$ and $\boldsymbol{G_B}$ indicate the stiffness ratio, of which the numerator is a summary of the stiffness of columns and the denominator is the summary of the stiffness of girders or beams, as given by

$$G = \frac{\sum_{i=1}^{n} \left(\frac{E_i I_i}{L_i} \right)_c}{\sum_{j=1}^{m} \left(\frac{E_j I_j}{L_j} \right)_g} \tag{2.8}$$

where, in the stiffness of column (numerator), E_c is its elastic modulus, I_c is its moment of inertia, and L_c is its unsupported length. In the girders connecting to either the top or bottom of the column, E_g is the elastic modulus of the girder, I_g is the moment of inertia of the girder, L_g is the unsupported length of the girder. Also, n and m indicate the number of columns and girders at the connection points. It is important to note that I_c and I_g are taken about axes perpendicular to the plane of buckling being considered.

Because the alignment charts are based on the assumptions previously discussed, and conditions of real structures cannot be in exact agreement with the idealized conditions of these assumptions, adjustments are often required. Duan and Chen (1988) suggested a simple modification of the alignment charts in order to take into account the effect of the boundary conditions at the far ends of beams, both above and below the column being investigated, in braced frames (Duan, Chen 1988). In their study, far end conditions have a significant effect on the *K*-factor of the column under consideration. As reported by their research, depending on the boundary conditions at the far end, the following adjustments should be applied:

Adjustments for Girders With Differing End Conditions. For sidesway inhibited frames, these adjustments for different girder end conditions may be made:

(a) If the far end of a girder is fixed, multiply the $(EI/L)_g$ *of the member by 2.*

(b) If the far end of the girder is pinned, multiply the $(EI/L)_g$ *of the member by 1 1/2.*

Adjustments for Girders With Differing End Conditions. For sidesway uninhibited frames, these adjustments for different girder end conditions may be made:

(a) If the far end of a girder is fixed, multiply the $(EI/L)_g$ *of the member by 2/3.*

(b) If the far end of the girder is pinned, multiply the $(EI/L)_g$ *of the member by 1/2.*

(AISC 2011)

2.4 Open Web Steel Joists

Open web steel joists are prefabricated truss-type flexural members typically used in floor and roof support systems. Generally, these joists are used in combination with metal decking to provide an economical system for light to moderately loaded structures. In terms of their bending resistance to weight ratio, open web steel joists are considered to be very efficient structural members when compared to conventional rolled steel sections.

Historically, steel joist manufacturing dates back to 1923, when Warren truss-type configurations comprised of top and bottom chords and a continuous round bar that was bent to form the web (SJI 2002). By 1928, the steel joist industry was thriving and various members of the steel joist industry came together to form the Steel Joist Institute (SJI), which would soon publish the first standard specification for steel joists. Since 1953, the Steel Joist Institute has developed adaptable joists depending on a purpose, span, minimum yielding stress steel, and property of the joist. In the standard specification of the SJI, the joists are comprised of compression web members, tension web members, vertical web member, and top and bottom chords. The SJI standard provides four types of open web steel joists, including K-Series, KCS-Series, LH-Series, and DLH-Series. A name of each joist series is made of two numbers and letters such as 16K6, 10KCS3, 28LH05, and 68DLH17. These numbers and letters indicate each character of the joists. The first number indicates the depth in inches, e.g., 16, and the letter represents the series designation, K. The third number signifies place within the series, e.g., 6.

The LH-Series joist was developed and introduced in 1966 by the SJI. The LH-Series joist is usually used in long spans and is designed for a uniformly distributed load. Similarly, the more recent DLH-Series joists are used in long span structures. However, DLH-Series joists are different from the LH-Series. The letter of D indicates “deep”. Moreover, these joists are intended primarily to support roof decks. DLH-Series joists start at a depth of 52 in. and go up to 120 in. Their strengths are tabulated for spans of 62 ft. up to 240 ft.

After the development of the LH-Series joists, the K-Series Joists were introduced in 1986 and eventually replaced many of the LH-Series joists. K-Series joists are generally used in many steel structures. K-series joists support uniformly distributed loads at spans up to 60 ft. and are available in depths from 10 in. to 30 in. The KCS-series joists are similar to the K-series joists. The difference between them is directly related to the moment and shear diagrams assumed in design. In the case of K-Series joists, the uniformly distributed load is applied on the joist resulting a moment diagram of parabolic shape. Corresponding to this bending diagram, the shear diagram appears as a triangular shape. However, the moment diagram of the KCS-series joist is a constant over all interior panel points and constant shear. Thus, the KCS-series joists can be used for special loading situations that combine distributed and concentrated loads.

This study considers two joist configurations, K-series and an LH-series. The following presents more detailed information regarding the K-series joist and the LH-series joists.

K-Series Joist

The K-Series joists are perhaps the most commonly used joists for floor and roof systems. The term "Open Web Steel Joist" refers to open web, with load-carrying members utilizing hot-rolled or cold-formed steel sections. The Steel Joist Institute uses Allowable Stress Design (ASD) or Load and Resistance Factor Design (LRFD) to proportion K-Series joists in accordance with their standard specification. The K-Series joists have been standardized in depths from 10 in. up to 30 in., for spans up to 60 ft. The maximum total safe uniformly distributed load carrying capacity of a K-Series joist is 550 plf in ASD or 825 plf in LRFD (SJI 2011).

The reasons for developing the K-Series Joists were: (1) to achieve greater economies by utilizing the load span design concept; (2) to meet the demand for roofs with lighter loads at depths from 18 inches to 30 inches; (3) to offer joists whose load carrying capacities at frequently used spans are those most commonly required; (4) to eliminate the very heavy joists in medium depths for which there was little, if any, demand. *(SJI Specification 2011)*

LH-Series Joist

The letters of "LH" refer to the "Long span steel joist". The LH-series joist has the same characteristics as the K-series joist except for available spans and load-carrying capacity. The LH-Series joists have been standardized in depths from 18 inches to 48 inches, for spans up to 96 feet. The maximum total safe uniformly distributed load-carrying capacity of 2400 plf in ASD and 3600 plf in LRFD has been established for this alternate LH-Series format.

2.5 Recent Research Related to *K*-factors of Web Members

In the 43rd SJI Specification, the effective length *K*-factor for in-plane and out-of-plane buckling of web members in K-series joists is taken as 1.0. In LH-series joists, however, the *K*-factor is taken as 0.75 for in-plane buckling and 1.0 for out-of-plane buckling. Although a *K*-factor of 1.0 is more conservative, it is not ideal value for computing the buckling strength due to economical considerations. Furthermore, given that the top and bottom chords of the joist are continuous and the ends of the web members are often welded between these chords, the joist is not a traditional truss comprised of two-force members joined at pinned connections. Under more realistic joist conditions, using a *K*-factor as 1.0 for web members will underestimate their capacity. To improve this situation, some researchers have conducted experimental studies. A brief overview of one of these projects is presented below.

Yost, Dinehart, Gross, Pote, and Gargan (2004) conducted an experimental study on open web steel joists to explore the strength and stiffness of K-Series joists with crimped-ended single angles for web members (Fig. 2.5) instead of traditional single angle or round bar compression web members. These researchers focused on monitoring the failure loads, deflections, top and bottom chord strains, and failure mechanisms of the joists. In their experimental studies, buckling often took place at the crimp location closest to either the top or bottom chord. The buckled shape of all samples in their research showed considerable rotational restraint at the joist, indicating that the actual effective length factor is less than its assumed value of K = 1.0.

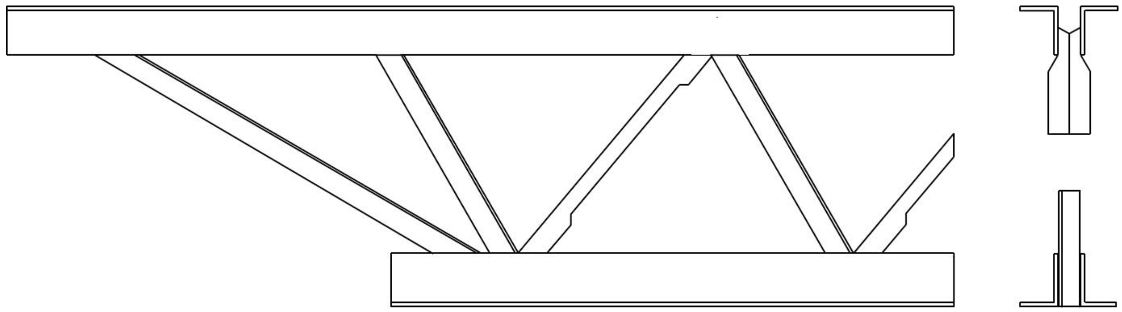


Figure 2.3. The crimped double angles shape (Yost, et al. 2004)

2.6 In-plane Buckling and Out-of-plane Buckling

In general, compression web members can buckle either in or out of the plane of the joist. As shown in Fig. 2.6, the left portion of the figure shows out-of-plane buckling and the right portion shows in-plane buckling. Depending on the direction in which the buckling occurs, the governing properties of the web member that should be taken into account are different. For the case of in-plane buckling, only the in-plane bending stiffness of the connected members need be considered. On the other hand, both the out-of-plane bending stiffness and torsional resistance of the connected members should be considered when investigating out-of-plane buckling.

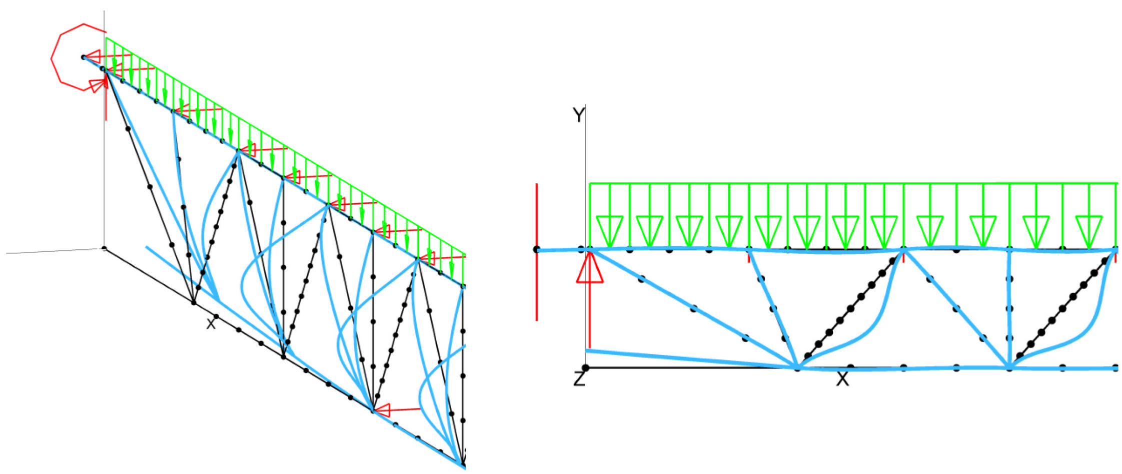


Figure 2.4. Buckling modes: out-of-plane (left) and in-plane (right).

Chapter 3: Derivation of the Key Stiffness Expressions Used in Hand Calculation Method

The primary objectives of this chapter are to:

Provide an overview of how induced compressive strains can be used to compute effective length *K*-factors. This includes details related to the novel self-equilibrating induced compression (SEIC) method, and its implications with respect to computing effective length factors in web members of joists.

Provide an overview of how the alignment charts can be used as a hand-method for estimating in-plane and out-of-plane effective length *K*-factors in compression web members of joists. Derivations of closed form expressions for calculating the restraining stiffness at the top and bottom ends of a compression web member are key parts of this process, and their accuracy is confirmed with computational analysis.

3.1 Induced Compressive Strains - Simple Frame Study

A simple frame study was conducted to develop one of the methods used in this research for computing effective length *K*-factors. The simple frame consists of a top beam, bottom beam, and a column (Fig. 3.1). To simulate an induced compressive strain in the column, its temperature is steadily increased thereby making it want to increase in length. Because the column is constrained by the top and bottom beams, the member is subject to compression and eventually buckles. Employing elastic critical load analyses (available within MASTAN2) and letting the top beam supports be either pinned or rollers, this approach can be used to study sidesway inhibited and sidesway uninhibited cases.

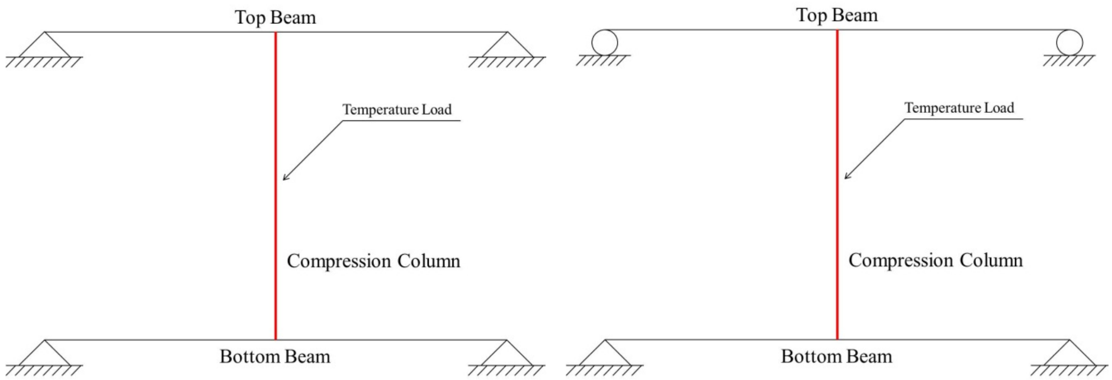


Figure 3.1. Simple frame study: sidesway inhibited (left) and sidesway not inhibited (right).

Horizontal movement of the top beam is prevented in Fig. 3.1 (left) by the beam's pinned supports, which result in the elastic strength of the column being calculated from a K-factor between 0.5 and 1.0. On the other hand, permitting horizontal movement of the top beam (right Fig. 3.1) will result in the column's compression strength computed using a K-factor larger than or equal to 1.0. In both cases, the exact value for K would depend on the relative column-to-beam stiffnesses.

When the beams are rigid, the method correctly computes effective length K-factors of 0.5 and 1.0 for the sidesway prevented and permitted cases, respectively (Fig. 3.2). On the other hand, making the top and bottom beams very flexible will essentially result in no rotational restraint at the ends of the column. Again, the analysis with temperature induced strains computes the correct K-factors of 1.0 for sidesway inhibited and infinity for sidesway not inhibited (Fig. 3.3).

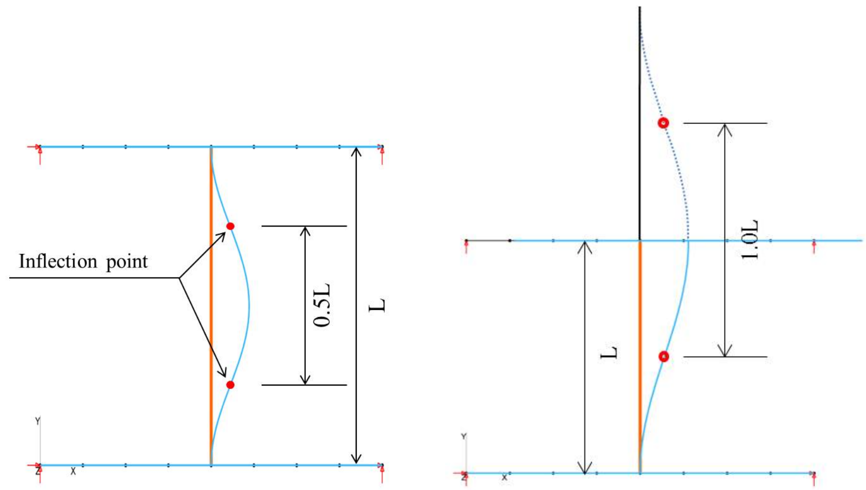


Figure 3.2. Buckled shapes with rigid beams for sidesway prevented (left) and sidesway permitted (right).

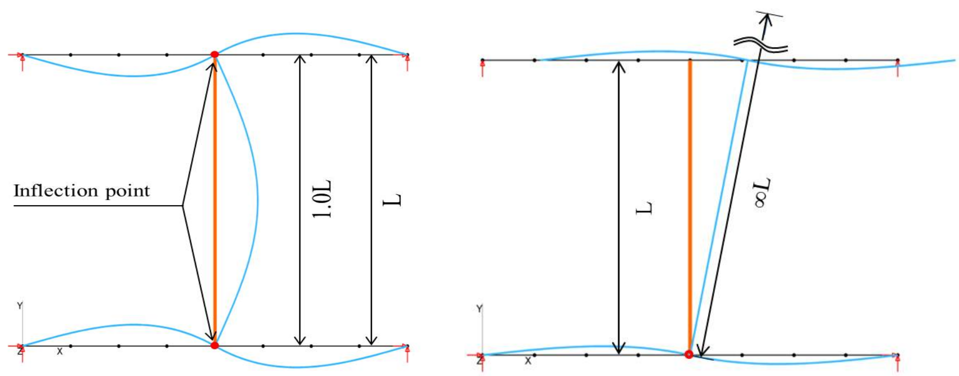


Figure 3.3. Buckled shapes with flexible beams for sidesway prevented (left) and sidesway permitted (right).

Furthermore, by varying the moment of inertia in both the top and bottom beams (and also requiring them to be equal), effective length *K*-factor curves can be produced as shown in Fig. 3.4. The plot is shown with the stiffness ratio $G=(EI/L)_{col}/(EI/L)_{bm}$ as the abscissa and the *K*-factor as the ordinate. Note that the coefficient k is used to represent the restraining stiffness of the beams $(EI/L)_{bm}$. The *K*-value curve appearing in Fig. 3.4 is in an exact agreement with *K*-factors determined from the alignment chart method (AISC 2011).

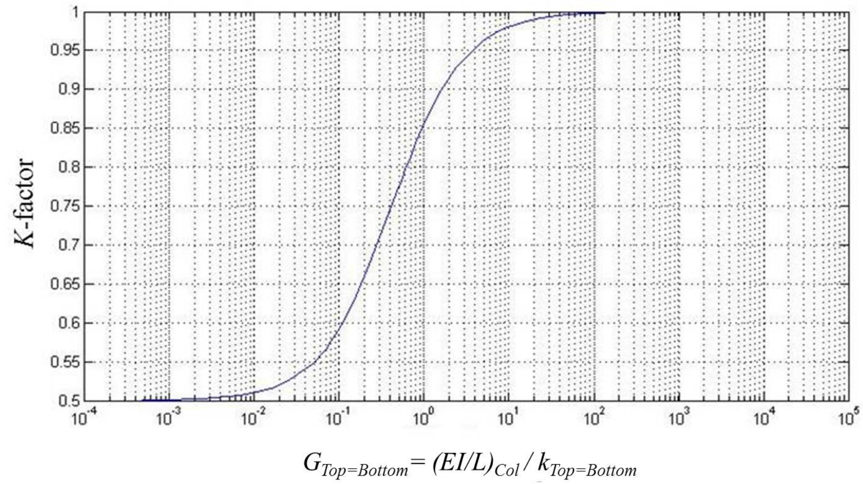


Figure 3.4. Curve for effective length *K*-factor as a function of the stiffness ratio *G* (sidesway inhibited case).

By allowing the moments of inertia in the top and bottom beams to differ, the plot in Fig. 3.5 is obtained. And, again these curves are identical to what is obtained using the alignment chart method. Given the results presented in Figs. 3.4 and 3.5, there is convincing evidence that the self-induced compressive strain method is a viable approach for determining *K*-factors.

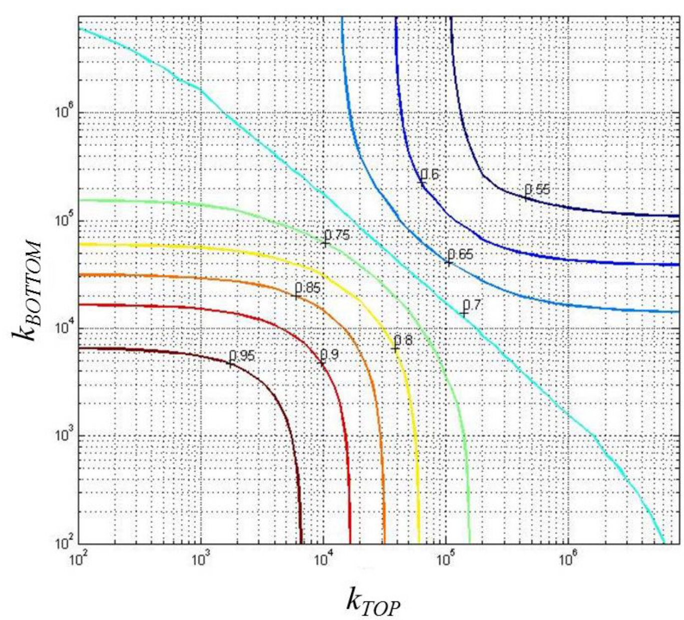


Figure 3.5. *K*-factors obtained varying top and bottom beam stiffnesses (sidesway inhibited case).

3.2 Using Induced Compressive Strains to compute *K*-factors in Steel Joists

In this study, all four joists (18K3, 28K10, 32LH06_L1, and 32LH06_L2) were studied using two- and three-dimensional critical load analyses, which is an option available in MASTAN2. To isolate the compressive strength and *K*-factors of a specific member, such as a joist web member, compressive strains were induced in this member using the approach outlined in the previous section. To assure that axial forces were not produced in neighboring members, a self-equilibrating induced compression (SEIC) method was developed. In this method, an artificial parallel element is provided so that any axial forces developed through the self-straining process could be equilibrated, thereby not distributing forces to any other members in the joist. The additional parallel element is of equal axial stiffness as the member being isolated and is pinned connected at its ends so as to not provide any additional flexural stiffness.

The simple frame example from the previous section is used to illustrate this concept (Fig. 3.6). As the temperature in the additional parallel member load is reduced, it will tend to contract. Because this parallel member is connected to the actual column, compatibility will result in the column being compressed by the same force that the parallel member feels as it is stretched. If both members are of equal stiffness (e.g., same length, area, and elastic modulus) and essentially lie on top of each other (in the finite element models, the members share the same end nodes), there will be no net displacement at the column ends. And, with no displacements occurring in the system, there will be no forces or moments developed in the beams. Similar behavior would occur if this parallel element is used to strain web members in joists. And, with this approach, the buckling load and *K*-factor for a specific member can be computed.

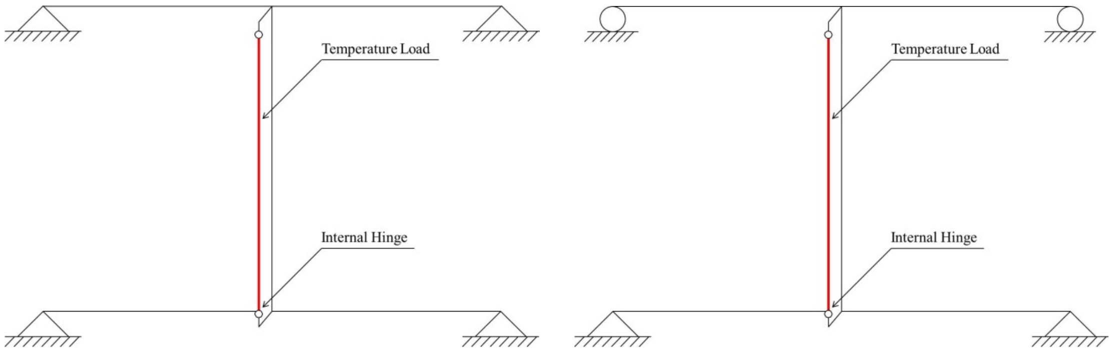


Figure 3.6. Self-equilibrating member attached to simple frames with and without sidesway inhibited.

An important attribute of this approach is that the compressive load being applied to the member of interest tracks (i.e., is aligned with) a chord defined by the ends of the buckled member. This concept of a member buckling force tracking its chord is demonstrated with the simple example shown in Fig. 3.7. The vertical member (column) is loaded at an end (top) where only vertical displacement is permitted (all other degrees of freedom are restrained). The bottom of the column is subject to two different support conditions; vertical displacement restrained by two in-plane cables (left) and vertical displacement restrained by a vertical support (right). In both cases, all other degrees of freedom at the base of the column are unrestrained.

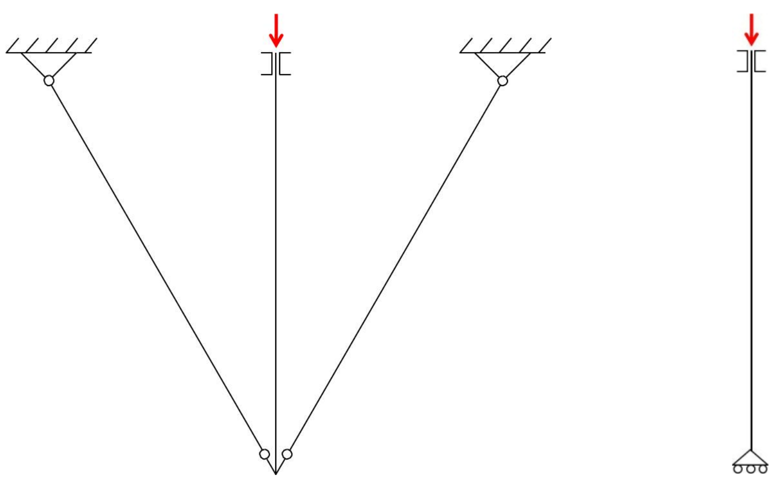


Figure 3.7. Examples of loading along axis of members: The compression member with cables (left) and vertical support (right).

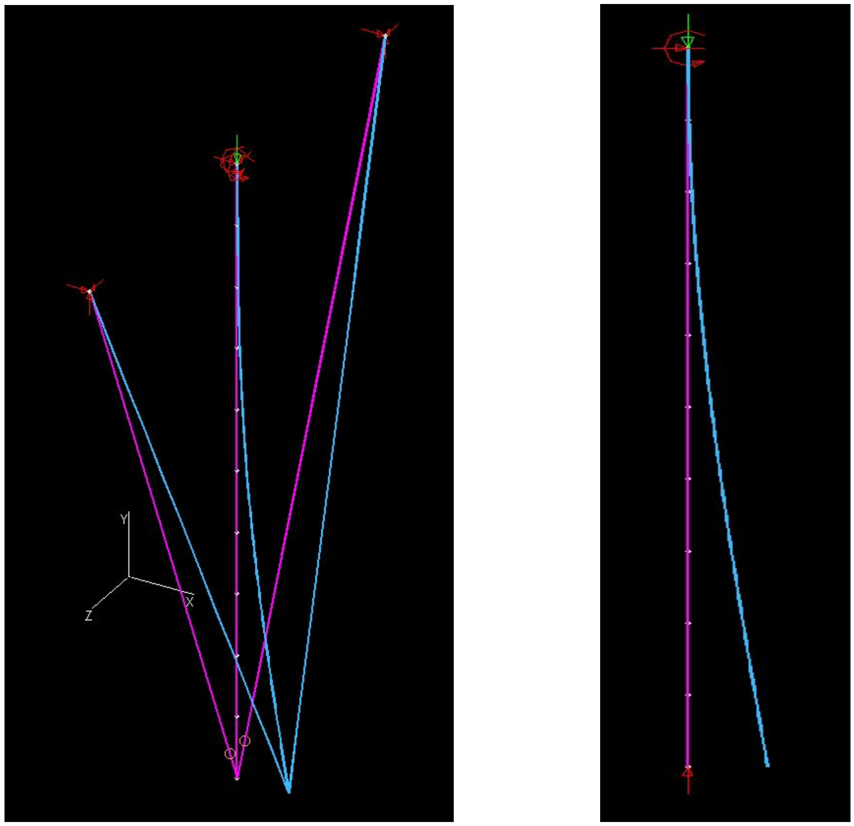


Figure 3.8. The deflected shapes (out-of-plane buckling modes) from the analyses.

The resulting out-of-plane elastic critical loads for the two configurations differ by a factor of 4.0 (regardless of the axial stiffness and/or steepness of the cables), with the cable system allowing for a greater buckling load. The *K*-factor for the cable-supported system is back-calculated to be 1.0, and as expected, the *K*-factor for the other system is 2.0 (Fig. 3.8). The difference is a result of the direction of the resisting force. For the cable system, the resisting force tracks (is aligned with) a chord defined by the columns ends. In contrast, the resisting force in the non-cable system is always vertical. Similar results are obtained by employing second-order elastic analyses on systems with small out-of-plane imperfections. As shown in Fig. 3.9, the applied loads at which instability occurs again differs by a factor of 4.0 (556 kip for cable supported system versus 144 kip for vertical support system). Note that these numerical values correspond to the vertical compression member being W10X60 of length 240 inches.

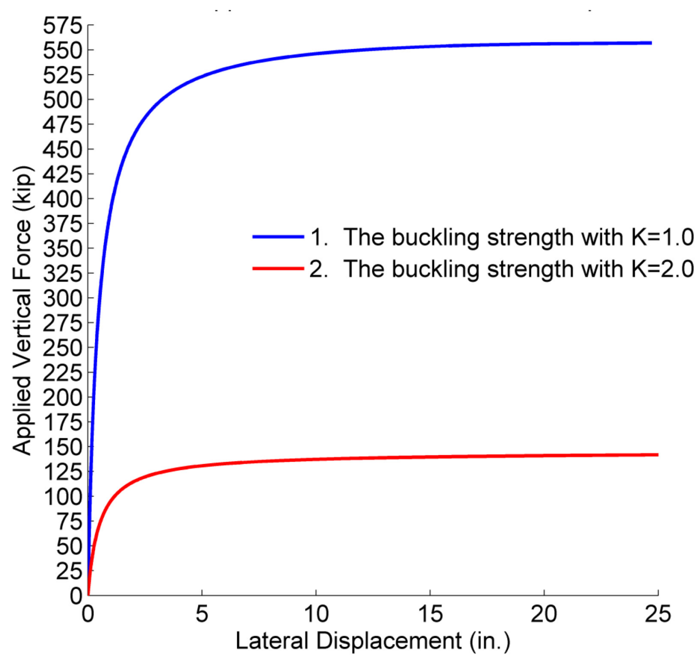


Figure 3.9. Response of cable- and noncable systems.

If the cables in this example were to provide some degree of rotational restraint to the bottom of the column, then the critical load increases and the out-of-plane effective length *K*-factor would be less than 1.0, but always greater than or equal to 0.5. The significance of this study is important for understanding why out-of-plane *K*-factors for compression web members in joists are bounded between 0.5 and 1.0. The cables in the simple example are representative of the bottom chord and neighboring tension web members.

With this in mind, the following is a summary for computing effective length *K*-factors in steel joists using the self-equilibrating induced compression (SEIC) method.

1. Subdivide the compression web member of interest into 10 elements along its length.
2. Define an element parallel to the web member of interest that has exactly the same geometric and material properties as the web member of interest.
3. Define connections at end of parallel element as pinned (hinges).

4. Apply a positive temperature load to the parallel member, thereby compressing web member of interest. No other loads, including self-weight, are to be applied to the joist.
5. Perform three-dimensional elastic critical load analysis of the joist system.
6. Observe and confirm buckling mode and load of the web member of interest.
7. Back-calculate effective length *K*-factor from buckling force in the web member of interest.

3.3 Use of Alignment Charts to Compute *K*-factors for Web Members

Since its introduction in the AISC Specification in 1961, the effective length *K*-factor method of design has been widely accepted for use in the stability assessment of columns in various types of structures. In the more recent editions of the AISC specification, the direct analysis method has been introduced as an alternate method because the effective length method has many assumptions with regard to system behavior and has at times proven to be less accurate (Surovek et al. 2005). Although the direct analysis method may be the preferred approach, the effective length method can provide a simple method for computing the strength of compression web members. As indicated earlier in this thesis, use of the alignment chart to determine *K*-factors requires an estimate of the relative stiffness *G* ratio at each end of the web member of interest.

3.3.1 Stiffness Equation for Using Alignment Chart in Joists

As explained in Chapter 2, the alignment chart consists of two simple steps, which includes computing the stiffness *G* ratios and from these obtaining the effective length *K*-factor. Using these charts for a three-dimensional system requires that the effective length *K*-factor depend on relative stiffness *G* ratios that not only includes the flexural stiffness, but may also include the torsional resistance provided by neighboring members (Duan 1989). In routine building frames, the G_A and G_B ratios include the flexural stiffnesses of the connected columns and girders at top and bottom joints A and B, and are defined by

$$G_{A\ or\ B} = \frac{\sum_{i=1}^{n}\left(\frac{E_i I_i}{L_i}\right)_c}{\sum_{j=1}^{m}\left(\frac{E_j I_j}{L_j}\right)_g} \tag{3.1}$$

where Σ indicates a summation of all members rigidly connected to the joint and lying in the plane in which buckling of the column is being considered. E is the elastic modulus and I is the columns' and girders' moments of inertia taken about the axis perpendicular to the plane of the buckling, and L is the corresponding length of the members.

In a three-dimensional analysis of a steel joist in which the out-of-plane K-factor of a web member is desired, the sum of the "girder" stiffnesses (denominator of Eq. (3.1)) should include the flexural and torsional resistances of the neighboring members, especially the top and bottom chords. The torsional and flexural stiffnesses can be defined by

$$k_{flexure} = \frac{4EI}{L} \tag{3.2}$$

$$k_{torsion} = \frac{GJ}{L} \tag{3.3}$$

in which G is the shear modulus, and J is the torsional constant.

For several members framing into the end of a compression member, the total resisting bending and torsional stiffness is

$$k_{flexure} = \sum_{i=1}^{N} \frac{4E_i I_i}{L_i} \tag{3.4}$$

$$k_{torsion} = \sum_{i=1}^{N} \frac{G_i J_i}{L_i} \tag{3.5}$$

Technically, Eqs. (3.4) and (3.5) only apply when all of the neighboring members are orthogonal to the web member of interest, which is obviously not the case for a typical joist as shown in Fig. 2.1. For members that are not orthogonal, only their component of stiffness that is perpendicular to the axis of the web member of interest should be included in Eq. (3.1). In essence, this requires a transformation of the stiffness from the reference axes of the top chord, bottom chord, and/or the neighboring tensile member to a consistent stiffness component that is perpendicular to the compression web member of interest.

Furthermore, and as explained previously in Section 2.1, determining the *K*-factor by using the alignment chart under the fixed far end conditions (which for simplicity was assumed in this study) the following adjustments should be applied to the girder stiffnesses.

Adjustment for Girders With Differing End Conditions. For sideway inhibited frames, these adjustments for different girder end conditions may be made:

(a) If the far end of a girder is fixed, multiply the $(EI/L)_g$ *of the member by 2.*

(AISC 2011)

With this in mind, the flexural stiffness contribution is multiplied by 2.0, which results in the denominator of Eq. (3.1) equaling

$$2\sum_{i=1}^{N}\left(\frac{E_i I_i}{L_i} + \frac{G_i J_i}{L_i}\right) = 2\left(\frac{k_{flexure}}{4} + k_{torsion}\right) = \frac{k_{flexure}}{2} + 2k_{torsion} \tag{3.6}$$

Substituting this into Eq. (3.1), the stiffness *G* ratio becomes

$$G_{Top/Bottom} = \frac{\sum_{i=1}^{N}\left(\frac{E_i I_i}{L_i}\right)_c}{\sum_{i=1}^{N}\left(\frac{E_i I_i}{L_i}\right)_g} = \frac{\sum_{i=1}^{N}\left(\frac{E_i I_i}{L_i}\right)_{web\ member}}{\left(\frac{k_{flexure}}{2} + 2k_{torsion}\right)_{neighbor\ members}} \tag{3.7}$$

As a result, the *K*-factor for a compression member can be determined by using the alignment chart method when the flexural and torsional stiffness contributions are transformed accordingly.

3.3.2 Closed-Form Expressions for Computing Stiffness *G* Values

In computing the stiffness *G*-values for a web member in joist, the term in the numerator of Eq. (3.7) reduces to a simple calculation of EI/L for the compression web member of interest. As indicated in the previous section, the denominator of Eq. (3.7) is more complex to compute because only the neighboring members' stiffness contributions that are perpendicular to the web member of interest should be included.

For each neighboring member, this stiffness contribution can be determined from key equations of the direct stiffness method, in which all element stiffness matrices are transformed from local element space to a single global space. By defining the local axis of the compression web member of interest to be the global coordinate system, the neighboring element stiffness contributions may each be transformed into this "global" coordinate system and summed accordingly.

Based on physics, stiffness is a measure of the resistance (force or moment) offered by a body to deformation (displacement or rotation), which can be represented mathematically as

$$k = \frac{F}{\delta} \tag{3.8}$$

where F is the resisting force, and δ is the applied displacement; noting that both the force and the displacement are in the same direction. By using this simple relationship and basic equations of solid mechanics, the elastic stiffness for a 12-degree of freedom finite element (line element) can be determined (Fig. 3.10). For this element, there are 6 independent displacement components at each end of the element, including three transitional and three rotational degrees of freedom. The coefficients of the matrix represent axial stiffness, flexural stiffness, and torsional stiffness.

$$\left[k'_e\right]=\begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-EA}{L} & 0 & 0 & 0 & 0 & 0 \\ & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\ & & \frac{12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 & 0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 \\ & & & \frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 \\ & & & & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L} & 0 & \frac{2EI_y}{L} & 0 \\ & & & & & \frac{4EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \\ & & & & & & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{-6EI_z}{L^2} \\ & & & & & & & & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ & & & Sym. & & & & & & \frac{GJ}{L} & 0 & 0 \\ & & & & & & & & & & \frac{4EI_y}{L} & 0 \\ & & & & & & & & & & & \frac{4EI_z}{L} \end{bmatrix}$$

Figure 3.10. Elastic stiffness matrix for a line element (McGuire, et al. 2000).

It is also important to note that this stiffness matrix references the element's local coordinate system, which may not necessarily align with the system's global coordinate system. To have this element stiffness matrix reference another orthogonal coordinate system, such as a system's global coordinate system, the following transformation is required.

$$[k] = [\Gamma]^T [k_e'][\Gamma] \tag{3.9}$$

where the transformation matrix [Γ] is defined as

$$[\Gamma] = \begin{bmatrix} [\gamma] & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\gamma] & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & [\gamma] & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & [\gamma] \end{bmatrix} \tag{3.10}$$

The repeating [γ] in Eq. (3.11) is defined by

$$[\gamma] = \begin{bmatrix} l_{x'} & m_{x'} & n_{x'} \\ l_{y'} & m_{y'} & n_{y'} \\ l_{z'} & m_{z'} & n_{z'} \end{bmatrix} \tag{3.11}$$

in which the rows of [γ] are defined by unit vectors of the element's local x', y', and z' axes as defined in defined in the global reference coordinate system.

Using the symbolic toolbox in MATLAB, the above local element stiffness matrix (Fig. 3.10) representative of a neighboring joist member (e.g., top chord) was transformed to a coordinate system that is aligned with the web member of interest's coordinate system. Given that z axes (out-of-plane axes) for both of these elements are aligned (parallel), Eq. (3.12) reduces to

$$\left[\gamma\right]=\begin{bmatrix} \cos\Phi_i & \sin\Phi_i & 0 \\ -\sin\Phi_i & \cos\Phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.12}$$

in which Φ is measured from the compression web member's length axis counter-clockwise to the adjacent neighboring member being considered (Fig. 3.11).

The resulting transformed stiffness matrix contains the in-plane and out-of-plane stiffness contributions of the neighboring element. The in-plane stiffness contribution is defined by the coefficient located at the 6th row and 6th column of the transformed element stiffness matrix. The desired out-of-plane stiffness contribution is defined by the coefficient located at the 4th row and 6th column.

By summing these stiffness contributions for all n of the neighboring elements, the resisting stiffness (denominator of Eq. (3.1)) for in-plane buckling of the compression web member is

$$k_{neighbor\ members}=\sum_{i=1}^{n}\left(\frac{4E_iI_{zi}}{L_i}\right) \tag{3.13}$$

Likewise, the total resisting stiffness for out-of-plane buckling of the compression web member is

$$k_{neighbor\ members}=\sum_{i=1}^{n}\frac{G_iJ_i\sin^2\Phi_i+4E_iI_{yi}\cos^2\Phi_i}{L_i} \tag{3.14}$$

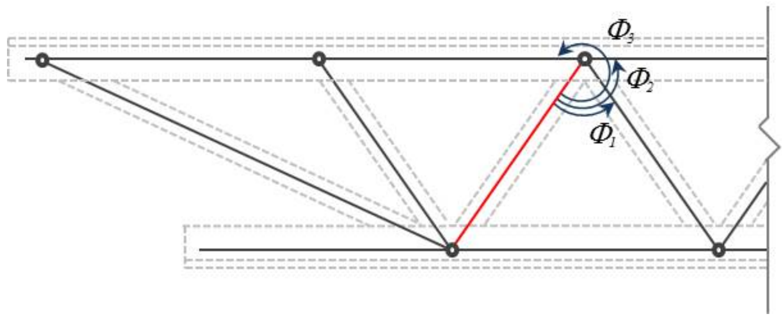


Figure 3.11. Definition of angles for computing stiffness contributions.

Using a first-order elastic analysis (MASTAN2) as described in the next section, the above stiffness values were checked for accuracy. The derived in-plane stiffness (Eq. (3.13)) was confirmed. Unfortunately, the out-of-plane stiffness computed by Eq. (3.14) is grossly over predicted when compared with "exact" MASTAN2 analysis. After an extensive investigation, it was determined that simply taking one coefficient from the resulting stiffness matrix (Eq. (3.9)) is only applicable, if the relative angle Φ is either 90, 180, or 270 degrees. For all other values of Φ, the link between flexure and twist is not accounted for properly. For example, when the neighboring member end rotates about one axis (e.g., global x axis), it also must be permitted to rotate about (twist) about the other axis (global y axis). In other words, these two degrees of freedom need to both be permitted to displace together, which is in contrast to the definition of a single coefficient of element stiffness matrix, which requires all other degrees of freedom be restrained.

To correct for this, the symbolic toolbox in MATLAB was used to complete a full structural analysis of a single element to determine the correct stiffness (e.g., moment resulting from a unit rotation). To do this, the direct stiffness method was used to analyze a single element at an arbitrary orientation. All degrees of freedom except for two were prescribed as zero (set equal to zero). Rotation about the global y-axis was defined as unrestrained (free) and rotation about the global x-axis was prescribed to be a value of one. The resulting reaction (moment) at the degree of freedom prescribed as unity is the desired and correct out-of-plane stiffness contributed by the element.

The sum of out-of-plane stiffness for all elements connected to the web member of interest is now determined as

$$k_{Top/Bottom} = \alpha - \frac{\beta}{\gamma} \tag{3.15}$$

$$\alpha = \sum_{i=1}^{n} \left[\frac{4E_i I_i \cos^2 \phi_i}{L_i} + \frac{G_i J_i \sin^2 \phi_i}{L_i} \right]$$

$$\beta = \sum_{i=1}^{n} \left[\frac{4E_i I_{yi} \cos \phi_i \sin \phi_i}{L_i} - \frac{G_i J_i \cos \phi_i \sin \phi_i}{L_i} \right]^2$$

$$\gamma = \sum_{i=1}^{n} \left[\frac{4E_i I_i \sin^2 \phi_i}{L_i} + \frac{G_i J_i \cos^2 \phi_i}{L_i} \right]$$

In terms of using the alignment charts to obtain effective length *K*-factors from stiffness *G* ratios, the above in-plane and out-of-plane stiffness equations are used within

$$G_{Top/\mathrm{Bottom}} = \frac{\left(\sum_{i=1}^{N} \frac{E_i I_i}{L_i} \right)_c}{2\left(\sum_{i=1}^{N} \frac{E_i I_i}{L_i} \right)_g} = \frac{\left(\frac{EI}{L} \right)_{web}}{\frac{k_{Top/Bottom}}{2}} \tag{3.16}$$

with the adjustment factor of 2 accounting for the far end fixed condition and with $k_{Top/Bottom}$ computed from Eq. (3.13) (in-plane buckling) and Eq. (3.15) (out-of-plane buckling).

3.4 Method for Confirming Stiffness Equations

As indicated earlier, the stiffnesses given by Eqs. (3.13) and (3.15) at both ends of the web member were confirmed with the structural analysis program MASTAN2. The process for doing so is provided below.

1. The joist is modeled at an incline, as shown in Fig. 3.12, so that the compressed web member of interest is vertical. This step is necessary because the MASTAN2 program is limited to only being able to prescribe displacements or rotations about one of the axes of the global coordinate system.

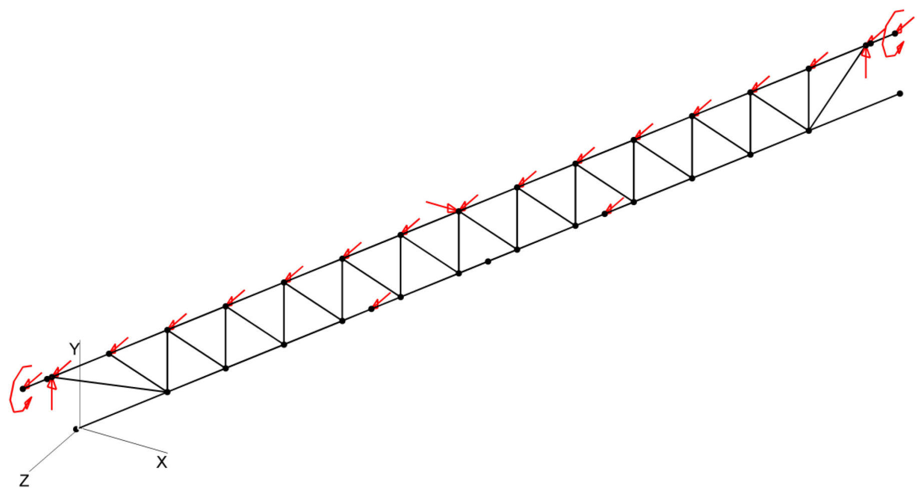


Figure 3.12: Inclined joist with web member of interest vertical; arrows indicate supports.

2. All members except for those connected to the top and bottom nodes of the web member of interest are removed (Fig. 3.13). The top portion of the model includes two elements representing the left and right parts of the top chord, and one element representing the connecting tension web member. Similarly, the bottom portion includes one element to represent the bottom chord, and two elements to model the diagonal tension members.

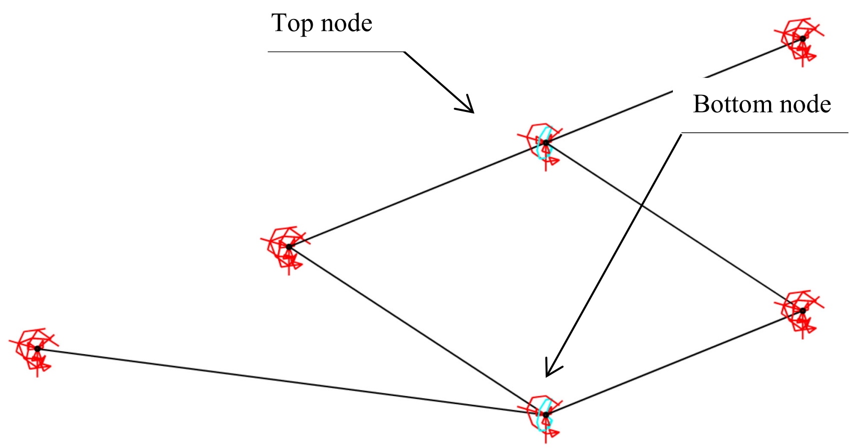


Figure 3.13. Sub-model for confirming the stiffness of the first compression web member in an 18K3 joist.

3. The support conditions are also shown in Fig. 3.13. All degrees of freedom except those described in the next sentences are set equal to zero.

    a. If the in-plane stiffnesses are desired, the rotational degrees of freedom about the out-of-plane axis (perpendicular to web member of interest) are then prescribed as unity. This is not shown in Fig. 3.13.
    b. If the out-of-plane stiffnesses are desired, the rotational degrees of freedom about the vertical axis at the nodes of where the web member's ends would exist are permitted to displace (i.e. are free degrees of freedom). The rotational degrees of freedom about the horizontal axis (perpendicular to web member of interest) are then prescribed as unity. This case corresponds to the support conditions shown in Fig. 3.13.

4. According to the definition of stiffness k (Eq. (3.17), which is analogous to Eq. (3.8)), the reactions (resulting moments) at the degrees of freedom prescribed to be unity, will equal the desired stiffnesses.

$$k = \frac{M}{\theta} \tag{3.17}$$

Chapter 4: Results of *K*-factor Studies

The main focus of this research is to determine accurate in-plane and out-of-plane effective length *K*-factors of compression web members. Four joist configurations (18K3, 28K10, 32LH06_L1, and 32LH06_L2) have been studied. These joists were selected because their member properties are known from previous experimental studies by Emerson (2001) and Schwarz (2002). Three general methods for computing *K*-factors are employed, including a hand calculation approach based on the alignment charts, a computation analysis method that uses a self-equilibrating induced compression (SEIC) loading procedure, and computation analysis employing a uniformly distributed load. In the latter two methods, effective length *K*-factors are back-calculated from the results of elastic critical load analyses. In these analyses, two scenarios are investigated: a) all compression web members buckling simultaneously and b) compression web members buckling individually. In this chapter, summaries of the calculated *K*-factors for each joist are provided along with descriptions of the joists, including geometric properties such as span length, depth, member size, and weight. Specific details for the 18K3 are provided in this chapter, and details related to the other joists studies are provided in the Appendices to this thesis. An overview of the cases presented is provided in Fig. 4.1.

In completing this study, the following assumptions have been made.

1) Section properties, including A, I_z, I_y, and J, are modeled.
2) Material is always assumed elastic with E = 29,000 ksi.
3) Top and bottom chords are continuous, and all web members are joined with fully restrained (rigid) connections.
4) No initial in-plane or out-of-plane imperfections are modeled.
5) Top chords of all joists are braced at all web member to top chord connection points.
6) Members are defined by centroid to centroid locations.

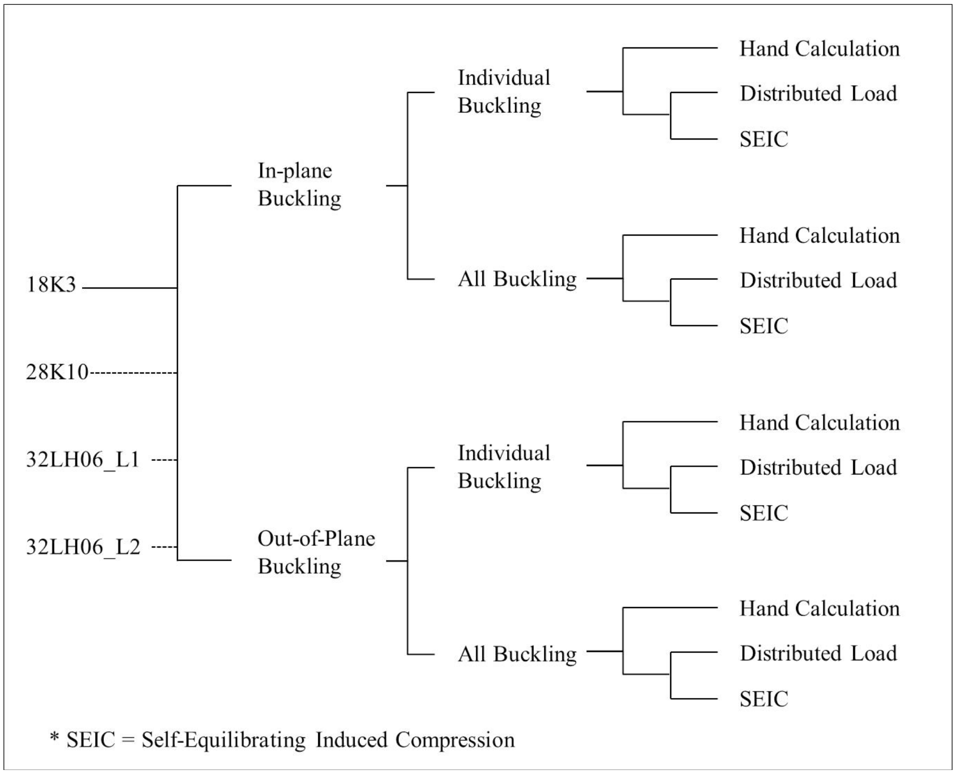
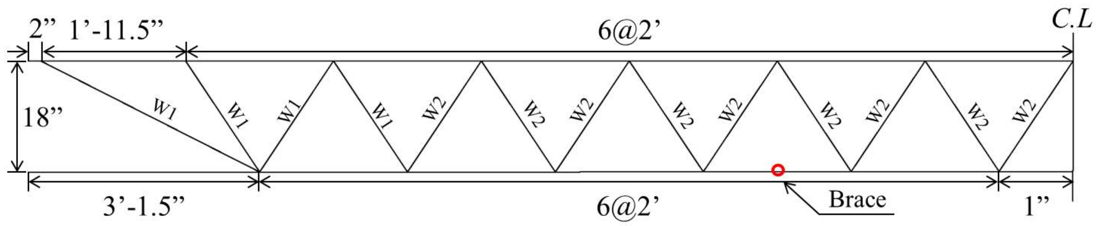


Figure 4.1. Cases and methods employed in study.

4.1 18K3 Joist

As shown in Fig. 4.2, an 18K3 joist consists of top and bottom chords, and diagonal tension and compression web members. In addition to the top chord being braced at the chord-web intersections, the bottom chord is braced at two symmetric locations over the span of the joist. The top and bottom chords are double-angles and all web members are round bars. The chords are made of two single-angle members—a leg of the first member is attached via the web members and spacers to the leg of the other single angle member, with a resulting 0.5” gap between the angles. In other words, the round diagonal web members are located in the gap and establish the gap between the two angles comprising the top and bottom chords. Specific member sizes are also provided in Fig. 4.2. Centroid to centroid dimensions are provided.



SPAN = 28’ – 3”

TOP CHORD (2L) = 1.5” x 1.5” x 0.123” with 0.5” GAP

BOTTOM CHORD (2L) = 1.25” x 1.25” x 0.109” with 0.5” GAP

W1 = 0.625” DIA. ROUND

W2 = 0.562” DIA. ROUND TOTAL WEIGHT = 165 lb

(The symbol 2L is used to indicate double angle members.)

Figure 4.2. Geometric configuration of 18K3 joist.

4.1.1 Comparison of *K*-factors for In-Plane Buckling

Based on all methods employed in this research, a summary of the in-plane effective length *K*-factors for the six compression web members in this 18K3 is provided in Table 4.1. Details specific to the results of each method are provided below. In general, the members that neighbor the compression web members (top or bottom chord, and tension web members) provide significant rotational restraint and thus result in *K*-factors for the compression web members that are very close 0.50 or a fixed-fixed end condition (Case *a*, Fig.2.3).

Individual web Buckling with given I_z	In-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web	6th Web
	I_z (in^4)		0.00749	0.00749	0.004897	0.004897	0.004897	0.004897
	K	Hand Calc.	0.52	0.51	0.51	0.51	0.51	0.51
		SEIC*	0.52	0.51	0.51	0.51	0.51	0.51
	Reduced I_z (in.^{4})		0.00675	0.0051	0.003967	0.002742	0.001714	0.000563
	K	Dist. Load	0.51	0.51	0.51	0.51	0.51	0.51
Simultaneous web buckling with reduced I_z	In-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web	6th Web
	I_z (in^4)		0.006225	0.0051	0.003967	0.00278	0.0017	0.000563
	K	Hand.Cal.	0.51	0.51	0.51	0.51	0.51	0.51
		Dist. Load	0.52	0.51	0.52	0.51	0.52	0.52
		SEIC*	0.51	0.51	0.51	0.50	0.50	0.50

* SEIC = Self-Equilibrating Induced Compression

Table 4.1. Results of in-plane *K*-factors (18K3).

4.1.1.1 Web members buckling individually

In the upper portion of Table 4.1, the analysis methods employ the compression web members actual section properties. As a result, compression web members would buckle independently. All methods are in near perfect agreement, and consistently provide an in-plane *K*-factor of 0.51 for the individual web member buckling.

4.1.1.1.1 Results of Hand Calculation Method

Table 4.2 provides the details for the hand calculation method that is based on the use of alignment charts with sidesway inhibited. The *K*-factors for the 3rd through 6th compression web members are equal (0.51) because all members in this vicinity possess the same relative stiffness properties (*EI*/*L*). The stiffness *G* ratios, which are values essential to using the alignment charts, are computed according to Section 3.3. The relatively small *G* values indicate that the bending stiffness of the compression web member is significantly less than that of the top or bottom chords. It is apparent from Table 4.1 that the hand calculation and computational methods are in close agreement. This shows promise that in-plane buckling *K*-factors may be computed for compression web members with only the properties of neighboring members and without the use of a computational analysis program. Numerical data for this stiffness is provided in Appendix A.

Web Member of interest	Location of *k*	*k (kip-in./rad.)*	*G*	*K*
1st Web Member	Top	1523.284	0.014	0.52
	Bottom	440.645	0.047	
2nd Web Member	Top	1508.957	0.014	0.51
	Bottom	795.868	0.026	
3rd Web Member	Top	1508.957	0.009	0.51
	Bottom	781.541	0.017	
4th Web Member	Top	1508.957	0.009	0. 51
	Bottom	781.541	0.017	
5th Web Member	Top	1508.957	0.009	0.51
	Bottom	781.541	0.017	
6th Web Member	Top	1508.957	0.009	0.51
	Bottom	781.541	0.017	

Table 4.2. Details for computing in-plane *K*-factors according to hand calculation method (18K3).

4.1.1.1.2 Results of Uniformly Distributed Loading Method

The details and results of K-factors computed from elastic critical load analyses of the joists with uniformly distributed loads are provided in Table 4.3. In this method, it is not possible to buckle a specific compression member without modifying (reducing) its moment of inertia. This is because the other members (e.g., top chord) may control the primary mode of buckling. To make the compression web member of interest buckle, this member's moment of inertia was artificially adjusted until it controlled the elastic critical load capacity of the joist. The computed buckling strengths of the controlling compression web members were used to back-calculate their K-factors. For in-plane buckling, K-factors of 0.51 were consistently obtained.

Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	20.9945	0.0065	16.09	0.51
2nd Web Member	20.9945	0.0051	12.81	0.51
3rd Web Member	20.9945	0.003967	9.998	0.51
4th Web Member	20.9945	0.002742	6.956	0.51
5th Web Member	20.9945	0.001714	4.355	0.51
6th Web Member	20.9945	0.0006	1.524	0.51

Table 4.3. Details for computing in-plane K-factors according to uniformly distributed load method (18K3).

4.1.1.1.3 Results of Self-Equilibrating Induced Compression Method

Specific details for the self-equilibrating induced compression (SEIC) method are provided in Table 4.4. This method employs self-straining to load only the compression web member of interest, and uses an eigenvalue analysis of the entire joist to compute the buckling load P_{cr} of the web member. The effective length K-factors are back-calculated from the member's buckling load P_{cr} according to Eq. (2.4). This data indicates in-plane K-factors from 0.51 up to 0.52, which is consistent with the other methods.

Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	20.9945	0.00749	18.22	0.52
2nd Web Member	20.9945	0.00749	18.59	0.51
3rd Web Member	20.9945	0.004897	12.35	0.51
4th Web Member	20.9945	0.004897	12.35	0.51
5th Web Member	20.9945	0.004897	12.35	0.51
6th Web Member	20.9945	0.004897	12.35	0.51

Table 4.4. Details for computing in-plane K-factors according to SEIC method (18K3).

4.1.1.2 All compressive web members buckling simultaneously

In the SEIC method, a single compression web member buckles with no force in the remaining members in the joist. Given that other web members may be in compression and as a result provide less restraining stiffness than when they are not loaded, the worst case scenario of all web members buckling simultaneously was investigated. To make this happen, all of the compression web members' moments of inertia were artificially adjusted until a uniformly distributed load on the joist caused this mode of failure (Fig. 4.3). Using the buckling forces in the compression web members and the adjusted moments of inertia, effective length *K*-factors were once again back-calculated. All three of these values are provided in the lower portion of Table 4.1, and in-plane effective length *K*-factors only slightly larger than the individual buckling values are obtained.

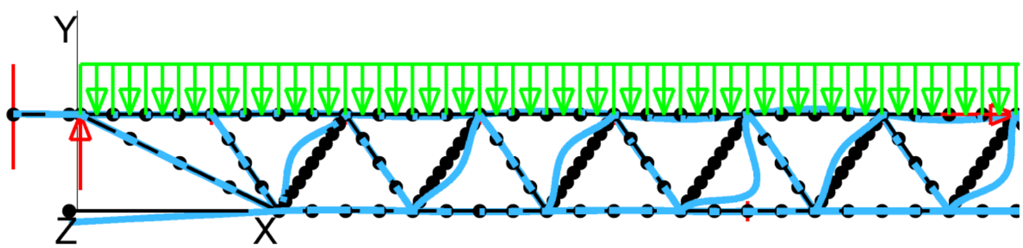


Figure 4.3. All compression web members buckling simultaneously under the uniformly distributed load condition (18K3).

4.1.1.2.1 Results of the Hand Calculation Method

Using the compression web members' adjusted moments of inertia, the hand method was once again used to compute their effective length *K*-factors. And again, these values are consistent with the results of computational analyses based on the uniformly distributed loading and SEIC loading. Numerical data corresponding to the hand method are provided in Table 4.5, and details are provided in Appendix A.

Web Member of interest	Location of k	k (kip-in/rad.)	G	K
1st Web Member	Top	1523.284	0.011	0.51
	Bottom	440.645	0.039	
2nd Web Member	Top	1508.957	0.009	0.51
	Bottom	795.868	0.018	
3rd Web Member	Top	1508.957	0.007	0.51
	Bottom	781.541	0.014	
4th Web Member	Top	1508.957	0.005	0.51
	Bottom	781.541	0.010	
5th Web Member	Top	1508.957	0.005	0.51
	Bottom	781.541	0.010	
6th Web Member	Top	1508.957	0.003	0.50
	Bottom	781.541	0.006	

Table 4.5. Effective length *K*-factors from the hand calculation method and considering simultaneous buckling web member properties (18K3).

4.1.1.2.2 Results of Uniformly Distributed Loading Method

In Table 4.6, *K*-factors resulting from the use of the uniformly distributed load method are provided. By adjusting the compression web members' moments of inertia, computational critical load analyses were employed to predict the compression web members' buckling strengths. Once again, *K*-factors of approximately 0.51 were back-calculated from the web member buckling strengths.

Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	20.9945	0.006225	15.25	0.51
2nd Web Member	20.9945	0.0051	12.25	0.51
3rd Web Member	20.9945	0.03967	9.584	0.51
4th Web Member	20.9945	0.00278	6.843	0.51
5th Web Member	20.9945	0.0017	4.105	0.51
6th Web Member	20.9945	0.000563	1.369	0.50

Table 4.6. Effective length *K*-factors from the uniformly distributed loading method and considering simultaneous buckling web member properties (18K3).

4.1.1.2.3 Results of Self-Equilibrating Induced Compression Method

Using the compression web members' adjusted moments of inertia, the SEIC method was also used to compute effective length *K*-factors. To do this, the SEIC loading was applied sequentially to each compression web member. Details for these computations are provided in Table 4.7. Comparing the *K*-factors from this method with the others, good agreement observed with *K*-factors ranging between 0.50 and 0.51.

Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	20.9945	0.006225	15.30	0.51
2nd Web Member	20.9945	0.0051	12.84	0.51
3rd Web Member	20.9945	0.003967	10.06	0.51
4th Web Member	20.9945	0.002742	7.006	0.50
5th Web Member	20.9945	0.001812	4.655	0.50
6th Web Member	20.9945	0.000563	1.458	0.50

Table 4.7. Effective length *K*-factors from the SEIC method and considering simultaneous buckling web member properties (18K3).

4.1.2 Comparison of *K*-factors for Out-of-Plane Buckling

Using an approach similar to that for in-plane buckling, a summary of the computed out-of-plane effective length *K*-factors is provided in Table 4.8. Because the out-of-plane resistance to buckling provided by the top and bottom chords is reduced from major axis bending to essentially the torsional resistance of the chords' double angles (which for open shapes is relatively small), the computed *K*-factors are larger (0.53 to 0.69) than their corresponding in-plane values. It is also noted that the effective length *K*-factors increase as the compression web member of interest is located further away from the bottom chord braces. This concept is illustrated in Fig. 4.4, which shows the case of uniformly distributed loading and all compression web members buckling simultaneously.

		Out-of-Plane	1st Web	2nd Web	3rd Web	4th Web	5th Web	6th Web
Individual web Buckling with given I_y		I_y (in^4)	0.00749	0.00749	0.004897	0.004897	0.004897	0.004897
	K	Hand Calc.	0.66	0.69	0.66	0.66	0.66	0.66
		SEIC*	0.69	0.69	0.66	0.65	0.65	0.65
		Reduced I_y (in.^{4})	0.00623	0.00465	0.00353	0.00235	0.00132	0.00039
	K	Dist. Load	0.63	0.61	0.60	0.58	0.56	0.53
Simultaneous web buckling with reduced I_y		Out-of-Plane	1st Web	2nd Web	3rd Web	4th Web	5th Web	6th Web
		I_y (in^4)	0.00667	0.00497	0.0036	0.00235	0.00125	0.00047
	K	Hand Calc.	0.69	0.64	0.63	0.59	0.55	0.52
		Dist. Load	0.67	0.65	0.62	0.60	0.56	0.60
		SEIC*	0.67	0.64	0.62	0.59	0.55	0.53

* SEIC = Self-Equilibrating Induced Compression

Table 4.8. Results of out-of-plane *K*-factors (18K3).

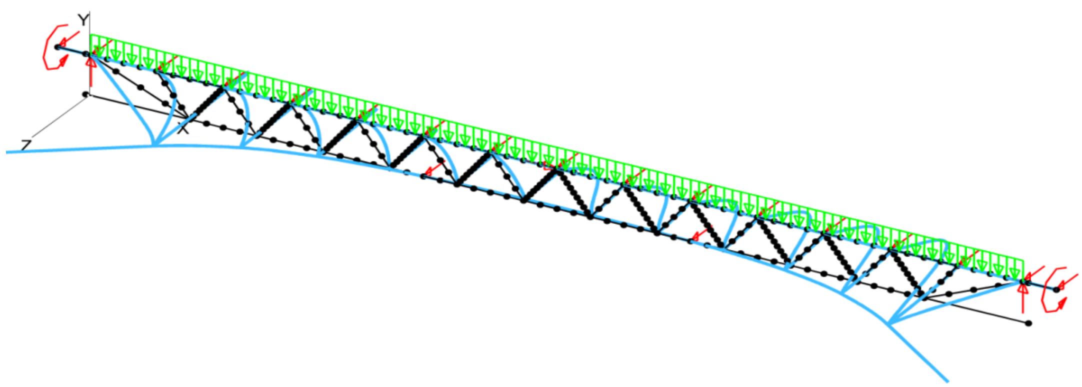


Figure 4.4. Deflected shape for simultaneous out-of-plane compression web buckling (18K3).

4.1.2.1 Web members buckling individually

The upper portion of Table 4.8 uses the original compression web members' moments of inertia (Hand Calc. and SEIC) and provides computed effective length *K*-factors for all three methods of analysis (hand, uniformly distributed loading, SEIC loading). In general, the results of these methods are in agreement.

4.1.2.1.1 Results of Hand Calculation Method

Details for the hand calculation method are provided in Table 4.9. Because the resisting stiffness *k* is reduced, the relative stiffness *G* ratios increase. And with this increase, the effective length *K*-factors also increase, and range from 0.66 to 0.69. Additional data for this method is provided in Appendix A.

Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	49.2824	0.420	0.66
	Bottom	55.6328	0.372	
2nd Web Member	Top	33.8860	0.611	0.69
	Bottom	47.9199	0.432	
3rd Web Member	Top	33.8860	0.399	0.66
	Bottom	32.7151	0.413	
4th Web Member	Top	33.8860	0.399	0.66
	Bottom	32.7151	0.413	
5th Web Member	Top	33.8860	0.399	0.66
	Bottom	32.7151	0.413	
6th Web Member	Top	33.8860	0.399	0.66
	Bottom	32.7151	0.413	

Table 4.9. Details for computing out-of-plane *K*-factors according to hand calculation method (18K3).

4.1.2.1.2 Results of Uniformly Distributed Loading Method

Results for the uniformly distributed load method are provided in Table 4.10. Similar to the in-plane buckling study, the compression web members' moments of inertia were adjusted so that they controlled the buckling capacity of the joist. Based on their buckling forces, *K*-factors ranging from 0.53 to 0.63 were back-calculated.

Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	20.9945	0.006225	10.13	0.63
2nd Web Member	20.9945	0.00465	8.169	0.61
3rd Web Member	20.9945	0.003526	6.402	0.60
4th Web Member	20.9945	0.002351	4.571	0.58
5th Web Member	20.9945	0.001322	2.743	0.56
6th Web Member	20.9945	0.000392	0.9144	0.53

Table 4.10. Details for computing out-of-plane *K*-factors according to uniformly distributed loading method (18K3).

4.1.2.1.3 Results of Self-Equilibrating Induced Compression Method

Table 4.11 provides the details in computing out-of-plane effective length *K*-factors according to the SEIC method. It is again apparent that the *K*-factor decreases when compression members are located near bottom brace points (it is noted that the top chord is fully braced at every web-chord intersection).

Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	20.9945	0.00749	10.36	0.69
2nd Web Member	20.9945	0.00749	10.33	0.69
3rd Web Member	20.9945	0.004897	7.40	0.66
4th Web Member	20.9945	0.004897	7.44	0.65
5th Web Member	20.9945	0.004897	7.56	0.65
6th Web Member	20.9945	0.004897	7.65	0.65

Table 4.11. Details for computing out-of-plane *K*-factors according to SEIC method (18K3).

To explore the impact of providing additional bottom chord bracing, the SEIC method was used in conjunction with the requirement that the out-of-plane displacement at the bottom of the compression member be restrained. As shown in Table 4.12, this modification surprisingly produces only a small decrease in the resulting *K*-factors.

Out-of-plane *K*-factors for individual web buckling							
Bottom Chord Panel Point	18K3 Joist	1st Web	2nd Web	3rd Web	4th Web	5th Web	6th Web
	Originally, unrestrained out-of-plane	0.69	0.69	0.66	0.65	0.65	0.65
	Restrained out-of-plane	0.66	0.67	0.64	0.64	0.64	0.65
	Difference (%)	3.43	2.29	1.83	1.93	0.87	0.04

Table 4.12. Impact of providing additional bottom chord bracing (18K3).

4.1.2.2 All compressive web members buckling simultaneously

Similar to the in-plane buckling study, all three of the methods for computing effective length *K*-factors were also employed for the case in which all compression web members buckle simultaneously. The results are provided in the lower portion of Table 4.8 and will be discussed in more detail below.

4.1.2.2.1 Results of the Hand Calculation Method

In comparing Tables 4.9 and 4.13, adjusting the compression members' moments of inertia to produce simultaneous buckling, produces only a small change in *K*-factors, with some values increased and others decreased. The reason for the decreasing *K*-factors may be attributed to the need to use smaller moments of inertia in the compression members so as to trigger simultaneous web buckling. As a result of these smaller values, the relative stiffness *G* values decrease, thereby decreasing the *K*-factors for some of the web members.

Web Member of interest	Location of *k*	*k (kip-in/rad.)*	*G*	*K*
1st Web Member	Top	49.2824	0.374	0.69
	Bottom	55.6328	0.311	
2nd Web Member	Top	33.8860	0.405	0.64
	Bottom	47.9199	0.286	
3rd Web Member	Top	33.8860	0.293	0.63
	Bottom	32.7151	0.304	
4th Web Member	Top	33.8860	0.192	0.59
	Bottom	32.7151	0.198	
5th Web Member	Top	33.8860	0.102	0.55
	Bottom	32.7151	0.106	
6th Web Member	Top	33.8860	0.038	0.52
	Bottom	32.7151	0.040	

Table 4.13. Out-of-plane effective length *K*-factors from the hand calculation method and considering simultaneous buckling web member properties (18K3).

4.1.2.2.2 Results of Uniformly Distributed Loading Method

The buckling forces and corresponding *K*-factors resulting from simultaneous buckling of all compression members are provided in Table 4.14. The *K*-factors tend to be smaller for web members closer to the center. Comparing Table 4.13 and Table 4.14, the *K*-factors show a good agreement with ranges between 0.56 and 0.67.

Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	20.9945	0.00667	9.573	0.67
2nd Web Member	20.9945	0.00497	7.691	0.65
3rd Web Member	20.9945	0.0036	6.016	0.62
4th Web Member	20.9945	0.00235	4.296	0.60
5th Web Member	20.9945	0.00125	2.578	0.56
6th Web Member	20.9945	0.00047	0.8594	0.60

Table 4.14. Out-of-plane effective length *K*-factors from the uniformly distributed loading method and considering simultaneous buckling web member properties (18K3).

4.1.2.2.3 Results of Self-Equilibrating Induced Compression Method

Effective length *K*-factors and the details used to calculate them according to the SEIC method and web member properties required for simultaneous buckling are provided in Table 4.15. Similar to the hand method and the results of employing a uniformly distributed load, there is only small impact in modifying the web members so that they all buckle simultaneously. Once again, the impact of bracing the bottom chord at the location of the compression web member being studied was explored. Similar to the previous study, the *K*-factors were reduced by only a small amount which varied from none to 3% (Table 4.16).

Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	20.9945	0.00667	9.615	0.67
2nd Web Member	20.9945	0.00497	7.897	0.64
3rd Web Member	20.9945	0.0036	6.021	0.62
4th Web Member	20.9945	0.00235	4.464	0.59
5th Web Member	20.9945	0.00125	2.732	0.55
6th Web Member	20.9945	0.00047	1.099	0.53

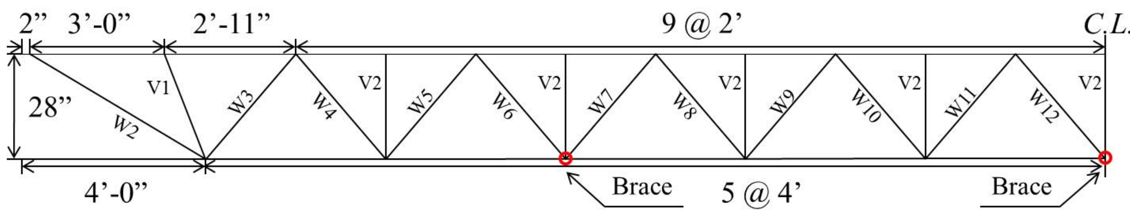
Table 4.15. Details for computing out-of-plane *K*-factors according to SEIC method and requiring simultaneous compression web buckling (18K3).

Out-of-plane *K*-factors For Simultaneous Buckling							
Bottom Chord Panel Point	18K3 Joist	1st Web Member	2nd Web Member	3rd Web Member	4th Web Member	5th Web Member	6th Web Member
	Originally, unrestrained out-of-plane	0.67	0.64	0.62	0.59	0.55	0.53
	Restrained out-of-plane	0.65	0.63	0.61	0.58	0.54	0.53
	Difference (%)	3.50	2.15	1.70	1.43	0.40	0.00

Table 4.16. Impact of providing additional bottom chord bracing and requiring simultaneous compression web buckling (18K3).

4.2 28K10 Joist

Detailed studies similar to the above were also performed on several other joist configurations. The description of the 28K10 joist investigated is provided in Fig. 4.5. The top and bottom chords are comprised of back-to-back double angles with a 1” gap between them. Instead of round bars for web members, crimped- and non-crimped angles are used for the diagonal web members. In addition to assuming the top chord is braced out-of-plane at the joist’s panel points, the bottom chord is also braced at three locations as shown in Fig. 4.5.



SPAN = 48’ – 3”
TOP CHORD (2L) = 2.0” x 2.0” x 0.216” with 1.0” GAP
BOTTOM CHORD (2L) = 2.0” x 2.0” x 0.163” with 1.0” GAP

Compression web members

W3 = 2.0” x 2.0” x 0.156” CRIMPED
W5 = 1.75” x 1.75” x 0.155” CRIMPED
W7 = 1.5” x 1.5” x 0.0.155” CRIMPED
W9 = 1.5” x 1.5” x 0.109” CRIMPED
W11 = 1.5” x 1.5” x 0.0.109” CRIMPED
WEIGHT = 573 lb

Tension members and vertical members

W2 = 0.9375” DIA. ROUND
W4 = 1.25” x 1.25” x 0.109” CRIMPED
W6 = 1.0” x 1.0” x 0.109” STRAIGHT
W8 = 1.0” x 1.0” x 0.0.109” STRAIGHT
W10 = 1.0” x 1.0” x 0.0.109” STRAIGHT
W12 = 1.0” x 1.0” x 0.0.109” STRAIGHT
V1 = 1.0” x 1.0” x 0.0.109” STRAIGHT
V2 = 1.0” x 1.0” x 0.0.109” STRAIGHT

Figure 4.5. Geometric configuration of 28K10 joist.

Crimping the ends of the web members (Fig. 4.6) allows for them to be aligned symmetrically with respect to the centerline of the joist, thereby eliminating the effects of eccentric loading in the web members. The 1” width at the crimped ends is used to define the gap between the top and bottom chords. Crimped angles are most often employed as the compression web members in steel joists. According to Yost et al. (2004), crimped web members are found to have smaller *K*-factors than uncrimped members.

The remaining vertical and tension members in the joists are uncrimped single angles with 1” legs to match the crimped width and assist with providing the top and bottom chord gaps.

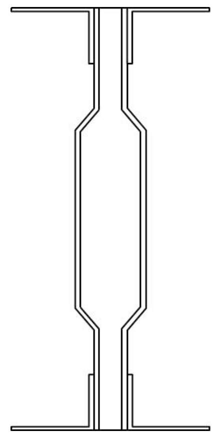


Figure 4.6. Crimped compression web member (Yost et al. 2004)

4.2.1 Comparison of in-plane *K*-factors for 28K10 Joist

Using the same methods employed for the 18K3, a summary of the in-plane effective length *K*-factors is provided in Table 4.17. The upper portion of the table presents results using the original moments of inertia for the compression web members and thus considers individual buckling capacities of each web member assuming all other members in the joist are not loaded. The lower portion of the table considers all compression web members buckling simultaneously, which is accomplished by varying the compression web members moments of inertia. All of the methods are in general agreement, and again indicate that the resisting stiffness of the chords and tension members is significantly greater than the stiffness of the compression web member of interest. These results indicate *K*-factors close to 0.5, which is representative of the fixed-fixed column end condition for sidesway inhibited.

			1st Web	2nd Web	3rd Web	4th Web	5th Web
Individual web Buckling with given I_z	In-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_z (in^4)		0.1294	0.08171	0.04766	0.03561	0.03561
	K	Hand Calc.	0.55	0.52	0.51	0.51	0.51
		SEIC*	0.55	0.52	0.51	0.51	0.51
	Reduced I_z (in.^{4})		0.08411	0.06373	0.04528	0.02849	0.01175
	K	Dist. Load	0.53	0.52	0.51	0.51	0.51
Simultaneous web buckling with reduced I_z	In-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_z (in^4)		0.079534	0.060465	0.045277	0.028844	0.01179
	K	Hand Calc.	0.53	0.52	0.51	0.51	0.50
		Dist. Load	0.53	0.52	0.52	0.52	0.51
		SEIC*	0.53	0.52	0.51	0.51	0.50

* SEIC = Self-Equilibrating Induced Compression

Table 4.17. Results of in-plane K-factors (28K10).

4.2.2 Comparison of out-of-plane K-factors for 28K10 Joist

A summary of the out-of-plane buckling results for the 28K10 joist is provided in Table 4.18. As expected, the resisting stiffness of the top and bottom chords is reduced and the K-factors increase to predicted values between 0.55 and 0.72. In all cases, the SEIC method provides the most conservative values (0.66 to 0.80 and 0.72 to 0.83) and the uniformly distributed loading provides the more liberal results (0.55 to 0.72 and 0.55 to 0.77). The difference, however, is not significant. Just as importantly, the results of the hand calculation method lie between the two computational results.

			1st Web	2nd Web	3rd Web	4th Web	5th Web
Individual web Buckling with given I_y	Out-of-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_y (in^4)		0.1003	0.08156	0.06341	0.04974	0.04974
	K	Hand Calc.	0.80	0.74	0.68	0.66	0.66
		SEIC*	0.83	0.78	0.75	0.72	0.72
	Reduced I_y (in.^{4})		0.0703	0.046	0.0315	0.016	0.00625
	K	Dist. Load	0.72	0.66	0.64	0.59	0.55
Simultaneous web buckling with reduced I_y	Out-of-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_y (in^4)		0.072	0.042	0.030	0.016	0.0058
	K	Hand Calc.	0.73	0.66	0.64	0.58	0.53
		Dist. Load	0.77	0.66	0.66	0.60	0.55
		SEIC*	0.79	0.70	0.67	0.60	0.54

* SEIC = Self-Equilibrating Induced Compression

Table 4.18. Results of out-of-plane K-factors (28K10).

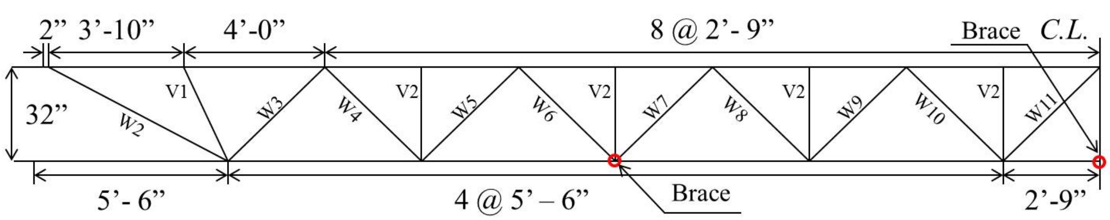
As shown in Table 4.19, restraining the bottom chord at the location of the web member interest continues to not significantly impact the results—a similar pattern to the 18K3 compression web members.

Out-of-plane *K*-factors For Individual Buckling						
Bottom Chord Panel Point	28K10 Joist	1st Web	2nd Web	3rd Web	4th Web	5th Web
	Originally unrestrained out-of-plane	0.83	0.78	0.75	0.72	0.72
	Restrained out-of-plane	0.80	0.76	0.75	0.72	0.72
	Difference (%)	3.29	1.91	0.00	0.00	0.00

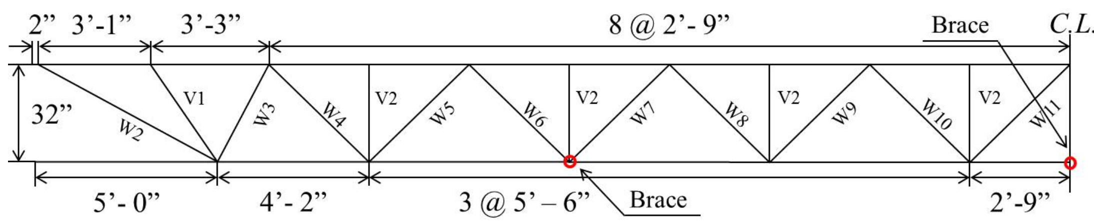
Table 4.19. Impact of providing additional bottom chord bracing (28K10).

4.3 32LH06 Joists

In addition to short-span joists, two variations of a long span LH-series joist were investigated. As shown in Fig. 4.7, both variations are employ the same member sizes, but differ in the steepness and length of the first compression member (W3). Similar to the previously described 28K10, these joists employ a mix of crimped compression and un-crimped tension and vertical web members. The length-to-height ratio of the 32LH06's investigated is about 10% larger than that of the 28K10. In both variations, the bottom chord is braced in three locations as shown in Fig. 4.7.



(a) Variation 32LH06_L1



(b) Variation 32LH06_L2

SPAN = 60’ – 0”
TOP CHORD (2L) = 2.5” x 2.5” x 0.212” with 1.0” GAP
BOTTOM CHORD (2L) = 2.0” x 2.0” x 0.216” with 1.0” GAP

Compression Web Members	Tension and vertical members
W3 = 2.0” x 2.0” x 0.187” CRIMPED	W2 = 1.5” x 1.5” x 0.109” DOUBLE ANGLE
W5 = 1.75” x 1.75” x 0.155” CRIMPED	W4 = 1.25” x 1.25” x 0.109” CRIMPED
W7 = 1.5” x 1.5” x 0.0.155” CRIMPED	W6 = 1.0” x 1.0” x 0.109” STRAIGHT
W9 = 1.5” x 1.5” x 0.155” CRIMPED	W8 = 1.0” x 1.0” x 0.0.109” STRAIGHT
W11 = 1.5” x 1.5” x 0.0.155” CRIMPED	W10 = 1.0” x 1.0” x 0.0.109” STRAIGHT
WEIGHT = 840 lb	V1 =1.0” x 1.0” x 0.0.109” STRAIGHT
	V2 = 1.0” x 1.0” x 0.0.109” STRAIGHT

Figure 4.7. Geometric configurations of 32LH06 joists.

4.3.1 Comparison of in-plane *K*-factors for 32LH06 Joists

Tables 4.20 and 4.21 provide a comparison of the in-plane effective length *K*-factors for the two variations of 32LH06 joist investigated. Consistent with the previous studies, values close to 0.5 are obtained. The two variations appear to provide nearly identical values, with only slight differences due to the steepness of the first web member.

Individual web Buckling with given I_z	In-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_z (in^4)		0.1512	0.07844	0.04544	0.04544	0.04544
	K	Hand Calc.	0.54	0.51	0.51	0.51	0.51
		SEIC*	0.54	0.52	0.51	0.51	0.51
	Reduced I_z (in.^{4})		0.08769	0.06589	0.05317	0.02408	0.00504
	K	Dist. Load	0.52	0.51	0.51	0.50	0.50
Simultaneous web buckling with reduced I_z	In-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_z (in^4)		0.08650	0.06389	0.043642	0.024013	0.005004
	K	Hand Calc.	0.53	0.51	0.51	0.50	0.50
		Dist. Load	0.52	0.51	0.51	0.50	0.50
		SEIC*	0.53	0.51	0.51	0.51	0.50

* SEIC = Self-Equilibrating Induced Compression

Table 4.20. Results of *K*-factor in-plane buckling (32LH06_L1)

Individual web Buckling with given I_z	In-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_z (in^4)		0.1512	0.07844	0.04544	0.04544	0.04544
	K	Hand. Calc.	0.54	0.51	0.51	0.51	0.51
		SEIC*	0.55	0.52	0.51	0.51	0.51
	Reduced I_z (in.^{4})		0.04082	0.06275	0.04364	0.02272	0.004998
	K	Dist. Load	0.51	0.51	0.51	0.50	0.50
Simultaneous web buckling with reduced I_z	In-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_z (in^4)		0.03995	0.06389	0.043642	0.024013	0.004977
	K	Hand. Cal.	0.51	0.51	0.51	0.50	0.50
		Uniform Load	0.51	0.51	0.51	0.50	0.50
		SEIC*	0.51	0.51	0.51	0.51	0.50

* SEIC = Self-Equilibrating Induced Compression

Table 4.21. Results of *K*-factors for in-plane buckling (32LH06_L2)

4.3.2 Comparison of out-of-plane *K*-factors for 32LH06 Joists

The out-of-plane *K*-factors for the two variations are provided in Table 4.22 and 4.23. As with the previous studies, three methods of analysis (uniformly loaded, hand method, and SEIC) were completed for each of the two cases, including members buckling individually (upper portions of the tables) and all web members buckling simultaneously (lower portion of the tables). In reviewing Tables 4.22 and 4.23, it is clear that these are the largest effective length *K*-factors (0.61 to 0.87) obtained in this research project. This data also show the largest variation between results obtained using uniformly distributed loading and the SEIC loading, with the latter being a conservative approximation of the former. In general, the hand method continues to provide results that are consistent with the distributed loading critical load analyses. It does not appear that the steepness and/or length of the first compression member has a dramatic impact on the computed effective length *K*-factors.

Individual web Buckling with given I_y	Out-of-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_y (in^4)		0.1107	0.08917	0.0672	0.0672	0.0672
	K	Hand Calc.	0.71	0.73	0.71	0.71	0.66
		SEIC*	0.81	0.77	0.74	0.74	0.69
	Reduced I_y (in.^{4})		0.09963	0.07134	0.04368	0.02083	0.005712
	K	Dist. Load	0.70	0.70	0.65	0.61	0.70
Simultaneous web buckling with reduced I_y	Out-of-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_y (in^4)		0.10516	0.075795	0.047041	0.0225	0.0059
	K	Hand Calc.	0.71	0.72	0.67	0.60	0.52
		Dist. Load	0.74	0.72	0.68	0.63	0.70
		SEIC*	0.80	0.75	0.70	0.62	0.55

* SEIC = Self-Equilibrating Induced Compression

Table 4.22. Results of *K*-factors for out-of-plane buckling (32LH06_L1).

Individual web Buckling with given I_y	Out-of-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_y (in^4)		0.1107	0.08917	0.0672	0.0672	0.0672
	K	Hand Calc.	0.78	0.73	0.71	0.71	0.66
		SEIC*	0.87	0.76	0.73	0.73	0.69
	Reduced I_y (in.^{4})		0.04871	0.07134	0.0504	0.02218	0.00612
	K	Dist. Load	0.71	0.69	0.69	0.61	0.70
Simultaneous web buckling with reduced I_y	Out-of-Plane		1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_y (in^4)		0.048708	0.075	0.0504	0.0215	0.006
	K	Hand Calc.	0.69	0.71	0.67	0.60	0.53
		Dist. Load	0.72	0.71	0.70	0.61	0.71
		SEIC*	0.78	0.75	0.70	0.61	0.55

* SEIC = Self-Equilibrating Induced Compression

Table 4.23. Results of *K*-factors for out-of-plane buckling (32LH06_L2).

As shown in Tables 4.24 and 4.25, bracing the bottom chord at the panel point located at the base of the first compression web member does have a more significant effect than in the previous studies. The reduction in the effective length *K*-factor, however, is still only about 5% to 8%.

Out-of-plane *K*-factors For Individual Buckling						
Bottom Chord Panel Point	32LH06_L1 Joist	1st Web Member	2nd Web Member	3rd Web Member	4th Web Member	5th Web Member
	Originally, unrestrained out-of-plane	0.81	0.77	0.74	0.74	0.69
	Restrained out-of-plane	0.75	0.76	0.74	0.74	0.69
	Difference (%)	7.93	1.40	0.39	0.12	0.05

Table 4.24. Impact of providing additional bottom chord bracing (32LH06_L1).

Out-of-plane *K*-factors For Individual Buckling						
Bottom Chord Panel Point	32LH06_L2 Joist	1st Web Member	2nd Web Member	3rd Web Member	4th Web Member	5th Web Member
	Originally, unrestrained out-of-plane	0.87	0.76	0.73	0.73	0.69
	Restrained out of plane	0.82	0.75	0.74	0.73	0.68
	Difference (%)	5.42	1.52	1.26	0.00	1.12

Table 4.25. Impact of providing additional bottom chord bracing (32LH06_L2).

4.4 Summary of 1st Web Member Data for All Joists Investigated

In general, the effective length *K*-factor used in designing the first compression web member is often the largest when compared to the remaining compression web members. Tables 4.26 and 4.27 provide a summary of the in-plane and out-of-plane *K*-factors obtained by the three different methods of analysis employed in this study.

In regard to in-plane buckling (Table 4.26), this research suggests the use of an effective length of approximately $K = 0.55$. Given that the ends of the compression web members have been assumed as fully restrained, which may not be consistent with field conditions, the true value is most likely significantly larger than 0.55, but also much less than K = 1.0 which conservatively assumes pinned-pinned connections.

In regard to out-of-plane buckling (Table 4.27), the computed effective length K-factors are much larger and in the range of 0.65 to 0.90. Although the analyses also assumed fully restrained connections in these cases, this assumption is probably fairly accurate given that the ends of the web members are welded (sandwiched) between the angle comprising the top and bottom chords. It is also important to note that the SEIC did provide the largest effective length K-factor (0.87), where the other methods resulted in values more consistently in the area of K = 0.80. It should also be noted that Yost, et al. (2004) suggested this value as a result of their experimental testing.

In-plane buckling K-factors for 1st web member							
web members buckling individually				all web members buckling simultaneously			
Joist	Hand Calc.	Dist. Load	SEIC*	Joist	Hand Calc.	Dist. Load	SEIC*
18K3	0.52	0.51	0.52	18K3	0.51	0.52	0.51
28K3	0.55	0.53	0.55	28K3	0.53	0.53	0.53
32LH06_L1	0.54	0.52	0.54	32LH06_L1	0.53	0.52	0.53
32LH06_L2	0.54	0.51	0.55	32LH06_L2	0.51	0.51	0.51

* SEIC : Self-Equilibrating Induced-Compression

Table 4.26. Summary of in-plane buckling K-factors for 1st web member.

Out-of-plane buckling K-factors for 1st web member							
web members buckling individually				all web members buckling simultaneously			
Joist	Hand Cal.	Dist. Load	SEIC*	Joist	Hand Cal.	Dist. Load	SEIC*
18K3	0.66	0.63	0.69	18K3	0.69	0.67	0.67
28K3	0.80	0.72	0.83	28K3	0.73	0.77	0.79
32LH06_L1	0.71	0.71	0.81	32LH06_L1	0.71	0.74	0.80
32LH06_L2	0.78	0.71	0.87	32LH06_L2	0.69	0.73	0.78

* SEIC : Self-Equilibrating Induced-Compression

Table 4.27. Summary of in-plane buckling K-factors for 1st web member.

Chapter 5: Summary and Conclusions

5.1 Summary

This study focuses on determining effective length K-factors in compression web members of open web steel joists. Current design provisions of the Steel Joist Institute (SJI) indicate that in many cases a value of $K = 1.0$ should be used, but they are aware that a limited number of recent experimental studies suggest that smaller values could be used. Given that the effective length K-factor is inversely proportional to the design strength of theses members, the potential for reducing these values could lead to larger capacities and perhaps for the design of more efficient systems that are still safe and reliable.

The research presented in this thesis employs three different methods for computing K-factors, including a hand calculation method based on the alignment charts, and two methods based on computational finite element procedure to perform critical load (eigenvalue) analyses. The latter two methods differ in the means by which the joists are loaded; one uses uniformly distributed loading and the other uses a novel approach called the self-equilibrating induced compression (SEIC) method. All three of these methods are described in detail within this thesis. The key to the hand calculation method is the accurate computation of the resisting stiffness of the joist members neighboring the ends of the compression web member of interest. The SEIC method is also well described in this thesis and essentially loads the compression web member of interest to the point of buckling through the use of an additional element that is parallel and connected to this web member. By subjecting this additional element to a thermal cooling, the actual web member of interest is compressed without loading any other members in the joist. By using this approach, the buckling behavior of the web member of interest is isolated and controlled allowing for the direct calculation of its effective length K-factor. The remaining computational method employed applies a uniformly distributed load to the joist and calculates the compressive force required to buckle the web member of interest. The effective length K-factor is then back-calculated from this force. This method is also used to determine the forces for two cases, one that only considers the possibility of a single member buckling, and the other accounts for all compression web members in the joist buckling simultaneously.

Unfortunately, the time required to develop and fine-tune the above methods for computing *K*-factors significantly limited the number of joist configurations that could be studied to four, including 18K3 and 28K10 short span joists, and two variations of a long span 32LH06 joist. These joists include a mix of compression web members fabricated from round bar or crimped angles. Using these joists and the above methods for calculating effective length *K*-factors, both in-plane and out-of-plane buckling of the compression web members were investigated. All results are provided in tabular form, with specific details provided in the appendices to this thesis.

5.2 Conclusions

Based on the methods and joists studied in this research the following conclusions are made. It should be emphasized that only a limited number of joists were investigated in this work, and specific results and recommended values will most likely change to some degree when a comprehensive study of many more SJI joists configurations is undertaken.

1) All three methods of analysis provided similar results. Given that the hand calculation method developed in this research is in close agreement with the computational results, it appears that this method could be used to compute effective length *K*-factors for compression web members without the need to perform detailed finite element analyses.
2) In-plane effective length *K*-factors for the compression web members were consistently calculated in the range of 0.51 to 0.60. Given that such values closely resemble those of a compression member with its ends restrained from rotation, it is evident that the in-plane flexural stiffness of the web members is significantly less than the resisting flexural stiffness offered by its neighboring top and bottom chords and tension web members.
3) In regard to out-of-plane buckling, it appears that again *K*-factors much less than unity prevail. For such a condition, this study produced values that ranged between K = 0.6 to 0.9, with most

values between 0.65 and 0.75. The increase in effective length K-factors (compared to the in-plane values) can be attributed to a reduction in the resisting stiffness provided by the top and bottom chords. This reduction is the result of relying on the relatively small torsional resistance instead of the more significant flexural resistance of these chord members.

4) For the out-of-plane buckling studies, this research also investigated the impact of providing additional bottom chord bracing. Somewhat surprisingly, this modification seems to only have a small influence of 1% to 7% on K-factors computed for the compression web members.

5) In general, the largest K-factors were computed for the compression web member located closest to the ends of the joists, with the first compression member nearly always providing the largest values.

6) The possibility of all compression web members buckling simultaneously versus a single web member buckling individually seems to not impact the values of the computed in-plane and out-of-plane K-factors.

7) Based on the very few joists investigated, it appears promising that an in-plane value of approximately $K = 0.75$ and an out-of-plane value of $K = 0.85$ could be used in computing the flexural compressive strength of compression web members. These values are very conservative when compared to the values computed by all the methods employed in this research.

8) The reason for the author's conservatism stems from the assumption that all of the web member to chord member connections have been modeled as fully restrained (rigid). In actual field conditions, these connections will most likely permit some degree of relative rotation between these members, which would result in a loss of stiffness provided by the resisting members (chords and tension members). This loss in connection restraint is probably less severe for the out-of-plane case because the ends of the web members are assembled within (sandwiched) between the double angles comprising the top and bottom chords. At the other extreme, these in-plane and out-of-plane connections are clearly not pinned, which would result in the very conservative assumption of $K = 1.0$. To counteract the inclination that lack of connection restraint should increase the resulting K-factors, it is noted that all of the analyses performed in this

research assumed elastic behavior. It is expected that the compression web members may yield to some extent before the joist achieves a strength limit state. Yielding in the compression web member could decrease the relative stiffness between this member and it neighbors (top and bottom chords), thereby decreasing its effective length *K*-factor. In fact, this latter concept is the basis for the stiffness reduction factors often employed with the use of the alignment charts for computing effective length factors in building columns.

9) The research and additional future studies related to this topic could have several implications related to the design of open web steel joists. For cases in which the top or bottom chord controls the capacity of the joist, a reduction in the compression web members *K*-factors (thereby increasing their flexural buckling strengths) could permit the use of smaller section sizes, thereby resulting in a more efficient design. It should also be noted that using smaller compression web members will most likely have a minimal impact on the overall bending stiffness of the joist, and thus, service live load deflection requirements should continue to be met.

5.3 Recommendations for Future Research

As with any analytical study, several assumptions were made in this research that should be carefully reviewed for their implications. These assumptions are provided and discussed throughout the chapters of this thesis and are the basis for the below recommendations for future work.

1) The most important recommendation for future work is the need to investigate many more than the four joist configurations that were studied in detail as part of this research. Such a study should focus on a selection of joists for which there is large variation in the relative stiffness of the web members to the neighboring chord and tension members, which provide the stiffness that resists web member buckling.
2) The impact of assuming all connections to be fully restrained needs to be carefully assessed. This could be accomplished by using a few standard joist configurations such as the ones used in this study and then varying the web to chord connection stiffness from pinned to fully restrained.

3) Given that the web members in joists may be relatively short and stocky, they will most likely experience inelastic buckling. Through the use of a series of inelastic critical load analyses, the influence of yielding on the prediction of *K*-factors for web members should be assessed.

4) In all cases investigated in this study the bottom chord was braced out-of-plane at two or more panel points. This study did find that additional bracing of this type did not significantly change the effective length *K*-factors for the compression web members. The question remains, however, on what the computed *K*-factors would be if the bottom chord had no bracing. A systematic study investigating this topic could be useful.

Appendix A: Data for 18K3

This appendix includes detailed data related to the determination of *K*-factors from the three methods employed in this study.

***K*-factors by Hand Calculation Method, 18K3, In-Plane Buckling, Individually**				
Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	1523.284	0.014	0.52
	Bottom	440.6446	0.047	
2nd Web Member	Top	1508.957	0.014	0.51
	Bottom	795.8675	0.026	
3rd Web Member	Top	1508.957	0.009	0.51
	Bottom	781.5405	0.017	
4th Web Member	Top	1508.957	0.009	0.51
	Bottom	781.5405	0.017	
5th Web Member	Top	1508.957	0.009	0.51
	Bottom	781.5405	0.017	
6th Web Member	Top	1508.957	0.009	0.51
	Bottom	781.5405	0.017	

1st Compression Web Member, In-Plane Buckling, Individually (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.00749	0.00749	0.01498	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	41.3842				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1523.284	10.346	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498
Elem. #2	39.4591	29000	11153.85	0.00749	0.00749	0.01498
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	41.3842				
Elem. #2	4	22.0187		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		440.645	10.346	0.05

2nd Compression Web Member, In-Plane Buckling, Individually (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.00749	0.00749	0.01498	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	10.346	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	41.3842				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		795.868	10.346	0.03

3rd Compression Web Member, In-Plane Buckling, Individually (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.004897	0.004897	0.009794	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	6.764	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	*Bending*		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		781.541	6.764	0.02

4th Compression Web Member, In-Plane Buckling, Individually (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.004897	0.004897	0.009794	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	6.764	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		781.541	6.764	0.02

5th Compression Web Member, In-Plane Buckling, Individually (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.004897	0.004897	0.009794	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	6.764	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		781.541	6.764	0.02

6th Compression Web Member, In-Plane Buckling, Individually (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.004897	0.004897	0.009794	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	6.764	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		781.541	6.764	0.02

K-factors by Hand Calculation Method, 18K3, In-Plane Buckling, Simultaneously				
Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	1523.284	0.011	0.51
	Bottom	440.6446	0.039	
2nd Web Member	Top	1508.957	0.009	0.51
	Bottom	795.8675	0.018	
3rd Web Member	Top	1508.957	0.007	0.51
	Bottom	781.5405	0.014	
4th Web Member	Top	1508.957	0.005	0.51
	Bottom	781.5405	0.010	
5th Web Member	Top	1508.957	0.005	0.51
	Bottom	781.5405	0.010	
6th Web Member	Top	1508.957	0.003	0.50
	Bottom	781.5405	0.006	

1st Compression Web Member, In-Plane Buckling, Simultaneously (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.006225	0.00749	0.01498	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	41.3842				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1523.284	8.599	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498
Elem. #2	39.4591	29000	11153.85	0.00749	0.00749	0.01498
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	41.3842				
Elem. #2	4	22.0187		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		440.645	8.599	0.04

2nd Compression Web Member, In-Plane Buckling, Simultaneously (18K3)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
20.9945	29000	11153.85	0.0051	0.00749	0.01498	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	7.045	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	41.3842				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		795.868	7.045	0.02

3rd Compression Web Member, In-Plane Buckling, Simultaneously (18K3)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
20.9945	29000	11153.85	0.0039666	0.004897	0.009794	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	5.479	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		781.541	5.479	0.01

4th Compression Web Member, In-Plane Buckling, Simultaneously (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.0027423	0.004897	0.009794	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	3.788	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		781.541	3.788	0.01

5th Compression Web Member, In-Plane Buckling, Simultaneously (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.0018119	0.004897	0.009794	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	2.503	0.00
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		781.541	2.503	0.01

6^{th} Compression Web Member, In-Plane Buckling, Simultaneously (18K3)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
20.9945	29000	11153.85	0.0005632	0.004897	0.009794	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	740.9500		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	740.9500		1508.957	0.778	0.001
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	27.0572				
Elem. #2	4	377.2417		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	377.2417		781.541	0.778	0.002

K-factors by Hand Calculation Method, 18K3, Out-of-Plane Buckling, Individually				
Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	49.28244	0.42	0.66
	Bottom	55.63276	0.37	
2nd Web Member	Top	33.88596	0.61	0.70
	Bottom	47.91987	0.43	
3rd Web Member	Top	33.88596	0.40	0.66
	Bottom	32.71513	0.41	
4th Web Member	Top	33.88596	0.40	0.66
	Bottom	32.71513	0.41	
5th Web Member	Top	33.88596	0.40	0.66
	Bottom	32.71513	0.41	
6th Web Member	Top	33.88596	0.40	0.66
	Bottom	32.71513	0.41	

1st Compression Web Member, Out-of-Plane Buckling, Individually (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9945	29000	11153.85	0.00749	0.00749	0.01498		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498	69.7206
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	7.9585	41.38
Elem. #2	-0.572	0.821		Elem. #2	4	1.5239	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.5239	2278.92

Stiffness of Top Node [Unit: kip-in./rad.]						
α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}
1503.07	4517147.73	3107.15		49.282	10.346	0.42

Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498	69.7207
Elem. #2	39.4591	29000	11153.85	0.00749	0.00749	0.01498	98.9745
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8504
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	7.9585	41.38
Elem. #2	-0.156	0.988		Elem. #2	4	4.2344	22.02
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43

Stiffness of Bottom Node [Unit: kip-in./rad.]						
α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
439.68	356982.30	929.53		55.633	10.346	0.37

2^{nd} Compression Web Member, Out-of-Plane Buckling, Individually (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9967	29000	11153.85	0.00749	0.00749	0.01498		

Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.72
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.86
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.86
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.5239	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.5239	2278.92

Stiffness of Top Node [Unit: kip-in./rad.]						
α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}
1498.93	4533153.74	3094.21		33.886	10.345	0.61

Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498	69.72
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.86
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.86
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	7.9585	41.38
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43

Stiffness of Bottom Node [Unit: kip-in./rad.]						
α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
858.05	1440831.38	1778.53		47.920	10.345	0.43

3^{rd} Compression Web Member, Out-of-Plane Buckling, Individually (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9966	29000	11153.85	0.004897	0.004897	0.009794		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.5239	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.5239	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
1498.93	4533153.7	3094.21			33.886	6.764	0.40
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.8603
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
853.90	1449877.3	1765.59			32.715	6.764	0.41

4th Compression Web Member, Out-of-Plane Buckling, Individually (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9959	29000	11153.85	0.004897	0.004897	0.009794		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.20	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.52	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.52	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
1498.93	4533153.98	3094.21			33.886	6.764	0.40
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.8603
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
853.90	1449877.73	1765.59			32.715	6.764	0.41

5th Compression Web Member, Out-of-Plane Buckling, Individually (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9949	29000	11153.85	0.004897	0.004897	0.009794		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.20	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.52	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.52	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
1498.93	4533153.74	3094.21			33.886	6.764	0.40
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.8603
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
853.90	1449877.27	1765.59			32.715	6.764	0.41

6th Compression Web Member, Out-of-Plane Buckling, Individually (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9945	29000	11153.85	0.004897	0.004897	0.009794		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.20	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.52	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.52	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}	
1498.93	4533153.74	3094.21		33.886	6.764	0.40	
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.8603
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}	
853.90	1449877.27	1765.59		32.715	6.764	0.41	

***K*-factors by Hand Calculation Method, 18K3, Out-of-Plane Buckling, Simultaneously**				
Web Member of interest	Location of *k*	*k* (kip-in./rad.)	*G*	*K*
1st Web Member	Top	49.28244	0.374	0.69
	Bottom	55.63276	0.331	
2nd Web Member	Top	33.88596	0.405	0.64
	Bottom	47.91987	0.286	
3rd Web Member	Top	33.88596	0.293	0.63
	Bottom	32.71513	0.304	
4th Web Member	Top	33.88596	0.192	0.59
	Bottom	32.71513	0.198	
5th Web Member	Top	33.88596	0.102	0.55
	Bottom	32.71513	0.106	
6th Web Member	Top	33.88596	0.038	0.52
	Bottom	32.71513	0.040	

1st Compression Web Member, Out-of-Plane Buckling, Simultaneously (18K3)							
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})		
20.9945	29000	11153.85	0.00749	0.00667	0.01498		
Properties of Top Members							
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498	69.7206
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			*n*	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	7.96	41.38
Elem. #2	-0.572	0.821		Elem. #2	4	1.52	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.52	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
1503.07	4517147.73	3107.15			49.282	9.213	0.37
Properties of Bottom Members							
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498	69.7207
Elem. #2	39.4591	29000	11153.85	0.00749	0.00749	0.01498	98.9745
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8504
	cos(Φ)	sin(Φ)			*n*	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	7.9585	41.38
Elem. #2	-0.156	0.988		Elem. #2	4	4.2344	22.02
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
439.68	356982.30	929.53			55.633	9.213	0.33

2nd Compression Web Member, Out-of-Plane Buckling, Simultaneously (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9967	29000	11153.85	0.00749	0.00497	0.01498		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.72
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.86
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.86
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.5239	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.5239	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
1498.93	4533153.7	3094.21			33.886	6.864	0.41
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.00749	0.00749	0.01498	69.72
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.86
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.86
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	7.9585	41.38
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
858.05	1440831.4	1778.53			47.920	6.864	0.29

3rd Compression Web Member, Out-of-Plane Buckling, Simultaneously (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9966	29000	11153.85	0.004897	0.0036	0.009794		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.5239	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.5239	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
1498.93	4533153.74	3094.21			33.886	4.972	0.29
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.8603
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.0572
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
853.90	1449877.27	1765.59			32.715	4.972	0.30

4^{th} Compression Web Member, Out-of-Plane Buckling, Simultaneously (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9959	29000	11153.85	0.004897	0.00235	0.009794		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.20	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.52	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.52	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}	
1498.93	4533153.7	3094.21		33.886	3.246	0.19	
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.8603
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.0572
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}	
853.90	1449877.3	1765.59		32.715	3.246	0.20	

5th Compression Web Member, Out-of-Plane Buckling, Simultaneously (18K3)							
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})		
20.9949	29000	11153.85	0.004897	0.00125	0.009794		
Properties of Top Members							
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			*n*	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.20	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.52	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.52	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}	
1498.93	4533153.74	3094.21		33.886	1.727	0.10	
Properties of Bottom Members							
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.8603
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8603
	cos(Φ)	sin(Φ)			*n*	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.0572
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}	
853.90	1449877.27	1765.59		32.715	1.727	0.11	

6th Compression Web Member, Out-of-Plane Buckling, Simultaneously (18K3)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
20.9945	29000	11153.85	0.004897	0.00047	0.009794		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.1533	0.4715	0.003279	124.8603
Elem. #3	24	29000	11153.85	0.1533	0.4715	0.003279	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.20	27.06
Elem. #2	-0.572	0.821		Elem. #2	4	1.52	2278.92
Elem. #3	0.572	-0.821		Elem. #3	4	1.52	2278.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
1498.93	4533153.7	3094.21			33.886	0.649	0.04
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	20.9945	29000	11153.85	0.004897	0.004897	0.009794	69.7208
Elem. #2	24	29000	11153.85	0.07805	0.2674	0.002542	124.8603
Elem. #3	24	29000	11153.85	0.07805	0.2674	0.002542	304.8603
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.347	0.938		Elem. #1	4	5.2033	27.0572
Elem. #2	-0.572	0.821		Elem. #2	4	1.1814	1292.43
Elem. #3	0.572	-0.821		Elem. #3	4	1.1814	1292.43
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
853.90	1449877.3	1765.59			32.715	0.649	0.04

Adjusted Moments of Inertia for In-Plane Buckling (18K3)						
Buckling Mode	1st Web	2nd Web	3rd Web	4th Web	5th Web	6th Web
	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})
Web	0.006600	0.004950	0.003869	0.002644	0.001616	0.000514
Web	0.006675	0.005025	0.003918	0.002693	0.001665	0.000539
Web (Control)	0.006750	0.005100	0.003967	0.002742	0.001714	0.000563
Top Chord	0.006825	0.005175	0.004016	0.002791	0.001763	0.000588
Top Chord	0.006900	0.005250	0.004065	0.002840	0.001812	0.000612

Adjusted Moments of Inertia for Out-of-Plane Buckling (18K3)						
Buckling Mode	1st Web	2nd Web	3rd Web	4th Web	5th Web	6th Web
	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})
Web	0.006075	0.004500	0.0034279	0.0022526	0.0012243	0.0002938
Web	0.006150	0.004575	0.0034769	0.0023016	0.0012732	0.0003428
Web (Control)	0.006225	0.004650	0.0035258	0.0023506	0.0013222	0.0003918
Top Chord	0.006300	0.004725	0.0035748	0.0023995	0.0013712	0.0004407
Top Chord	0.006375	0.004800	0.0036238	0.0024485	0.0014201	0.0004897

Appendix B: Data for 28K10

This appendix includes detailed data related to the determination of *K*-factors from the three methods employed in this study.

***K*-factors by Hand Calculation Method, 28K10, In-Plane Buckling, Individually**				
Web Member of interest	Location of *k*	*k* (kip-in./rad.)	*G*	*K*
1st Web Member	Top	5071.437	0.04	0.55
	Bottom	1323.463	0.16	
2nd Web Member	Top	6008.308	0.02	0.52
	Bottom	2473.2	0.05	
3rd Web Member	Top	6008.308	0.01	0.51
	Bottom	2475.717	0.03	
4th Web Member	Top	6008.295	0.01	0.51
	Bottom	2475.722	0.02	
5th Web Member	Top	6008.295	0.01	0.51
	Bottom	2475.724	0.02	

1st Web Member, In-Plane Buckling, Individually (28K10)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
36.0212	29000	11153.85	0.1294	0.1003	0.004659	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	36.0206	29000	11153.85	0.01859	0.03593	0.000981
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	35.0001	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	59.8669				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2038.6113		5071.437	104.178	0.04
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	47.9997	29000	11153.85	0.482	1.8882	0.01088
Elem. #2	29.0266	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	54.1338	29000	11153.85	0.0379	0.0379	0.0758
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	1164.8406				
Elem. #2	4	77.4090		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	81.2136		1323.463	104.178	0.16

2nd Compression Web Member, In-Plane Buckling, Individually (28K10)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
36.0212	29000	11153.85	0.08171	0.08156	0.003942	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	36.0206	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	62.3787				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2972.9709		6008.308	65.783	0.02
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	36.0206	29000	11153.85	0.01859	0.03593	0.000981
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088
	n	Bending				
Elem. #1	4	83.6499		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	59.8669				
Elem. #3	4	1164.8406		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1164.8430		2473.200	65.783	0.05

3rd Compression Web Member, In-Plane Buckling, Individually (28K10)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
36.0212	29000	11153.85	0.04766	0.06341	0.003315	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	36.0208	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	62.3784				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2972.9709		6008.308	38.370	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	36.0206	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	47.9996	29000	11153.85	0.482	1.8882	0.01088
Elem. #4	47.9995	29000	11153.85	0.482	1.8882	0.01088
	n	Bending				
Elem. #1	4	83.6499		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	62.3787				
Elem. #3	4	1164.8430		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1164.8455		2475.717	38.370	0.03

4th Compression Web Member, In-Plane Buckling, Individually (28K10)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
36.0211	29000	11153.85	0.03561	0.04974	0.001199	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	36.0209	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	24.0002	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	62.3782				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2972.9586		6008.295	28.669	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	36.0208	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	47.9995	29000	11153.85	0.482	1.8882	0.01088
Elem. #4	47.9994	29000	11153.85	0.482	1.8882	0.01088
	n	Bending				
Elem. #1	4	83.6499		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	62.3784				
Elem. #3	4	1164.8455		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1164.8479		2475.722	28.669	0.02

5th Compression Web Member, In-Plane Buckling, Individually (28K10)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
36.0211	29000	11153.85	0.03561	0.04974	0.001199	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	36.021	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	24.0002	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	62.3781				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2972.9586		6008.295	28.669	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	36.0209	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	47.9994	29000	11153.85	0.482	1.8882	0.01088
Elem. #4	47.9994	29000	11153.85	0.482	1.8882	0.01088
	n	Bending				
Elem. #1	4	83.6499		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	62.3782				
Elem. #3	4	1164.8479		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1164.8479		2475.724	28.669	0.02

K-factors by Hand Calculation Method, 28K10, In-Plane Buckling, Simultaneously				
Web Member of interest	Location of *k*	*k* (kip-in./rad.)	*G*	*K*
1st Web Member	Top	5071.437	0.03	0.53
	Bottom	1323.463	0.10	
2nd Web Member	Top	6008.308	0.02	0.52
	Bottom	2473.2	0.04	
3rd Web Member	Top	6008.308	0.01	0.51
	Bottom	2475.717	0.03	
4th Web Member	Top	6008.295	0.01	0.51
	Bottom	2475.722	0.02	
5th Web Member	Top	6008.295	0.00	0.50
	Bottom	2475.724	0.01	

1st Compression Web Member, In-Plane Buckling, Simultaneously (28K10)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
36.0212	29000	11153.85	0.079534	0.1003	0.004659	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	36.0206	29000	11153.85	0.01859	0.03593	0.000981
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	35.0001	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	59.8669				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2038.6113		5071.437	64.031	0.03
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	47.9997	29000	11153.85	0.482	1.8882	0.01088
Elem. #2	29.0266	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	54.1338	29000	11153.85	0.0379	0.0379	0.0758
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	1164.8406				
Elem. #2	4	77.4090		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	81.2136		1323.463	64.031	0.10

2nd Compression Web Member, In-Plane Buckling, Simultaneously (28K10)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
36.0212	29000	11153.85	0.060465	0.08156	0.003942	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	36.0206	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	62.3787				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2972.9709		6008.308	48.679	0.02
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	36.0206	29000	11153.85	0.01859	0.03593	0.000981
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088
	n	Bending				
Elem. #1	4	83.6499		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	59.8669				
Elem. #3	4	1164.8406		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1164.8430		2473.200	48.679	0.04

3rd Compression Web Member, In-Plane Buckling, Simultaneously (28K10)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
36.0212	29000	11153.85	0.045277	0.06341	0.003315	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	36.0208	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	62.3784				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2972.9709		6008.308	36.452	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	36.0206	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	47.9996	29000	11153.85	0.482	1.8882	0.01088
Elem. #4	47.9995	29000	11153.85	0.482	1.8882	0.01088
	n	Bending				
Elem. #1	4	83.6499		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	62.3787				
Elem. #3	4	1164.8430		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1164.8455		2475.717	36.452	0.03

4th Compression Web Member, In-Plane Buckling, Simultaneously (28K10)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
36.0211	29000	11153.85	0.028844	0.04974	0.001199	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	36.0209	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	24.0002	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	62.3782				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2972.9586		6008.295	23.222	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	36.0208	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	47.9995	29000	11153.85	0.482	1.8882	0.01088
Elem. #4	47.9994	29000	11153.85	0.482	1.8882	0.01088
	n	Bending				
Elem. #1	4	83.6499		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	62.3784				
Elem. #3	4	1164.8455		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1164.8479		2475.722	23.222	0.02

5th Compression Web Member, In-Plane Buckling, Simultaneously (28K10)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
36.0211	29000	11153.85	0.01179	0.04974	0.001199	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	36.021	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482
Elem. #3	24.0002	29000	11153.85	0.6151	2.5199	0.02482
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	62.3781				
Elem. #2	4	2972.959		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2972.9586		6008.295	9.492	0.00
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	36.0209	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	47.9994	29000	11153.85	0.482	1.8882	0.01088
Elem. #4	47.9994	29000	11153.85	0.482	1.8882	0.01088
	n	Bending				
Elem. #1	4	83.6499		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	62.3782				
Elem. #3	4	1164.8479		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1164.8479		2475.724	9.492	0.01

***K*-factors by Hand Calculation Method, 28K10, Out-of-Plane Buckling, Individually**				
Web Member of interest	Location of *k*	*k* (kip-in./rad.)	*G*	*K*
1st Web Member	Top	149.3942	1.39	0.80
	Bottom	173.0999	1.20	
2nd Web Member	Top	103.5639	1.27	0.74
	Bottom	268.2717	0.49	
3rd Web Member	Top	103.5639	0.74	0.68
	Bottom	218.2549	0.35	
4th Web Member	Top	103.5639	0.55	0.66
	Bottom	218.2549	0.26	
5th Web Member	Top	103.5633	0.55	0.66
	Bottom	218.2549	0.26	

1st Compression Web Member, Out-of-Plane Buckling, Individually (28K10)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
36.0223	29000	11153.85	0.1294	0.1003	0.004659		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	36.0216	29000	11153.85	0.01859	0.03593	0.000981	83.5576
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7786
Elem. #3	35.0003	29000	11153.85	0.6151	2.5199	0.02482	311.7786
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.3036	115.71
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	7.9096	8351.60
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
9126.17	103594643.80	11540.30			149.394	104.174	1.39
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	47.9997	29000	11153.85	0.482	1.8882	0.01088	311.78
Elem. #2	29.0266	29000	11153.85	0.01937	0.01937	0.000799	64.05
Elem. #3	54.1338	29000	11153.85	0.0379	0.0379	0.0758	102.03
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.666	-0.746		Elem. #1	4	2.5282	4563.1785
Elem. #2	0.438	0.899		Elem. #2	4	0.3069	77.4090
Elem. #3	-0.208	0.978		Elem. #3	4	15.6180	81.2136
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
2060.68	5057911.32	2679.57			173.100	104.174	1.20

2nd Compression Web Member, Out-of-Plane Buckling, Individually (28K10)							
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})		
36.0228	29000	11153.85	0.08171	0.08156	0.003942		
Properties of Top Members							
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	Φ (degree)
Elem. #1	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5602
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7805
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482	311.7805
	cos(Φ)	sin(Φ)			*n*	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.2473	62.38
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	11.5349	12179.47
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
10827.43	146028460.79	13617.15			103.564	65.780	1.27
Properties of Bottom Members							
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	Φ (degree)
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799	41.7804
Elem. #2	36.0206	29000	11153.85	0.01859	0.03593	0.000981	83.5614
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088	131.7816
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088	311.7816
	cos(Φ)	sin(Φ)			*n*	Torsion	Bending
Elem. #1	0.746	0.666		Elem. #1	4	0.3317	83.65
Elem. #2	0.112	0.994		Elem. #2	4	0.3036	115.71
Elem. #3	-0.666	0.746		Elem. #3	4	2.5282	4563.18
Elem. #4	0.666	-0.746		Elem. #4	4	2.5282	4563.19
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4102.85	20049342.80	5228.57			268.272	65.780	0.49

3rd Compression Web Member, Out-of-Plane Buckling, Individually (28K10)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
36.0228	29000	11153.85	0.04766	0.06341	0.003315		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5602
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7805
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482	311.7805
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.2473	62.38
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	11.5349	12179.47
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}	
10827.43	146028460.79	13617.15		103.564	38.368	0.74	
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799	41.7804
Elem. #2	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5614
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088	131.7816
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088	311.7816
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.746	0.666		Elem. #1	4	0.3317	83.65
Elem. #2	0.112	0.994		Elem. #2	4	0.2473	62.38
Elem. #3	-0.666	0.746		Elem. #3	4	2.5282	4563.18
Elem. #4	0.666	-0.746		Elem. #4	4	2.5282	4563.19
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}	
4102.12	20102539.29	5175.91		218.255	38.368	0.35	

4th Compression Web Member, Out-of-Plane Buckling, Individually (28K10)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
36.0228	29000	11153.85	0.03561	0.04974	0.001199		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5602
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7805
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482	311.7805
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.2473	62.38
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	11.5349	12179.47
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
10827.43	146028460.79	13617.15			103.564	28.668	0.55
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799	41.7804
Elem. #2	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5614
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088	131.7816
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088	311.7816
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.746	0.666		Elem. #1	4	0.3317	83.65
Elem. #2	0.112	0.994		Elem. #2	4	0.2473	62.38
Elem. #3	-0.666	0.746		Elem. #3	4	2.5282	4563.18
Elem. #4	0.666	-0.746		Elem. #4	4	2.5282	4563.19
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4102.12	20102539.29	5175.91			218.255	28.668	0.26

5th Compression Web Member, Out-of-Plane Buckling, Individually (28K10)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
36.0228	29000	11153.85	0.03561	0.04974	0.001199		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	36.0209	29000	11153.85	0.01937	0.01937	0.000799	83.5602
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7805
Elem. #3	24.0002	29000	11153.85	0.6151	2.5199	0.02482	311.7805
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.2473	62.3782
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	11.5348	12179.42
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
10827.41	146027853.39	13617.12			103.563	28.668	0.55
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799	41.7804
Elem. #2	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5614
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088	131.7816
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088	311.7816
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.746	0.666		Elem. #1	4	0.3317	83.65
Elem. #2	0.112	0.994		Elem. #2	4	0.2473	62.38
Elem. #3	-0.666	0.746		Elem. #3	4	2.5282	4563.18
Elem. #4	0.666	-0.746		Elem. #4	4	2.5282	4563.19
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4102.12	20102539.29	5175.91			218.255	28.668	0.26

K-factors by Hand Calculation Method, 28K10, In-Plane Buckling, Simultaneously				
Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	149.3942	0.78	0.73
	Bottom	173.0999	0.67	
2nd Web Member	Top	103.5639	0.65	0.66
	Bottom	268.2717	0.25	
3rd Web Member	Top	103.5639	0.47	0.64
	Bottom	218.2549	0.22	
4th Web Member	Top	103.5639	0.25	0.58
	Bottom	218.2549	0.12	
5th Web Member	Top	103.5633	0.09	0.53
	Bottom	218.2549	0.04	

1st Compression Web Member, Out-of-Plane Buckling, Simultaneously (28K10)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
36.0223	29000	11153.85	0.1294	0.072	0.004659		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	36.0216	29000	11153.85	0.01859	0.03593	0.000981	83.5576
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7786
Elem. #3	35.0003	29000	11153.85	0.6151	2.5199	0.02482	311.7786
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.3036	115.71
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	7.9096	8351.60
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
9126.17	103594643.77	11540.29			149.394	57.964	0.78
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	47.9997	29000	11153.85	0.482	1.8882	0.01088	311.78
Elem. #2	29.0266	29000	11153.85	0.01937	0.01937	0.000799	64.05
Elem. #3	54.1338	29000	11153.85	0.0379	0.0379	0.0758	102.03
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.666	-0.746		Elem. #1	4	2.5282	4563.18
Elem. #2	0.438	0.899		Elem. #2	4	0.3069	77.41
Elem. #3	-0.208	0.978		Elem. #3	4	15.6180	81.21
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
2060.68	5057911.3	2679.57			173.100	57.964	0.67

2^{nd} Compression Web Member, Out-of-Plane Buckling, Simultaneously (28K10)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
36.0228	29000	11153.85	0.08171	0.042	0.003942		

Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5602
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7805
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482	311.7805
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.2473	62.38
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	11.5349	12179.47

Stiffness of Top Node [Unit: kip-in./rad.]						
α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}
10827.43	146028460.79	13617.15		103.564	33.812	0.65

Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799	41.7804
Elem. #2	36.0206	29000	11153.85	0.01859	0.03593	0.000981	83.5614
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088	131.7816
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088	311.7816
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.746	0.666		Elem. #1	4	0.3317	83.65
Elem. #2	0.112	0.994		Elem. #2	4	0.3036	115.71
Elem. #3	-0.666	0.746		Elem. #3	4	2.5282	4563.18
Elem. #4	0.666	-0.746		Elem. #4	4	2.5282	4563.19

Stiffness of Bottom Node [Unit: kip-in./rad.]						
α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4102.85	20049342.80	5228.57		268.272	33.812	0.25

3rd Compression Web Member, Out-of-Plane Buckling, Simultaneously (28K10)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
36.0228	29000	11153.85	0.04766	0.03	0.003315		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5602
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7805
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482	311.7805
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.2473	62.3787
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	11.5349	12179.47
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
10827.43	146028460.79	13617.15			103.564	24.151	0.47
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799	41.7804
Elem. #2	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5614
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088	131.7816
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088	311.7816
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.746	0.666		Elem. #1	4	0.3317	83.65
Elem. #2	0.112	0.994		Elem. #2	4	0.2473	62.38
Elem. #3	-0.666	0.746		Elem. #3	4	2.5282	4563.18
Elem. #4	0.666	-0.746		Elem. #4	4	2.5282	4563.19
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4102.12	20102539.29	5175.91			218.255	24.151	0.22

4th Compression Web Member, Out-of-Plane Buckling, Simultaneously (28K10)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
36.0228	29000	11153.85	0.03561	0.016	0.001199		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5602
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7805
Elem. #3	24.0001	29000	11153.85	0.6151	2.5199	0.02482	311.7805
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.2473	62.3787
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	11.5349	12179.47
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
10827.43	146028460.79	13617.15			103.564	12.881	0.25
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799	41.7804
Elem. #2	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5614
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088	131.7816
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088	311.7816
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.746	0.666		Elem. #1	4	0.3317	83.65
Elem. #2	0.112	0.994		Elem. #2	4	0.2473	62.38
Elem. #3	-0.666	0.746		Elem. #3	4	2.5282	4563.18
Elem. #4	0.666	-0.746		Elem. #4	4	2.5282	4563.19
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4102.12	20102539.29	5175.91			218.255	12.881	0.12

5th Compression Web Member, Out-of-Plane Buckling, Simultaneously (28K10)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
36.0228	29000	11153.85	0.03561	0.0058	0.001199		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	36.0209	29000	11153.85	0.01937	0.01937	0.000799	83.5602
Elem. #2	24.0002	29000	11153.85	0.6151	2.5199	0.02482	131.7805
Elem. #3	24.0002	29000	11153.85	0.6151	2.5199	0.02482	311.7805
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.112	0.994		Elem. #1	4	0.2473	62.3782
Elem. #2	-0.666	0.746		Elem. #2	4	11.5348	12179.42
Elem. #3	0.666	-0.746		Elem. #3	4	11.5348	12179.42
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
10827.41	146027853.39	13617.12			103.563	4.669	0.09
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	26.861	29000	11153.85	0.01937	0.01937	0.000799	41.7804
Elem. #2	36.0206	29000	11153.85	0.01937	0.01937	0.000799	83.5614
Elem. #3	47.9997	29000	11153.85	0.482	1.8882	0.01088	131.7816
Elem. #4	47.9996	29000	11153.85	0.482	1.8882	0.01088	311.7816
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.746	0.666		Elem. #1	4	0.3317	83.65
Elem. #2	0.112	0.994		Elem. #2	4	0.2473	62.38
Elem. #3	-0.666	0.746		Elem. #3	4	2.5282	4563.18
Elem. #4	0.666	-0.746		Elem. #4	4	2.5282	4563.19
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4102.12	20102539.29	5175.91			218.255	4.669	0.04

Adjusted Moments of Inertia for In-Plane Buckling (28K10)

Buckling Mode	1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})
Web	0.081522	0.06210	0.044324	0.027776	0.011039
Web	0.082816	0.062917	0.044800	0.028132	0.011395
Web (Control)	0.084110	0.063734	0.045277	0.028488	0.011751
Top Chord	0.085404	0.064551	0.045754	0.028844	0.012107
Top Chord	0.086698	0.065368	0.046230	0.029200	0.012464

Adjusted Moments of Inertia for Out-of-Plane Buckling (28K10)

Buckling Mode	1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})
Web	0.0505	0.0240	0.0217	0.0125	0.0005
Web	0.0606	0.0320	0.0248	0.0150	0.0025
Web (Control)	0.0703	0.0400	0.0279	0.0175	0.0050
Top Chord	0.0707	0.0480	0.0310	0.0200	0.0075
Top Chord	0.0808	0.0560	0.0341	0.0225	0.0100

Details for computing in-plane *K*-factors according to hand method and requiring individual compression web buckling (28K10)

Web Member of interest	Location of *k*	*k (kip-in./rad.)*	*G*	*K*
1st Web Member	Top	5071.437	0.04	0.55
	Bottom	1323.463	0.16	
2nd Web Member	Top	6008.308	0.02	0.52
	Bottom	2473.200	0.05	
3rd Web Member	Top	6008.308	0.01	0.51
	Bottom	2475.717	0.03	
4th Web Member	Top	6008.295	0.01	0.51
	Bottom	2475.722	0.02	
5th Web Member	Top	6008.295	0.01	0.51
	Bottom	2475.724	0.02	

Details for computing in-plane *K*-factors according to uniformly distributed loading method and requiring individual compression web buckling (28K10)

Web Member of interest	*L* (in.)	I_z (in.^{4})	P_{cr} (kip)	*K*
1st Web Member	36.0212	0.08411	66.83	0.53
2nd Web Member	36.0212	0.06373	52.85	0.52
3rd Web Member	36.0212	0.04528	37.87	0.51
4th Web Member	36.0212	0.02849	23.75	0.51
5th Web Member	36.0212	0.01175	9.99	0.51

Details for computing in-plane *K*-factors according to SEIC method and requiring individual compression web buckling (28K10)

Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	36.0212	0.1294	93.41	0.55
2nd Web Member	36.0212	0.08171	66.18	0.52
3rd Web Member	36.0212	0.04766	40.01	0.51
4th Web Member	36.0212	0.03561	30.33	0.51
5th Web Member	36.0212	0.03561	30.27	0.51

Details for computing in-plane *K*-factors according to hand method and requiring simultaneous compression web buckling (28K10)

Web Member of interest	Location of k	k *(kip-in./rad.)*	G	K
1st Web Member	Top	5071.437	0.03	0.53
	Bottom	1323.463	0.10	
2nd Web Member	Top	6008.308	0.02	0.52
	Bottom	2473.200	0.04	
3rd Web Member	Top	6008.308	0.01	0.51
	Bottom	2475.717	0.03	
4th Web Member	Top	6008.295	0.01	0.51
	Bottom	2475.722	0.02	
5th Web Member	Top	6008.295	0.00	0.50
	Bottom	2475.724	0.01	

Details for computing in-plane *K*-factors according to uniformly distributed loading method and requiring simultaneous compression web buckling (28K10)

Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	36.0212	0.079534	63.54	0.53
2nd Web Member	36.0212	0.060465	50.30	0.52
3rd Web Member	36.0212	0.045277	36.84	0.52
4th Web Member	36.0211	0.028844	23.17	0.52
5th Web Member	36.0211	0.011790	9.93	0.51

Details for computing in-plane *K*-factors according to SEIC and requiring simultaneous compression web buckling (28K10)

Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	36.0212	0.079534	61.43	0.53
2nd Web Member	36.0212	0.060465	50.03	0.52
3rd Web Member	36.0212	0.045277	38.01	0.51
4th Web Member	36.0211	0.028844	24.72	0.51
5th Web Member	36.0211	0.011790	10.28	0.50

Details for computing out-of-plane *K*-factors according to hand method and requiring individual compression web buckling (28K10)				
Web Member of interest	Location of k	k *(kip-in./rad.)*	G	K
1st Web Member	Top	149.3942	1.39	0.80
	Bottom	173.0999	1.20	
2nd Web Member	Top	103.5639	1.27	0.74
	Bottom	268.2717	0.49	
3rd Web Member	Top	103.5639	0.74	0.68
	Bottom	218.2549	0.35	
4th Web Member	Top	103.5639	0.55	0.66
	Bottom	218.2549	0.26	
5th Web Member	Top	103.5633	0.55	0.66
	Bottom	218.2549	0.26	

Details for computing out-of-plane *K*-factors according to uniformly distributed loading method and requiring individual compression web buckling (28K10)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	36.0223	0.07030	29.59	0.72
2nd Web Member	36.0228	0.04600	23.35	0.66
3rd Web Member	36.0218	0.03150	17.16	0.64
4th Web Member	36.0212	0.01600	10.14	0.59
5th Web Member	36.0220	0.00625	4.54	0.55

Details for computing out-of-plane *K*-factors according to SEIC method and requiring individual compression web buckling (28K10)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	36.0223	0.10030	32.34	0.83
2nd Web Member	36.0228	0.08156	29.76	0.78
3rd Web Member	36.0218	0.06341	24.62	0.75
4th Web Member	36.0212	0.04974	21.28	0.72
5th Web Member	36.0220	0.04974	20.91	0.72

Details for computing out-of-plane *K*-factors according to hand method and requiring simultaneous compression web buckling (28K10)				
Web Member of interest	Location of k	k *(kip-in./rad.)*	G	K
1st Web Member	Top	149.3942	0.78	0.73
	Bottom	173.0999	0.67	
2nd Web Member	Top	103.5639	0.65	0.66
	Bottom	268.2717	0.25	
3rd Web Member	Top	103.5639	0.47	0.64
	Bottom	218.2549	0.22	
4th Web Member	Top	103.5639	0.25	0.58
	Bottom	218.2549	0.12	
5th Web Member	Top	103.5633	0.09	0.53
	Bottom	218.2549	0.04	

Details for computing out-of-plane *K*-factors according to uniformly distributed loading method and requiring simultaneous compression web buckling (28K10)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	36.0223	0.0720	26.62	0.77
2nd Web Member	36.0228	0.0420	21.08	0.66
3rd Web Member	36.0218	0.0300	15.44	0.65
4th Web Member	36.0212	0.0160	9.71	0.60
5th Web Member	36.0220	0.0058	4.16	0.55

Details for computing out-of-plane *K*-factors according to SEIC method and requiring simultaneous compression web buckling (28K10)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	36.0223	0.0720	25.42	0.79
2nd Web Member	36.0228	0.0420	18.96	0.70
3rd Web Member	36.0218	0.0300	14.96	0.67
4th Web Member	36.0212	0.0160	9.79	0.60
5th Web Member	36.0220	0.0058	4.37	0.54

Appendix C: Data for 32LH06_L1

This appendix includes detailed data related to the determination of *K*-factors from the three methods employed in this study.

***K*-factors by hand calculation method, 32LH06_L1, In-Plane buckling, individually**				
Web Member of interest	Location of *k*	*k* (kip-in./rad.)	*G*	*K*
1st Web Member	Top	7274.391	0.03	0.54
	Bottom	1376.548	0.14	
2nd Web Member	Top	8616.008	0.01	0.51
	Bottom	2278.264	0.04	
3rd Web Member	Top	8616.008	0.01	0.51
	Bottom	2281.267	0.03	
4th Web Member	Top	8616.008	0.01	0.51
	Bottom	2281.267	0.03	
5th Web Member	Top	8681.796	0.01	0.51
	Bottom	2281.267	0.03	

1st Web Member, In-Plane Buckling, Individually (32LH06_L1)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
45.9674	29000	11153.85	0.1512	0.1107	0.007871	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.01818	0.03799	0.00098
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	48	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	45.878				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2944.9500		7274.391	95.389	0.03
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	35.3412	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	68.884	29000	11153.85	0.1377	0.6658	0.002331
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	63.5779				
Elem. #2	4	231.8855		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	1081.0848		1376.548	95.389	0.14

2nd Compression Web Member, In-Plane Buckling, Individually (32LH06_L1)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
45.9674	29000	11153.85	0.07844	0.08917	0.003942	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	48.8807				
Elem. #2	4	4283.564		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.564		8616.008	49.486	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.9674	29000	11153.85	0.01818	0.03799	0.00098
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	70.2163		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	45.8777				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2278.264	49.486	0.04

3rd Compression Web Member, In-Plane Buckling, Individually (32LH06_L1)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
45.9674	29000	11153.85	0.04544	0.0672	0.003314	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	48.8807				
Elem. #2	4	4283.564		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.564		8616.008	28.667	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	70.2163		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	48.8807				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2281.267	28.667	0.03

4th Compression Web Member, In-Plane Buckling, Individually (32LH06_L1)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
45.9674	29000	11153.85	0.04544	0.0672	0.003314	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	48.8807				
Elem. #2	4	4283.564		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.008	28.667	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	70.2163		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	48.8807				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2281.267	28.667	0.03

5th Compression Web Member, In-Plane Buckling, Individually (32LH06_L1)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
45.9674	29000	11153.85	0.04544	0.0672	0.003314	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.04544	0.0672	0.003314
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	114.6691				
Elem. #2	4	4283.564		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8681.796	28.667	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	70.2163		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	48.8807				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2281.267	28.667	0.03

***K*-factors by Hand Calculation Method, 32LH06_L1, In-Plane Buckling, Simultaneously**				
Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	7274.391	0.020	0.53
	Bottom	1376.548	0.080	
2nd Web Member	Top	8616.008	0.010	0.51
	Bottom	2278.264	0.040	
3rd Web Member	Top	8616.008	0.010	0.51
	Bottom	2281.267	0.020	
4th Web Member	Top	8616.008	0.004	0.50
	Bottom	2281.267	0.013	
5th Web Member	Top	8681.796	0.001	0.50
	Bottom	2281.267	0.003	

1st Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L1)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
45.9674	29000	11153.85	0.086495	0.1107	0.007871	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.01818	0.03799	0.00098
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	48	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	45.878				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	2944.9500		7274.391	54.568	0.02
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	35.3412	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	68.884	29000	11153.85	0.1377	0.6658	0.002331
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	63.5779				
Elem. #2	4	231.8855		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	1081.0848		1376.548	54.568	0.08

2nd Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L1)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
45.9674	29000	11153.85	0.06389	0.08917	0.003942	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	48.8807				
Elem. #2	4	4283.564		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.008	40.307	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.9674	29000	11153.85	0.01818	0.03799	0.00098
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	70.2163		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	45.8777				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2278.264	40.307	0.04

3rd Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L1)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
45.9674	29000	11153.85	0.043642	0.0672	0.003314	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	48.8807				
Elem. #2	4	4283.564		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.008	27.533	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	70.2163		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	48.8807				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2281.267	27.533	0.02

4^{th} Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L1)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
45.9674	29000	11153.85	0.024013	0.0672	0.003314	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	48.8807				
Elem. #2	4	4283.564		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.008	15.149	0.00
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	70.2163		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	48.8807				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2281.267	15.149	0.01

5^{th} Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L1)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
45.9674	29000	11153.85	0.005004	0.0672	0.003314	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	45.9674	29000	11153.85	0.04544	0.0672	0.003314
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	114.6691				
Elem. #2	4	4283.564		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8681.796	3.157	0.00
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	70.2163		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	48.8807				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2281.267	3.157	0.00

K-factors by Hand Calculation Method, 32LH06_L1, Out-of-Plane Buckling, Individually				
Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	119.5673	1.17	0.71
	Bottom	420.5478	0.33	
2nd Web Member	Top	90.54967	1.24	0.73
	Bottom	251.2449	0.45	
3rd Web Member	Top	90.54967	0.94	0.71
	Bottom	207.4227	0.41	
4th Web Member	Top	90.54967	0.94	0.71
	Bottom	207.4227	0.41	
5th Web Member	Top	209.7138	0.40	0.66
	Bottom	207.4227	0.41	

1st Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.1512	0.1107	0.007871		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	66	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #2	45.9674	29000	11153.85	0.01818	0.03799	0.00098	91.7628
Elem. #3	48	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.718	0.696		Elem. #1	4	5.0649	7308.35
Elem. #2	-0.031	1.000		Elem. #2	4	0.2378	95.87
Elem. #3	0.718	-0.696		Elem. #3	4	6.9642	10048.98
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}	
8951.80	75194719.82	8513.67		119.567	69.839	1.17	
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	35.3412	29000	11153.85	0.01937	0.01937	0.000799	71.00
Elem. #2	68.884	29000	11153.85	0.1377	0.6658	0.002331	108.20
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	315.88
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.326	0.945		Elem. #1	4	0.2521	63.58
Elem. #2	-0.312	0.950		Elem. #2	4	0.3774	1121.20
Elem. #3	0.718	-0.696		Elem. #3	4	4.1915	4428.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}	
2401.30	6372501.89	3217.22		420.548	69.839	0.33	

2nd Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.07844	0.08917	0.003942		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.031	0.999		Elem. #1	4	0.1938	48.88
Elem. #2	-0.718	0.696		Elem. #2	4	10.1297	14616.70
Elem. #3	0.718	-0.696		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15076.40	213193804.54	14226.34			90.550	56.256	1.24
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799	45.8814
Elem. #2	45.9674	29000	11153.85	0.01818	0.03799	0.00098	91.7628
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	135.8814
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.696	0.718		Elem. #1	4	0.2784	70.22
Elem. #2	-0.031	1.000		Elem. #2	4	0.2378	95.87
Elem. #3	-0.718	0.696		Elem. #3	4	4.1915	4428.92
Elem. #4	0.718	-0.696		Elem. #4	4	4.1915	4428.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4603.72	19277523.38	4429.10			251.245	56.256	0.45

3rd Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.04544	0.0672	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.031	0.999		Elem. #1	4	0.1938	48.8807
Elem. #2	-0.718	0.696		Elem. #2	4	10.1297	14616.70
Elem. #3	0.718	-0.696		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15076.40	213193804.54	14226.34			90.550	42.395	0.94
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799	45.8814
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	135.8814
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.696	0.718		Elem. #1	4	0.2784	70.22
Elem. #2	-0.031	1.000		Elem. #2	4	0.1938	48.88
Elem. #3	-0.718	0.696		Elem. #3	4	4.1915	4428.92
Elem. #4	0.718	-0.696		Elem. #4	4	4.1915	4428.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4603.63	19264850.62	4382.15			207.423	42.395	0.41

4th Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.04544	0.0672	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.031	0.999		Elem. #1	4	0.1938	48.88
Elem. #2	-0.718	0.696		Elem. #2	4	10.1297	14616.70
Elem. #3	0.718	-0.696		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15076.40	213193804.54	14226.34			90.550	42.395	0.94
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799	45.8813
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	135.8814
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.696	0.718		Elem. #1	4	0.2784	70.22
Elem. #2	-0.031	1.000		Elem. #2	4	0.1938	48.88
Elem. #3	-0.718	0.696		Elem. #3	4	4.1915	4428.92
Elem. #4	0.718	-0.696		Elem. #4	4	4.1915	4428.92
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4603.63	19264850.58	4382.15			207.423	42.395	0.41

5th Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.04544	0.0672	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	45.9674	29000	11153.85	0.04544	0.0672	0.003314	91.7628
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.031	0.999		Elem. #1	4	0.8041	169.58
Elem. #2	-0.718	0.696		Elem. #2	4	10.1297	14616.70
Elem. #3	0.718	-0.696		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15077.12	213301645.89	14346.93			209.714	42.395	0.40
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799	45.8814
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	135.8814
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.696	0.718		Elem. #1	4	0.2784	70.22
Elem. #2	-0.031	1.000		Elem. #2	4	0.1938	48.88
Elem. #3	-0.718	0.696		Elem. #3	4	4.1915	4428.92
Elem. #4	0.718	-0.696		Elem. #4	4	4.1915	4428.92
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4603.63	19264850.62	4382.15			207.423	42.395	0.41

***K*-factors by Hand Calculation Method, 32LH06_L1, Out-of-Plane Buckling, Simultaneously**				
Web Member of interest	Location of k	k (kip-in./Rad.)	G	K
1st Web Member	Top	119.5673	1.11	0.71
	Bottom	420.5478	0.32	
2nd Web Member	Top	90.54967	1.06	0.72
	Bottom	251.2449	0.38	
3rd Web Member	Top	90.54967	0.66	0.67
	Bottom	207.4227	0.29	
4th Web Member	Top	90.54967	0.31	0.60
	Bottom	207.4227	0.14	
5th Web Member	Top	209.7138	0.04	0.52
	Bottom	207.4227	0.04	

1st Compression Web Member, Out-of-Plane Buckling, Simultaneously (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.1512	0.10516	0.007871		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	66	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #2	45.9674	29000	11153.85	0.01818	0.03799	0.00098	91.7628
Elem. #3	48	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.718	0.696		Elem. #1	4	5.0649	7308.35
Elem. #2	-0.031	1.000		Elem. #2	4	0.2378	95.87
Elem. #3	0.718	-0.696		Elem. #3	4	6.9642	10048.98
Stiffness of Top Node [unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
8951.797	75194719.8	8513.7			119.567	66.344	1.11
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	35.3412	29000	11153.85	0.01937	0.01937	0.000799	71.00
Elem. #2	68.884	29000	11153.85	0.1377	0.6658	0.002331	108.20
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	315.88
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.326	0.945		Elem. #1	4	0.2521	63.5779
Elem. #2	-0.312	0.950		Elem. #2	4	0.3774	1121.2009
Elem. #3	0.718	-0.696		Elem. #3	4	4.1915	4428.9152
Stiffness of Bottom Node [unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
2401.30	6372501.89	3217.22			420.548	66.344	0.32

2nd Compression Web Member, Out-of-Plane Buckling, Simultaneously (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.07844	0.075795	0.003942		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.031	0.999		Elem. #1	4	0.1938	48.8807
Elem. #2	-0.718	0.696		Elem. #2	4	10.1297	14616.70
Elem. #3	0.718	-0.696		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15076.40	213193804.54	14226.34			90.550	47.818	1.06
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799	45.8814
Elem. #2	45.9674	29000	11153.85	0.01818	0.03799	0.00098	91.7628
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	135.8814
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.696	0.718		Elem. #1	4	0.2784	70.22
Elem. #2	-0.031	1.000		Elem. #2	4	0.2378	95.87
Elem. #3	-0.718	0.696		Elem. #3	4	4.1915	4428.92
Elem. #4	0.718	-0.696		Elem. #4	4	4.1915	4428.92
Stiffness of Bottom Node [unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4603.72	19277523.38	4429.10			251.245	47.818	0.38

3^{rd} Compression Web Member, Out-of-Plane Buckling, Simultaneously (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.04544	0.047041	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.031	0.999		Elem. #1	4	0.1938	48.8807
Elem. #2	-0.718	0.696		Elem. #2	4	10.1297	14616.70
Elem. #3	0.718	-0.696		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15076.40	213193804.54	14226.34			90.550	29.677	0.66
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799	45.8814
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	135.8814
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.696	0.718		Elem. #1	4	0.2784	70.22
Elem. #2	-0.031	1.000		Elem. #2	4	0.1938	48.88
Elem. #3	-0.718	0.696		Elem. #3	4	4.1915	4428.92
Elem. #4	0.718	-0.696		Elem. #4	4	4.1915	4428.92
Stiffness of Bottom Node [unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4603.63	19264850.62	4382.15			207.423	29.677	0.29

4^{th} Compression Web Member, Out-of-Plane Buckling, Simultaneously (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.04544	0.0225	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.031	0.999		Elem. #1	4	0.1938	48.8807
Elem. #2	-0.718	0.696		Elem. #2	4	10.1297	14616.70
Elem. #3	0.718	-0.696		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [unit: kip-in/rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15076.40	213193804.54	14226.34			90.550	14.195	0.31
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799	45.8813
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	135.8814
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.696	0.718		Elem. #1	4	0.2784	70.22
Elem. #2	-0.031	1.000		Elem. #2	4	0.1938	48.88
Elem. #3	-0.718	0.696		Elem. #3	4	4.1915	4428.92
Elem. #4	0.718	-0.696		Elem. #4	4	4.1915	4428.92
Stiffness of Bottom Node [unit: kip-in/rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4603.63	19264850.58	4382.15			207.423	14.195	0.14

5th Compression Web Member, Out-of-Plane Buckling, Simultaneously (32LH06_L1)							
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})		
45.9674	29000	11153.85	0.04544	0.0059	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	45.9674	29000	11153.85	0.04544	0.0672	0.003314	91.7628
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	135.8814
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.031	0.999		Elem. #1	4	0.8041	169.5811
Elem. #2	-0.718	0.696		Elem. #2	4	10.1297	14616.70
Elem. #3	0.718	-0.696		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [unit: kip-in/rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15077.12	213301645.89	14346.93			209.714	3.722	0.04
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	Φ (degree)
Elem. #1	32	29000	11153.85	0.01937	0.01937	0.000799	45.8814
Elem. #2	45.9674	29000	11153.85	0.01937	0.01937	0.000799	91.7628
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	135.8814
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	315.8814
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.696	0.718		Elem. #1	4	0.2784	70.22
Elem. #2	-0.031	1.000		Elem. #2	4	0.1938	48.88
Elem. #3	-0.718	0.696		Elem. #3	4	4.1915	4428.92
Elem. #4	0.718	-0.696		Elem. #4	4	4.1915	4428.92
Stiffness of Bottom Node [unit: kip-in/rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4603.63	19264850.62	4382.15			207.423	3.722	0.04

Adjusted Moments of Inertia for In-Plane Buckling (32LH06_L1)					
Buckling Mode	1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})
Web	0.084672	0.064321	0.052256	0.023174	0.004589
Web	0.086184	0.065105	0.052710	0.023629	0.004817
Web (Control)	0.087696	0.065890	0.053165	0.024083	0.005044
Top Chord	0.089208	0.066674	0.053619	0.024538	0.005271
Top Chord	0.090720	0.067458	0.054074	0.024992	0.005498

Adjusted Moments of Inertia for Out-of-Plane Buckling (32LH06_L1)					
Buckling Mode	1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})
Web	0.08856	0.062419	0.036961	0.014112	0.005040
Web	0.094095	0.066878	0.040321	0.017472	0.005376
Web (Control)	0.099630	0.071336	0.043681	0.020832	0.005712
Top Chord	0.105165	0.075795	0.047041	0.024192	0.006048
Top Chord	0.110700	0.080253	0.050401	0.027552	0.006384

Details for computing in-plane *K*-factors according to hand method and requiring individual compression web buckling (28K10)				
Web Member of interest	Location of *k*	*k (kip-in./rad.)*	*G*	*K*
1st Web Member	Top	7274.391	0.03	0.54
	Bottom	1376.548	0.14	
2nd Web Member	Top	8616.008	0.01	0.51
	Bottom	2278.264	0.04	
3rd Web Member	Top	8616.008	0.01	0.51
	Bottom	2281.267	0.03	
4th Web Member	Top	8616.008	0.01	0.51
	Bottom	2281.267	0.03	
5th Web Member	Top	8681.796	0.01	0.51
	Bottom	2281.267	0.03	

Details for computing in-plane *K*-factors according to uniformly distributed loading method and requiring individual compression web buckling (32LH06_L1)				
Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	45.9674	0.08770	44.40	0.52
2nd Web Member	45.9674	0.06589	34.42	0.51
3rd Web Member	45.9674	0.05317	27.94	0.51
4th Web Member	45.9674	0.02408	12.87	0.50
5th Web Member	45.9674	0.00504	2.722	0.50

Details for computing in-plane *K*-factors according to SEIC method and requiring individual compression web buckling (32LH06_L1)				
Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	45.9674	0.1512	69.21	0.54
2nd Web Member	45.9674	0.07844	39.92	0.52
3rd Web Member	45.9674	0.04544	23.75	0.51
4th Web Member	45.9674	0.04544	23.75	0.51
5th Web Member	45.9674	0.04544	23.75	0.51

Details for computing in-plane *K*-factors according to hand method and requiring simultaneous compression web buckling (32LH06_L1)				
Web Member of interest	Location of k	k *(kip-in./rad.)*	G	K
1st Web Member	Top	7274.391	0.020	0.53
	Bottom	1376.548	0.080	
2nd Web Member	Top	8616.008	0.010	0.51
	Bottom	2278.264	0.040	
3rd Web Member	Top	8616.008	0.010	0.51
	Bottom	2281.267	0.020	
4th Web Member	Top	8616.008	0.004	0.50
	Bottom	2281.267	0.013	
5th Web Member	Top	8681.796	0.001	0.50
	Bottom	2281.267	0.003	

Details for computing in-plane *K*-factors according to uniformly distributed loading method and requiring simultaneous compression web buckling (32LH06_L1)				
Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	45.9674	0.086495	43.68	0.52
2nd Web Member	45.9674	0.063890	33.33	0.51
3rd Web Member	45.9674	0.043642	23.07	0.51
4th Web Member	45.9674	0.024013	12.83	0.50
5th Web Member	45.9674	0.005004	2.691	0.50

Details for computing in-plane *K*-factors according to SEIC and requiring simultaneous compression web buckling (32LH06_L1)				
Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	45.9674	0.086495	42.36	0.53
2nd Web Member	45.9674	0.063890	32.88	0.51
3rd Web Member	45.9674	0.043642	22.84	0.51
4th Web Member	45.9674	0.024013	12.76	0.51
5th Web Member	45.9674	0.005004	2.701	0.50

Details for computing out-of-plane *K*-factors according to hand method and requiring individual compression web buckling (32K06_L1)

Web Member of interest	Location of k	k *(kip-in./rad.)*	G	K
1st Web Member	Top	119.5673	1.17	0.71
	Bottom	420.5478	0.33	
2nd Web Member	Top	90.54967	1.24	0.73
	Bottom	251.2449	0.45	
3rd Web Member	Top	90.54967	0.94	0.71
	Bottom	207.4227	0.41	
4th Web Member	Top	90.54967	0.94	0.71
	Bottom	207.4227	0.41	
5th Web Member	Top	209.7138	0.40	0.66
	Bottom	207.4227	0.41	

Details for computing out-of-plane *K*-factors according to uniformly distributed loading method and requiring individual compression web buckling (32LH06_L1)

Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	45.9674	0.0895	24.71	0.70
2nd Web Member	45.9674	0.0730	20.29	0.70
3rd Web Member	45.9674	0.0418	13.23	0.65
4th Web Member	45.9674	0.0215	7.84	0.61
5th Web Member	45.9674	0.0059	1.64	0.70

Details for computing out-of-plane *K*-factors according to SEIC method and requiring individual compression web buckling (32LH06_L1)

Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	45.9674	0.11070	22.87	0.81
2nd Web Member	45.9674	0.08917	20.53	0.77
3rd Web Member	45.9674	0.06720	16.60	0.74
4th Web Member	45.9674	0.06720	16.73	0.74
5th Web Member	45.9674	0.06720	18.87	0.69

Details for computing out-of-plane *K*-factors according to hand method and requiring simultaneous compression web buckling (32LH06_L1)

Web Member of interest	Location of k	k *(kip-in./rad.)*	G	K
1st Web Member	Top	119.5673	1.11	0.71
	Bottom	420.5478	0.32	
2nd Web Member	Top	90.54967	1.06	0.72
	Bottom	251.2449	0.38	
3rd Web Member	Top	90.54967	0.66	0.67
	Bottom	207.4227	0.29	
4th Web Member	Top	90.54967	0.31	0.60
	Bottom	207.4227	0.14	
5th Web Member	Top	209.7138	0.04	0.52
	Bottom	207.4227	0.04	

Details for computing out-of-plane *K*-factors according to uniformly distributed loading method and requiring simultaneous compression web buckling (32LH06_L1)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	45.9674	0.105160	26.15	0.74
2nd Web Member	45.9674	0.075795	19.96	0.72
3rd Web Member	45.9674	0.047041	13.81	0.68
4th Web Member	45.9674	0.022500	7.68	0.63
5th Web Member	45.9674	0.005900	1.61	0.70

Details for computing out-of-plane *K*-factors according to SEIC method and requiring simultaneous compression web buckling (32LH06_L1)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	45.9674	0.10516	22.02	0.80
2nd Web Member	45.9674	0.07580	18.35	0.75
3rd Web Member	45.9674	0.04704	13.07	0.70
4th Web Member	45.9674	0.02250	8.01	0.62
5th Web Member	45.9674	0.00590	2.66	0.55

Appendix D: Data for 32LH06_L2

This appendix includes detailed data related to the determination of *K*-factors from the three methods employed in this study.

***K*-factors by Hand Calculation Method, 32LH06_L2, In-Plane Buckling, Individually**				
Web Member of interest	Location of *k*	*k* (kip-in./rad.)	*G*	*K*
1st Web Member	Top	7954.89	0.03	0.54
	Bottom	1726.64	0.14	
2nd Web Member	Top	8616.97	0.01	0.51
	Bottom	2628.04	0.04	
3rd Web Member	Top	8616.97	0.01	0.51
	Bottom	2285.16	0.03	
4th Web Member	Top	8616.97	0.01	0.51
	Bottom	2285.16	0.03	
5th Web Member	Top	8684.04	0.01	0.51
	Bottom	2285.16	0.03	

1st Web Member, In-Plane Buckling, Individually (32LH06_L2)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
35.1083	29000	11153.85	0.1512	0.1107	0.007871	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #2	45.0843	29000	11153.85	0.01818	0.03799	0.00098
Elem. #3	39	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	4283.56				
Elem. #2	4	46.78		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	3624.55		7954.894	124.894	0.03
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	37.7835	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	66.5176	29000	11153.85	0.1377	0.6658	0.002331
Elem. #3	50	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	59.47				
Elem. #2	4	240.13		k_{bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	1427.03		1726.635	124.894	0.14

2nd Compression Web Member, In-Plane Buckling, Individually (32LH06_L2)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
45.0843	29000	11153.85	0.07844	0.08917	0.003942	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	49.8382				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.97	50.456	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.0843	29000	11153.85	0.01818	0.03799	0.00098
Elem. #3	50	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	73.1467		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	46.7764				
Elem. #3	4	1427.0320		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2628.04	50.456	0.04

3rd Compression Web Member, In-Plane Buckling, Individually (32LH06_L2)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})	
45.0843	29000	11153.85	0.04544	0.0672	0.003314	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	49.8382				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.97	29.229	0.01
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in.^{4})	I_y (in.^{4})	*J* (in.^{4})
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	73.1467		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	49.8382				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2285.15	29.229	0.03

4th Compression Web Member, In-Plane Buckling, Individually (32LH06_L2)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
45.0843	29000	11153.85	0.04544	0.0672	0.003314	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	49.8382				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.965	29.229	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	73.1467		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	49.8382				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2285.155	29.229	0.03

5th Compression Web Member, In-Plane Buckling, Individually (32LH06_L2)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
45.0843	29000	11153.85	0.04544	0.0672	0.003314	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	45.0843	29000	11153.85	0.04544	0.0672	0.003314
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	116.9152				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8684.042	29.229	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	73.1467		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	49.8382				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2285.155	29.229	0.03

K-factors by Hand Calculation Method, 32LH06_L2, In-Plane Buckling, Simultaneously				
Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	7954.89	0.008	0.51
	Bottom	1726.64	0.038	
2nd Web Member	Top	8616.97	0.010	0.51
	Bottom	2628.04	0.031	
3rd Web Member	Top	8616.97	0.007	0.51
	Bottom	2285.16	0.025	
4th Web Member	Top	8616.97	0.004	0.50
	Bottom	2285.16	0.014	
5th Web Member	Top	8684.04	0.001	0.50
	Bottom	2285.16	0.003	

1st Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L2)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
35.1083	29000	11153.85	0.03995	0.1107	0.007871	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #2	45.0843	29000	11153.85	0.01818	0.03799	0.00098
Elem. #3	39	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	4283.56				
Elem. #2	4	46.78		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	3624.55		7954.894	32.999	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	37.7835	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	66.5176	29000	11153.85	0.1377	0.6658	0.002331
Elem. #3	50	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #1	4	59.47				
Elem. #2	4	240.13		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #3	4	1427.03		1726.635	32.999	0.04

2nd Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L2)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
45.0843	29000	11153.85	0.06389	0.08917	0.003942	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	49.8382				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.965	41.097	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.0843	29000	11153.85	0.01818	0.03799	0.00098
Elem. #3	50	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	73.1467		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	46.7764				
Elem. #3	4	1427.0320		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2628.040	41.097	0.03

3rd Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L2)						
L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})	
45.0843	29000	11153.85	0.043642	0.0672	0.003314	
Properties of Top Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	49.8382				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.97	28.072	0.01
Properties of Bottom Members						
	L (in.)	E (ksi)	G (ksi)	I_z (in.^{4})	I_y (in.^{4})	J (in.^{4})
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	73.1467		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	49.8382				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2285.16	28.072	0.02

4th Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L2)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in^4)	I_y (in^4)	*J* (in^4)	
45.0843	29000	11153.85	0.024013	0.0672	0.003314	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in^4)	I_y (in^4)	*J* (in^4)
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	49.8382				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8616.965	15.446	0.00
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in^4)	I_y (in^4)	*J* (in^4)
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	73.1467		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	49.8382				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2285.155	15.446	0.01

5th Compression Web Member, In-Plane Buckling, Simultaneously (32LH06_L2)						
L (in.)	*E* (ksi)	*G* (ksi)	I_z (in^4)	I_y (in^4)	*J* (in^4)	
45.0843	29000	11153.85	0.004977	0.0672	0.003314	
Properties of Top Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in^4)	I_y (in^4)	*J* (in^4)
Elem. #1	45.0843	29000	11153.85	0.04544	0.0672	0.003314
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997
	n	Bending		**Stiffness of Top Node [Unit: kip-in./rad.]**		
Elem. #1	4	116.9152				
Elem. #2	4	4283.5636		k_{top}	$(EI/L)_{web}$	G_{top}
Elem. #3	4	4283.5636		8684.042	3.201	0.001
Properties of Bottom Members						
	L (in.)	*E* (ksi)	*G* (ksi)	I_z (in^4)	I_y (in^4)	*J* (in^4)
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802
	n	Bending				
Elem. #1	4	73.1467		**Stiffness of Bottom Node [Unit: kip-in./rad.]**		
Elem. #2	4	49.8382				
Elem. #3	4	1081.0848		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
Elem. #4	4	1081.0848		2285.155	3.201	0.003

K-factors by Hand Calculation Method, 32LH06_L2, Out-of-Plane Buckling, Individually				
Web Member of interest	Location of k	k (kip-in/rad.)	G	K
1st Web Member	Top	83.53	2.19	0.78
	Bottom	304.63	0.60	
2nd Web Member	Top	93.33	1.23	0.73
	Bottom	269.45	0.43	
3rd Web Member	Top	93.33	0.93	0.71
	Bottom	221.21	0.39	
4th Web Member	Top	93.33	0.93	0.71
	Bottom	221.21	0.39	
5th Web Member	Top	214.75	0.40	0.66
	Bottom	221.21	0.39	

1st Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L2)							
L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
35.1083	29000	11153.85	0.1512	0.1107	0.007871		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	33	29000	11153.85	1.2186	4.1582	0.02997	118.9611
Elem. #2	45.08	29000	11153.85	0.01818	0.03799	0.00098	76.0122
Elem. #3	39	29000	11153.85	1.2186	4.1582	0.02997	298.9611
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.484	0.875		Elem. #1	4	10.13	14616.70
Elem. #2	0.242	0.970		Elem. #2	4	0.24	97.75
Elem. #3	0.484	-0.875		Elem. #3	4	8.57	12367.98
Stiffness at Top Node [Unit: kip-in./rad]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
6347.21	129997460.55	20754.16			83.532	91.440	2.19
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	37.7835	29000	11153.85	0.01937	0.01937	0.000799	64.57
Elem. #2	66.5176	29000	11153.85	0.1377	0.6658	0.002331	91.46
Elem. #3	50	29000	11153.85	0.6151	2.5199	0.024802	298.96
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.429	0.903		Elem. #1	4	0.24	59.47
Elem. #2	-0.025	1.000		Elem. #2	4	0.39	1161.09
Elem. #3	0.484	-0.875		Elem. #3	4	5.53	5846.17
Stiffness at Bottom Node [Unit: kip-in./rad]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
1387.26	6155415.29	5685.63			304.629	91.44	0.60

2nd Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L2)							
L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
45.0843	29000	11153.85	0.07844	0.08917	0.003942		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	137.05
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	317.05
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.072	0.997		Elem. #1	4	0.20	49.84
Elem. #2	-0.732	0.681		Elem. #2	4	10.13	14616.70
Elem. #3	0.732	-0.681		Elem. #3	4	10.13	14616.70
Stiffness at Top Node [Unit: kip-in./rad]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15672.23	212363300.41	13631.47			93.332	57.358	1.23
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799	47.05
Elem. #2	45.0843	29000	11153.85	0.01818	0.03799	0.00098	94.10
Elem. #3	50	29000	11153.85	0.6151	2.5199	0.024802	137.05
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	317.05
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.681	0.732		Elem. #1	4	0.29	73.15
Elem. #2	-0.072	0.997		Elem. #2	4	0.24	97.75
Elem. #3	-0.732	0.681		Elem. #3	4	5.53	5846.17
Elem. #4	0.732	-0.681		Elem. #4	4	4.19	4428.92
Stiffness at Bottom Node [Unit: kip-in./rad]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
5544.43	25909647.87	4911.81			269.453	57.358	0.43

3rd Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L2)							
L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
45.0843	29000	11153.85	0.04544	0.0672	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	137.0512
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	317.0512
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.072	0.997		Elem. #1	4	0.20	49.84
Elem. #2	-0.732	0.681		Elem. #2	4	10.13	14616.70
Elem. #3	0.732	-0.681		Elem. #3	4	10.13	14616.70
Stiffness at Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15672.23	212363300.41	13631.47			93.332	43.226	0.93
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799	47.0511
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	137.0511
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	317.0511
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.681	0.732		Elem. #1	4	0.29	73.15
Elem. #2	-0.072	0.997		Elem. #2	4	0.20	49.84
Elem. #3	-0.732	0.681		Elem. #3	4	4.19	4428.92
Elem. #4	0.732	-0.681		Elem. #4	4	4.19	4428.92
Stiffness at Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4784.20	19189610.33	4205.49			221.205	43.226	0.39

4th Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L2)							
L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
45.0843	29000	11153.85	0.04544	0.0672	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	137.0512
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	317.0512
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.072	0.997		Elem. #1	4	0.1976	49.83
Elem. #2	-0.732	0.681		Elem. #2	4	10.1297	14616.70
Elem. #3	0.732	-0.681		Elem. #3	4	10.1297	14616.70
Stiffness at Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15672.23	212363300.41	13631.47			93.332	43.226	0.93
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799	47.0511
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	137.0511
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	317.0511
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.681	0.732		Elem. #1	4	0.2900	73.15
Elem. #2	-0.072	0.997		Elem. #2	4	0.1976	49.84
Elem. #3	-0.732	0.681		Elem. #3	4	4.1915	4428.92
Elem. #4	0.732	-0.681		Elem. #4	4	4.1915	4428.92
Stiffness at Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4784.20	19189610.33	4205.49			221.205	43.226	0.39

5th Compression Web Member, Out-of-Plane Buckling, Individually (32LH06_L2)							
L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
45.0843	29000	11153.85	0.04544	0.0672	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem #1	45.0843	29000	11153.85	0.04544	0.0672	0.003314	94.1023
Elem #2	33	29000	11153.85	1.2186	4.1582	0.02997	137.0512
Elem #3	33	29000	11153.85	1.2186	4.1582	0.02997	317.0512
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem #1	-0.072	0.997		Elem #1	4	0.8199	172.90
Elem #2	-0.732	0.681		Elem #2	4	10.1297	14616.70
Elem #3	0.732	-0.681		Elem #3	4	10.1297	14616.70
Stiffness at Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{Top}
15673.48	212618013.26	13753.91			214.746	43.226	0.40
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem #1	30.718	29000	11153.85	0.01937	0.01937	0.000799	47.0511
Elem #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem #3	66	29000	11153.85	0.6151	2.5199	0.024802	137.0511
Elem #4	66	29000	11153.85	0.6151	2.5199	0.024802	317.0511
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem #1	0.681	0.732		Elem #1	4	0.2900	73.15
Elem #2	-0.072	0.997		Elem #2	4	0.1976	49.84
Elem #3	-0.732	0.681		Elem #3	4	4.1915	4428.92
Elem #4	0.732	-0.681		Elem #4	4	4.1915	4428.92
Stiffness at Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4784.20	19189610.33	4205.49			221.205	43.226	0.39

K-factors by Hand Calculation Method, 32LH06_L2, Out-of-Plane Buckling, Simultaneously				
Web Member of interest	Location of k	k (kip-in/rad.)	G	K
1st Web Member	Top	83.53	0.95	0.69
	Bottom	304.63	0.26	
2nd Web Member	Top	93.33	1.03	0.71
	Bottom	269.45	0.36	
3rd Web Member	Top	93.33	0.69	0.67
	Bottom	221.21	0.29	
4th Web Member	Top	93.33	0.30	0.60
	Bottom	221.21	0.13	
5th Web Member	Top	93.33	0.08	0.53
	Bottom	221.21	0.03	

1st Compression Web Member, Out-of-Plane Buckling, Simultaneously							
L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
35.1083	29000	11153.85	0.1512	0.04808	0.007871		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	33	29000	11153.85	1.2186	4.1582	0.02997	118.9611
Elem. #2	45.0843	29000	11153.85	0.01818	0.03799	0.00098	76.0122
Elem. #3	39	29000	11153.85	1.2186	4.1582	0.02997	298.9611
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.484	0.875		Elem. #1	4	10.1297	14616.70
Elem. #2	0.242	0.970		Elem. #2	4	0.2425	97.75
Elem. #3	0.484	-0.875		Elem. #3	4	8.5713	12367.98
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
6347.215	129997460.6	20754.2			83.532	39.715	0.95
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	37.7835	29000	11153.85	0.01937	0.01937	0.000799	64.57
Elem. #2	66.5176	29000	11153.85	0.1377	0.6658	0.002331	91.46
Elem. #3	50	29000	11153.85	0.6151	2.5199	0.024802	298.96
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.429	0.903		Elem. #1	4	0.2358	59.47
Elem. #2	-0.025	1.000		Elem. #2	4	0.3909	1161.09
Elem. #3	0.484	-0.875		Elem. #3	4	5.5328	5846.17
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
1387.26	6155415.29	5685.63			304.629	39.715	0.26

2nd Compression Web Member, Out-of-Plane Buckling, Simultaneously (32LH06_L2)

L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
45.0843	29000	11153.85	0.07844	0.075	0.003942		

Properties of Top Members

	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	137.0512
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	317.0512
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.072	0.997		Elem. #1	4	0.1976	49.8382
Elem. #2	-0.732	0.681		Elem. #2	4	10.1297	14616.70
Elem. #3	0.732	-0.681		Elem. #3	4	10.1297	14616.70

Stiffness of Top Node [Unit: kip-in./rad.]

α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}
15672.23	212363300.41	13631.47		93.332	48.243	1.03

Properties of Bottom Members

	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799	47.0511
Elem. #2	45.0843	29000	11153.85	0.01818	0.03799	0.00098	94.1023
Elem. #3	50	29000	11153.85	0.6151	2.5199	0.024802	137.0511
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	317.0511
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.681	0.732		Elem. #1	4	0.2900	73.15
Elem. #2	-0.072	0.997		Elem. #2	4	0.2425	97.75
Elem. #3	-0.732	0.681		Elem. #3	4	5.5328	5846.17
Elem. #4	0.732	-0.681		Elem. #4	4	4.1915	4428.92

Stiffness of Bottom Node [Unit: kip-in./rad.]

α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
5544.43	25909647.87	4911.81		269.453	48.243	0.36

3rd Compression Web Member, Out-of-Plane Buckling, Simultaneously (32LH06_L2)							
L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
45.0843	29000	11153.85	0.04544	0.0504	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	137.0512
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	317.0512
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.072	0.997		Elem. #1	4	0.1976	49.84
Elem. #2	-0.732	0.681		Elem. #2	4	10.1297	14616.70
Elem. #3	0.732	-0.681		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ		k_{top}	$(EI/L)_{web}$	G_{top}	
15672.23	212363300.41	13631.47		93.332	32.419	0.69	
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799	47.0511
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	137.0511
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	317.0511
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.681	0.732		Elem. #1	4	0.2900	73.15
Elem. #2	-0.072	0.997		Elem. #2	4	0.1976	49.84
Elem. #3	-0.732	0.681		Elem. #3	4	4.1915	4428.92
Elem. #4	0.732	-0.681		Elem. #4	4	4.1915	4428.92
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ		k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}	
4784.20	19189610.33	4205.49		221.205	32.419	0.29	

4th Compression Web Member, Out-of-Plane Buckling, Simultaneously (32LH06_L2)							
L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
45.0843	29000	11153.85	0.04544	0.0215	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	137.0512
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	317.0512
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.072	0.997		Elem. #1	4	0.1976	49.8382
Elem. #2	-0.732	0.681		Elem. #2	4	10.1297	14616.70
Elem. #3	0.732	-0.681		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15672.23	212363300.41	13631.47			93.332	13.830	0.30
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799	47.0511
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	137.0511
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	317.0511
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.681	0.732		Elem. #1	4	0.2900	73.15
Elem. #2	-0.072	0.997		Elem. #2	4	0.1976	49.84
Elem. #3	-0.732	0.681		Elem. #3	4	4.1915	4428.92
Elem. #4	0.732	-0.681		Elem. #4	4	4.1915	4428.92
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4784.20	19189610.33	4205.49			221.205	13.830	0.13

5th Compression Web Member, Out-of-Plane Buckling, Simultaneously (32LH06_L2)							
L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)		
45.0843	29000	11153.85	0.04544	0.006	0.003314		
Properties of Top Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #2	33	29000	11153.85	1.2186	4.1582	0.02997	137.0512
Elem. #3	33	29000	11153.85	1.2186	4.1582	0.02997	317.0512
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	-0.072	0.997		Elem. #1	4	0.1976	49.8382
Elem. #2	-0.732	0.681		Elem. #2	4	10.1297	14616.70
Elem. #3	0.732	-0.681		Elem. #3	4	10.1297	14616.70
Stiffness of Top Node [Unit: kip-in./rad.]							
α	β	γ			k_{top}	$(EI/L)_{web}$	G_{top}
15672.23	212363300.41	13631.47			93.332	3.859	0.08
Properties of Bottom Members							
	L (in.)	E (ksi)	G (ksi)	I_z (in^4)	I_y (in^4)	J (in^4)	Φ (degree)
Elem. #1	30.718	29000	11153.85	0.01937	0.01937	0.000799	47.0511
Elem. #2	45.0843	29000	11153.85	0.01937	0.01937	0.000799	94.1023
Elem. #3	66	29000	11153.85	0.6151	2.5199	0.024802	137.0511
Elem. #4	66	29000	11153.85	0.6151	2.5199	0.024802	317.0511
	cos(Φ)	sin(Φ)			n	Torsion	Bending
Elem. #1	0.681	0.732		Elem. #1	4	0.2900	73.15
Elem. #2	-0.072	0.997		Elem. #2	4	0.1976	49.84
Elem. #3	-0.732	0.681		Elem. #3	4	4.1915	4428.92
Elem. #4	0.732	-0.681		Elem. #4	4	4.1915	4428.92
Stiffness of Bottom Node [Unit: kip-in./rad.]							
α	β	γ			k_{Bottom}	$(EI/L)_{web}$	G_{Bottom}
4784.20	19189610.33	4205.49			221.205	3.859	0.03

Adjusted Moments of Inertia for In-Plane Buckling (32LH06_L2)					
Buckling Mode	1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})	I_z (in.^{4})
Web	0.03780	0.054908	0.040896	0.018176	0.044544
Web	0.039312	0.058830	0.043168	0.020448	0.004771
Web (Control)	0.040824	0.062752	0.045440	0.022720	0.004998
Top Chord	0.042336	0.066674	0.047712	0.024992	0.005226
Top Chord	0.043848	0.070596	0.049984	0.027264	0.005453

Adjusted Moments of Inertia for Out-of-Plane Buckling (32LH06_L2)					
Buckling Mode	1st Web	2nd Web	3rd Web	4th Web	5th Web
	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})	I_y (in.^{4})
Web	0.046494	0.062419	0.04368	0.020832	0.005981
Web	0.047601	0.066878	0.04704	0.021504	0.006048
Web (Control)	0.048708	0.071336	0.05040	0.022176	0.006115
Top Chord	0.049815	0.075795	0.05376	0.022848	0.006182
Top Chord	0.050922	0.080253	0.05712	0.023520	0.006250

Details for computing in-plane *K*-factors according to Hand Method and requiring individual compression web buckling (32K06_L2)				
Web Member of interest	Location of *k*	*k (kip-in./rad.)*	*G*	*K*
1st Web Member	Top	7954.894	0.03	0.54
	Bottom	1726.635	0.14	
2nd Web Member	Top	8616.965	0.01	0.51
	Bottom	2628.040	0.04	
3rd Web Member	Top	8616.965	0.01	0.51
	Bottom	2285.155	0.03	
4th Web Member	Top	8616.965	0.01	0.51
	Bottom	2285.155	0.03	
5th Web Member	Top	8684.042	0.01	0.51
	Bottom	2285.155	0.03	

Details for computing in-plane *K*-factors according to uniformly distributed loading method and requiring individual compression web buckling (32LH06_L2)				
Web Member of interest	*L* (in.)	I_z (in.^{4})	P_{cr} (kip)	*K*
1st Web Member	35.1083	0.03995	35.73	0.51
2nd Web Member	45.0843	0.06389	34.73	0.51
3rd Web Member	45.0843	0.04364	23.99	0.51
4th Web Member	45.0843	0.02401	13.34	0.50
5th Web Member	45.0843	0.00498	2.795	0.50

Details for computing in-plane *K*-factors according to SEIC method and requiring individual compression web buckling (32LH06_L2)				
Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	35.1083	0.15120	117.10	0.55
2nd Web Member	45.0843	0.07844	41.70	0.52
3rd Web Member	45.0843	0.04544	24.68	0.51
4th Web Member	45.0843	0.04544	24.67	0.51
5th Web Member	45.0843	0.04544	24.68	0.51

Details for computing in-plane *K*-factors according to hand method and requiring simultaneous compression web buckling (32LH06_L2)				
Web Member of interest	Location of k	k *(kip-in./rad.)*	G	K
1st Web Member	Top	7954.894	0.008	0.51
	Bottom	1726.635	0.038	
2nd Web Member	Top	8616.965	0.010	0.51
	Bottom	2628.040	0.031	
3rd Web Member	Top	8616.965	0.007	0.51
	Bottom	2285.155	0.025	
4th Web Member	Top	8616.965	0.004	0.50
	Bottom	2285.155	0.014	
5th Web Member	Top	8684.042	0.001	0.50
	Bottom	2285.155	0.003	

Details for computing in-plane *K*-factors according to uniformly distributed loading method and requiring simultaneous compression web buckling (32LH06_L2)				
Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	35.1083	0.039950	35.48	0.51
2nd Web Member	45.0843	0.063890	34.66	0.51
3rd Web Member	45.0843	0.043642	23.95	0.51
4th Web Member	45.0843	0.024013	13.32	0.50
5th Web Member	45.0843	0.004977	2.79	0.50

Details for computing in-plane *K*-factors according to SEIC and requiring simultaneous compression web buckling (32LH06_L2)				
Web Member of interest	L (in.)	I_z (in.^{4})	P_{cr} (kip)	K
1st Web Member	35.1083	0.039950	35.23	0.51
2nd Web Member	45.0843	0.063890	34.30	0.51
3rd Web Member	45.0843	0.043642	23.73	0.51
4th Web Member	45.0843	0.024013	13.27	0.51
5th Web Member	45.0843	0.004977	2.79	0.50

Details for computing out-of-plane *K*-factors according to hand method and requiring individual compression web buckling (32K06_L2)				
Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	83.53215	2.19	0.78
	Bottom	304.6288	0.60	
2nd Web Member	Top	93.33241	1.23	0.73
	Bottom	269.4534	0.43	
3rd Web Member	Top	93.33241	0.93	0.71
	Bottom	221.2053	0.39	
4th Web Member	Top	93.33241	0.93	0.71
	Bottom	221.2053	0.39	
5th Web Member	Top	214.746	0.40	0.66
	Bottom	221.2053	0.39	

Details for computing out-of-plane *K*-factors according to uniformly distributed loading method and requiring individual compression web buckling (32LH06_L2)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	35.1083	0.04871	22.73	0.71
2nd Web Member	45.0843	0.07134	21.28	0.69
3rd Web Member	45.0843	0.05040	15.07	0.69
4th Web Member	45.0843	0.02218	8.41	0.61
5th Web Member	45.0843	0.00612	1.78	0.70

Details for computing out-of-plane *K*-factors according to SEIC method and requiring individual compression web buckling (32LH06_L2)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	35.1083	0.11070	33.85	0.87
2nd Web Member	45.0843	0.08917	21.56	0.76
3rd Web Member	45.0843	0.06720	17.86	0.73
4th Web Member	45.0843	0.06720	17.60	0.73
5th Web Member	45.0843	0.06720	19.78	0.69

Details for computing out-of-plane *K*-factors according to hand method and requiring simultaneous compression web buckling (32LH06_L2)				
Web Member of interest	Location of k	k (kip-in./rad.)	G	K
1st Web Member	Top	83.53215	0.95	0.69
	Bottom	304.6288	0.26	
2nd Web Member	Top	93.33241	1.03	0.71
	Bottom	269.4534	0.36	
3rd Web Member	Top	93.33241	0.69	0.67
	Bottom	221.2053	0.29	
4th Web Member	Top	93.33241	0.30	0.60
	Bottom	221.2053	0.13	
5th Web Member	Top	93.33241	0.08	0.53
	Bottom	221.2053	0.03	

Details for computing out-of-plane *K*-factors according to uniformly distributed loading method and requiring simultaneous compression web buckling (32LH06_L2)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	35.1083	0.0481	21.37	0.72
2nd Web Member	45.0843	0.0750	20.9	0.71
3rd Web Member	45.0843	0.0504	14.44	0.70
4th Web Member	45.0843	0.0215	8.03	0.61
5th Web Member	45.0843	0.0060	1.681	0.71

Details for computing out-of-plane *K*-factors according to SEIC method and requiring simultaneous compression web buckling (32LH06_L2)				
Web Member of interest	L (in.)	I_y (in.^{4})	P_{cr} (kip)	K
1st Web Member	35.1083	0.0481	18.40	0.78
2nd Web Member	45.0843	0.0750	18.99	0.75
3rd Web Member	45.0843	0.0504	14.35	0.70
4th Web Member	45.0843	0.0215	8.13	0.61
5th Web Member	45.0843	0.0060	2.82	0.55

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